ON DYNAMICS OF THE MINIMAL SU(5) INFLATION*

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We analyze evolution of the Higgs field in the minimal SU(5) grand unified theory and we show that only in a narrow range of parameters symmetry is broken directly to $SU(3)\times SU(2)\times U(1)$ but there is no inflation. Inflation is possible but then the SU(5) group is broken to $SU(4)\times U(1)$. During the inflationary era temperature never becomes very low and after releasing the latent heat the universe is reheated to temperature comparable to the critical temperature.

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The original idea of Guth [1] that at very early stage of evolution the universe went through an epoch of exponential expansion — called the inflationary model, can solve several problems of the standard Big Bang scenario. Guth suggested that the GUT's phase transition is first order and that the universe could supercool in the symmetric phase much below the transition temperature $T_c \approx 10^{14}$ GeV. The false vacuum energy density plays a role of the cosmological constant and the universe expands exponentially. Transition to the true vacuum state proceeds via formation of bubbles of the asymmetric phase. Universe is reheated by collisions of bubble walls. This process creates however excessive inhomogeneity.

The inflationary model was substantially improved independently by Linde [2], and Albrecht and Steinhardt [3]. In the new inflationary model a scalar field trapped in the

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false vacuum state slowly rolls over a flat part of the potential. During this slow roll-over a bubble of false vacuum expands exponentially and if the process of roll-over lasts sufficiently long the bubble becomes larger than the presently observed part of the universe.

In first realizations of the new inflationary model based on GUT's a multicomponent Higgs field played the role of the scalar field. It was pointed out by Breit et al. [4], and Sato and Kodama [5] among others that this leads to difficulties because the transition from the false vacuum state to the true vacuum state might go through intermediate steps. In their analysis they explored only limited spectrum of initial conditions.

To determine the range of initial conditions and parameters which allow inflation we consider the minimal SU(5) grand unified theory and investigate evolution of the Higgs field belonging to the adjoint representation of SU(5). Initially the Higgs field is supercooled in the symmetric phase, so $\langle\phi\rangle_{in} \approx 0$ and we also assume that $\langle\dot{\phi}\rangle_{in} \approx 0$. Without loosing generality we take ϕ in the diagonal form $\phi = \mathrm{Diag}\,(\phi_1, \phi_2, \phi_3, \phi_4, \phi_5); \sum_{i=1}^{5} \phi_i = 0$. In this parametrization the Coleman-Weinberg potential with the one-loop quantum corrections assumes the form

$$V = \frac{3g^4}{256\pi^2}\left\{b\left[\sum_{i=1}^{5}\phi_i^4 - \tfrac{7}{30}\left(\sum_{i=1}^{5}\phi_i^2\right)^2\right] + \sum_{i,j=1}^{5}(\phi_i-\phi_j)^4\left[\ln\left(\frac{\phi_i-\phi_j}{\mu}\right) - \tfrac{1}{2}\right]\right\} + V_0. \quad (1)$$

V_0 is determined from the requirement that $V = 0$ when $\phi = 0.4\mu\,\mathrm{Diag}\,(1, 1, 1, -\tfrac{3}{2}, -\tfrac{3}{2})$ corresponding to the $SU(3)\times SU(2)\times U(1)$ minimum. The coupling constant g was taken to be $g = \sqrt{4\pi\alpha}$, where α is the fine structure constant. At energies corresponding to GUT's unification scale $\alpha = 1/45$. We assume that the coupling constant g is constant. $SU(3)\times SU(2)\times U(1)$ is the global minimum of the potential when $-15 \ln 1.5 < b < 15$. We have performed our calculations taking $b = 1$, and $b = -6$.

The classical equations of motion for ϕ's are

$$\ddot{\phi}_i + 3\frac{\dot{R}}{R}\dot{\phi}_i + \Gamma\dot{\phi}_i + \frac{\partial V}{\partial \phi_i} + \lambda = 0, \quad (2)$$

where λ is a Lagrange multiplier introduced to enforce the condition $\sum_{i=1}^{5}\phi_i = 0$, and Γ is a friction coefficient. The energy density of radiation and relativistic particles ϱ_r obeys the equation

$$\dot{\varrho}_r = -4\frac{\dot{R}}{R}\varrho_r + q, \quad (3)$$

where q describes the increase of heat energy due to friction. The expansion rate is related to the total energy density by the Friedman equation

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3m_{Pl}^2}\left(\varrho_r + \tfrac{1}{2}\sum_{i=1}^{5}\dot{\phi}_i^2 + V(\phi)\right). \quad (4)$$

In our calculations we used two forms of the friction coefficient $\Gamma_1 = ag^2|\phi| = ag^2(\sum_{i=1}^{5}\phi_i^2)^{1/2}$, and $\Gamma_{2i} = ag^2|\phi_i|$, corresponding q's have the form $q_1 = \Gamma_1(\sum_{i=1}^{5}\dot{\phi}_i^2)$, $q_2 = ag^2\sum_{i=1}^{5}|\phi_i|\dot{\phi}_i^2$.

We have numerically solved the set of Eqs (2)–(4) with fairly wide range of initial conditions. The initial amplitude of the field $\langle\phi\rangle$ was varied in the range $\langle\phi_i\rangle_{\text{in}} = (10^{-7} - 10^{-5})\mu$, and $\langle\dot{\phi}\rangle_{\text{in}}/\langle\phi\rangle_{\text{in}}$ was taken to be 0, 10^{-15}, 10^{-10}, 10^{-9}, ..., and up to 1. We have restricted ourselves to study the case when initially $\dot{\phi}_i$ is proportional to ϕ_i because only then it is possible to simultaneously diagonalize ϕ_i and $\dot{\phi}_i$ and the equations of motion preserve their diagonal form. Of course this is not the most general case. When initially $\dot{\phi}_i$ is not parallel to ϕ_i then one has to consider the evolution of all 24 components of ϕ.

The initial direction of the field was taken to be

a) random,

b) in the direction of SU(3)×SU(2)×U(1) minimum with small (1%) deviations,

c) continuously changing from the direction of SU(3)×SU(2)×U(1) minimum to the nearest SU(4)×U(1) minimum.

With special attention we have studied evolution of the field ϕ with initial orientation restricted to the plane $\phi_1 = \phi_2 = \phi_3$.

The amplitude of the friction coefficient was varied by changing the value of a in the range 1, 10^{-1}, ..., 10^{-10}.

The initial value of the energy density of radiation and relativistic particles have been also varied in a wide range. However we disregarded temperature corrections to the potential V. The initial effective temperature $\left(\varrho_r = \frac{\pi^2}{30} T_{\text{eff}}^4\right)$ was changed from 0 to the critical temperature T_c, and in several most interesting cases we assumed that $T_{\text{in}} = 0.1\langle\phi\rangle_{\text{in}}$. At non-zero temperature ($T < T_c$) the Coleman-Weinberg potential acquires a small bump, it appears at $\langle\phi\rangle \approx T$. The initial value of the field ϕ was taken to be outside this bump.

The new inflationary scenario can solve some of the conundrums of the standard Big Bang model if the symmetry is broken to SU(3)×SU(2)×U(1) and the transition to the asymmetric true vacuum is slow enough so that the scale factor can grow exponentially by at least twenty orders of magnitude ($\int_{t_{\text{in}}}^{t_f} H(t)dt \approx 50$). We have found that it is very difficult to simultaneously satisfy these conditions.

Transition to the SU(3)×SU(2)×U(1) global minimum is possible when the initial direction of ϕ is very close to the SU(3)×SU(2)×U(1) minimum. For example, if we allow 1% deviation from the direction of the SU(3)×SU(2)×U(1) minimum and take $\langle\phi\rangle_{\text{in}} = 10^{-6}\mu$ then the field settles at the SU(3)×SU(2)×U(1) minimum only when $\langle\dot{\phi}\rangle_{\text{in}}/\langle\phi\rangle_{\text{in}}$ is greater than 10^{-2} and the amplitude of the friction coefficient a is larger or equal to 0.1. In this case however the transition is very fast and the scale factor does not grow sufficiently ($\int_{t_{\text{in}}}^{t_f} H(t) \approx 1$). For smaller values of $\langle\dot{\phi}\rangle_{\text{in}}/\langle\phi\rangle_{\text{in}}$ the field diverts towards the SU(4)×U(1) minimum and its further evolution is very complicated and it strongly depends on the value of a (see Fig. 1).

We noticed that when $\langle\dot{\phi}\rangle_{in}$ is larger than $\sim 10^{-8}\langle\phi\rangle_{in}$ it is initially dumped by the effects of dynamical friction, in Eq. (4) $\frac{1}{2}\sum_{i=1}^{5}\dot{\phi}_i^2$ is greater than ϱ_r and V (see Fig. 2). When $\langle\dot{\phi}\rangle_{in}$ is very large this effect is not very important because the field ϕ very quickly approaches value close to μ (no inflation). When $\langle\dot{\phi}\rangle_{in}$ is not very large $\dot{\phi}$ is quickly dumped and approaches a constant "equilibrium" value which could have been attained by $\dot{\phi}$ if initially $\dot{\phi}$ was zero or smaller than $10^{-8}\langle\phi\rangle_{in}$ (inflation is possible).

When the friction parameter a is smaller than 10^{-1} the field ϕ moves towards the SU(3)×SU(2)×U(1) minimum but it does not settle there and it moves away to other local minimum. If a is only slightly smaller than 10^{-1} then the field ϕ drifts around and before finally settling down visits several or even all minima.

For every other choice of the initial direction (cases a and c) the field ϕ evolves towards SU(4)×U(1) minimum and its further evolution strongly depends on the value of a. When friction is very large ($a \geqslant 10^{-1}$) ϕ is trapped in the first SU(4)×U(1) minimum and it can tunnel to the SU(3)×SU(2)×U(1) minimum but this creates problems which appeared in the original Guth model. When friction is small ($a < 10^{-1}$) evolution of the field ϕ is very similar to the evolution of the field which was initially aligned almost directly into the SU(3)×SU(2)×U(1) minimum. In this case changes of the initial value of $\langle\dot{\phi}\rangle$ lead to effects discussed above.

The field ϕ evolves directly to the SU(3)×SU(2)×U(1) minimum only if initially it almost exactly points towards the SU(3)×SU(2)×U(1) minimum. This strong dependence on the initial direction is connected with the topography of the SU(5) group space. The minima corresponding to SU(4)×U(1) subgroup are closer to the origin ($\langle\phi\rangle = \frac{1}{5}\mu e^{-b/60}$) than the SU(3)×SU(2)×U(1) minima ($\langle\phi\rangle = \frac{2}{5}\mu$) and the path leading directly to the SU(3)×SU(2)×U(1) minimum traverses a ridge separating two SU(4)×U(1) minima.

We can also vary the initial amplitude of the field ϕ. It turns out that the initial value of the field ϕ influences mainly the duration of the inflationary era but it does not change the qualitative features of evolution of ϕ. The inflationary era lasts sufficiently long ($\int_{t_{in}}^{t_f} H(t)dt \approx 50$) when the initial value of ϕ is small, for $\langle\phi\rangle_{in} = 10^{-5}\,\mu$, $\int H(t)dt \approx 1$, and for $\langle\phi\rangle_{in} = 10^{-6}\,\mu$, $\int H(t)dt \approx 100$.

We have also followed changes in the effective temperature. Evolution of the effective temperature is governed by Eq. (3). If the initial conditions are specified in such a way that $(\dot{\varrho}_r)_{in} < 0$ then at first the temperature drops until the source term q counterbalances the effects of expansion (see Fig. 3). This can happen in two different situations:

a) if the inflationary era last so long that ϱ_r becomes very small and there is a moment when $\varrho_r = q/4H$. Even though the universe expands further the effective temperature does not decrease. When inflation lasts sufficiently long the minimal temperature drops below the Hawking temperature. When the initial conditions are such that the inflationary period is short then the minimal temperature exceeds the Hawking temperature, if initially it was higher;

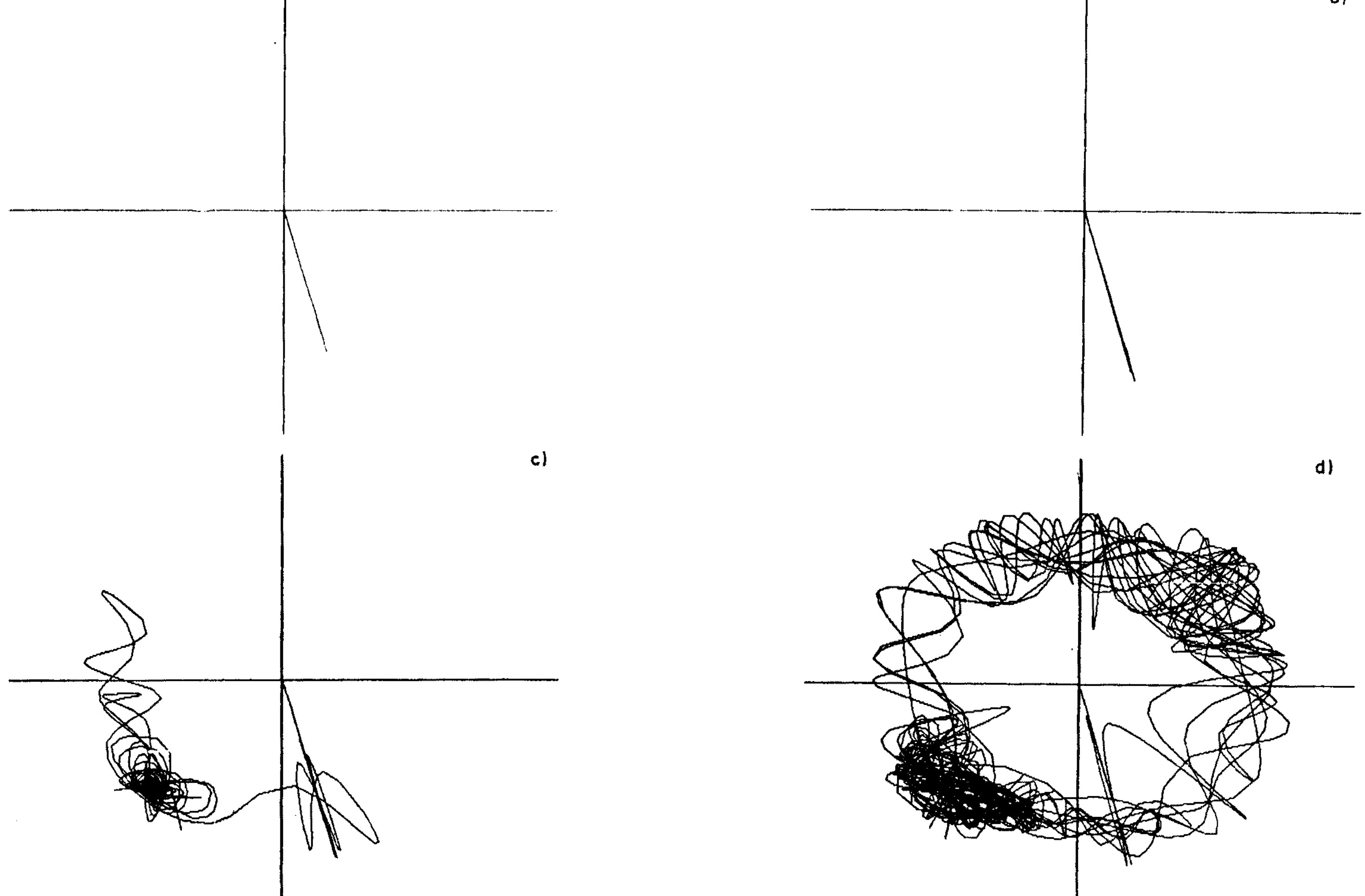

Fig. 1. Evolution of the Higgs field ϕ in the plane $\phi_1 = \phi_2 = \phi_3$ for different values of friction parameter. Initially the field ϕ was pointing towards SU(3)×SU(2)×U(1) minimum, and $\langle\phi\rangle_{in} = 10^{-6}\,\mu$, $\langle\dot{\phi}\rangle_{in} = 0$, $T_{in} = 0.1\,\langle\phi\rangle_{in}$, $b = 1$. The friction parameter was $a = 1$(a), $a = 0.1$ (b), $a = 0.01$ (c), and $a = 0.001$ (d)

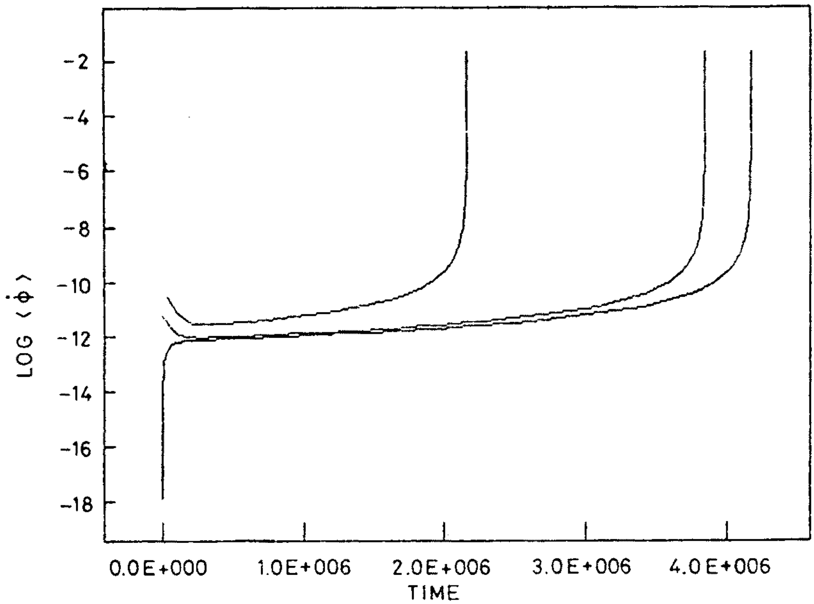

Fig. 2. Evolution of $\dot{\phi}$ for different initial values, $\langle\dot{\phi}\rangle_{in} = 0$ (bottom curve), $\langle\dot{\phi}\rangle_{in} = 10^{-6}\,\langle\phi\rangle_{in}$ (central curve), $\langle\dot{\phi}\rangle_{in} = 10^{-5}\,\langle\phi\rangle_{in}$ (top curve). Other parameters were taken to be $a = 0.1$, $\langle\phi\rangle = 10^{-6}\,\mu$, $T_{in} = 0.1\,\langle\phi\rangle_{in}$

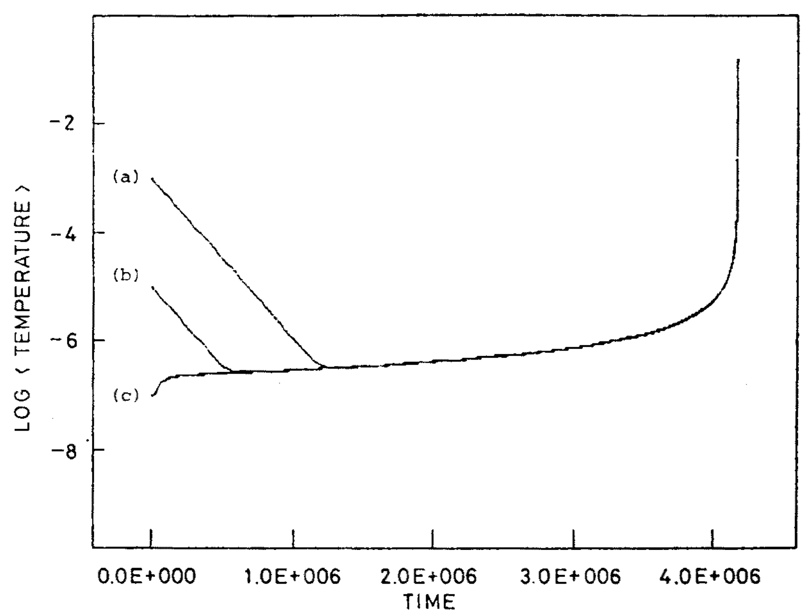

Fig. 3. Evolution of the effective temperature for different initial values of T_{in}, $T_{in} = 10^{-3}\,\mu$ (*a*), $T_{in} = 10^{-5}\,\mu$ (*b*), $T_{in} = 10^{-7}\,\mu$ (c). Other parameters were taken to be $a = 0.1$, $\langle\phi\rangle_{in} = 10^{-6}\,\mu$, $\langle\dot{\phi}\rangle_{in} = 0$

b) when the field ϕ oscillates around some minimal point and the effects of friction are large.

The effective temperature to which the universe reheats after inflation depends on the amplitude of the friction coefficient and it does not depend on the initial temperature. When friction is large ($a \sim 1$) the universe is reheated to $T_r = 0.17\ \mu$ which is of the order of the critical temperature T_c. The reheating temperature only weakly depends on the amplitude of the friction coefficient and even for very small friction reheating is very efficient, for example when $a = 10^{-10}$, $T_r \approx 10^{-3}\ \mu$.

In realistic models the reheating temperature cannot be too small because to create matter we relay on the process of baryogenesis. This process is effective only when the reheating temperature is larger than $0.1m$, when m is the typical mass of gauge bosons ($m \approx 0.4\ \mu$). This condition restricts also the amplitude of the friction coefficient which should not be smaller than 10^{-6}.

Conclusions

We have analyzed evolution of the Higgs field in the minimal SU(5) model and we confirm results of previous investigations by Breit et al., and Sato and Kodama. It turns out that it is not possible to simultaneously have inflation and symmetry breaking directly to $SU(3) \times SU(2) \times U(1)$. We have found range of parameters and initial conditions which guarantee direct symmetry breaking to $SU(3) \times SU(2) \times U(1)$ but this transition does not last long enough and the universe does not expand sufficiently to solve basic problems of the standard Big Bang model. It is possible to choose free parameters and specify initial conditions in such a way that the inflationary era lasts long enough but then the transition is either directly to $SU(4) \times U(1)$ minimum or the field ϕ visits several minima. It could finally settle into a $SU(3) \times SU(2) \times U(1)$ minimum or tunnel into it but this behaviour is not natural and creates excessive inhomogeneity. Inflation is possible only when the initial value of the field ϕ is smaller than $10^{-5}\ \mu$ and if $\langle\dot{\phi}\rangle_{in}$ is not too large. When inflation occurs it is always possible to adjust the friction coefficient so that the universe reheats to sufficiently high temperature to allow baryogenesis. When initial temperature is high (larger then the Hawking temperature) then the minimal temperature reached during the exponential expansion can be larger than the Hawking temperature.

After we finished all our calculations we have learned that the similar problem was considered by Sakagami and Hosoya [6]. They assumed that there is an additional temperature dependent friction generated by the reheating. With this additional friction the $SU(3) \times SU(2) \times U(1)$ minimum is attained for wider range of parameters. Therefore the new inflation is more probable. This modification of the friction term does not significantly alter our other conclusions.

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