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TRANSPORT IN STRANGE STARS

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We calculate thermal conductivity, electric conductivity and shear viscosity of hypothetical strange star matter. Transport coefficients are calculated using variational solutions of kinetic equation for a normal degenerate quark plasma. All considered transport processes are dominated by quarks. Color screened QCD scattering largely dominates over the Coulomb scattering of quarks. At the same density and temperature, transport coefficients of strange star matter are about an order of magnitude larger than those of normal neutron star matter. The influence of strong magnetic field presumably accompanying strange stars on transport processes is studied.

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1. *Introduction*

It is currently believed that at sufficiently high density matter undergoes some kind of transition from a state where quarks are localized (confined) inside baryons to a deconfined state of quark matter. Quark matter in equilibrium with respect to weak interactions contains, apart from the massless u and d quarks, also the massive s ones. The strange quark matter, with the strangeness per unit baryon number close to -1, is energetically preferred over the non-strange one, composed of the u and d quarks only. This is due to the fact that the exclusion principle effect, favoring the appearance of the s quark, prevails over the non-zero mass effects. Recently Witten [1] has pointed out an intriguing possibility that such a *strange matter* might be an absolute ground state of matter at zero pressure. This would not be in conflict with reality: ordinary nuclei could not convert to the strange state because of the difficulty in making transition to the strange configuration via a very high order weak interaction.

The existence of the self-bound strange matter might have important consequences for physics of neutron stars. The appearance of the nucleus of the strange matter during the implosion of a massive star or during the evolution of a massive normal neutron star could lead to the formation of the *strange stars*, composed entirely, or predominantly, of

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strange matter. Possible scenarios of conversion of neutron stars into strange stars have been considered by Olinto [2]. Some of the static properties of strange stars have been recently studied in [3, 4] (for a review, see [5]). It should be stressed, however, that the very *possibility* of existence of strange stars is a matter of a lively debate. Arguments against the existence of strange stars have been presented by Bethe et al. [6].

In order to study the astronomically relevant properties of strange stars one should know, apart from the equation of state, also the transport properties of strange matter. Knowledge of thermal conductivity of strange matter is necessary for studying the cooling of strange stars. Electric conductivity is needed if one is studying the electrodynamics of these objects. Hydrodynamics of strange stars involves viscosity of strange matter.

In the present paper we calculate some of the transport coefficients of strange matter starting from the relativistic Boltzmann equation for degenerate quark plasma and assuming that the system is *normal.* The formulation of the problem is given in Sections 2, 3. In Section 4 we calculate the transition rates relevant for the considered transport processes. Our numerical results in absence of a strong magnetic field are presented in Section 5. In Section 6 the influence of strong magnetic field on the transport coefficients is studied. Finally, Section 7 contains a discussion of our results and conclusions.

2. *Formulation of the problem*

2.1. Introductory remarks

For reasonably small values of the quark-gluon coupling constant and $m_s c^2 \lesssim 200$ MeV strange matter is a nearly flavor symmetric mixture of the u, d and s quarks. Thus, the baryon number density of strange matter, n_B, is to a very good approximation equal to the number density of quarks of a given flavor, $n_B = n_f$ (f = u, d, s). The strangeness per unit baryon number is close to -1. The electron fraction $Y_e = n_e/n_B$ is small, $Y_e \lesssim 10^{-4}$. Detailed models of strange matter have been studied by Farhi and Jaffe [7], see also Haensel et al. [3].

In the simplest approximation strange matter is an ideal gas of massless quarks. For $T < 10^{12}$ K strange quark matter is degenerate, with

$$T/T_F = 3.68 \cdot 10^{-3} (n_B/n_{B0})^{-1/3} T_{10}, \tag{2.1}$$

where $T_{10} = T/10^{10}$ K, normal nuclear matter density $n_{B0} = 0.17\ \mathrm{fm}^{-3}$ and the Fermi temperature $T_F = \mu/k_B$, μ being the quark chemical potential. Finite strange quark mass and quark-quark interaction imply only minor corrections to Eq. (2.1). In what follows we shall thus consider a gas of massless quarks. Moreover, we shall neglect weak and electromagnetic interactions between quarks. In this approximation, strange matter will be completely symmetric with respect to all internal degrees of freedom (flavor, color, spin). Corrections resulting from the electromagnetic interactions between quarks will be estimated in Sections 3, 4.

Quark matter is a relativistic system, in which pressure is comparable to energy density. Indeed, for noninteracting massless quarks $P = 1/3\,\varepsilon$. In view of this, we have to use relativistic description of kinetics of quark plasma, as well as the formalism of relativistic hydrodynamics (for a detailed presentation of relativistic kinetics and hydrodynamics, see the monograph of de Groot et al. [8]). We assume that strange matter is a *normal* system.

We shall use Greek letters μ, ν for the relativistic four-indices, with the summation convention over repeated indices. Full sets of internal degrees of freedom of quarks will be denoted by $\alpha = (\mathrm{f, c, s})$, where f, c, s stand for flavor, color and spin, respectively. Infinitesimal element of momentum space will be denoted by $d\Gamma = ((2\pi\hbar)^3 p^0)^{-1} d^3p$. We shall use the metric tensor $g_{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -1)$.

2.2. Boltzmann equation and hydrodynamics

Assuming, that the screened quark-quark interaction is relatively weak, we describe the nonequilibrium behavior of quark plasma using relativistic Boltzmann equation (cf., eg., [8, 9]),

$$\frac{\partial}{\partial x^\mu} f_{\alpha_1}(x, p_1) p_1^\mu + F^{\mu\nu} q_{\alpha_1} p_{1\nu} \frac{\partial}{\partial p_{1\nu}} f_{\alpha_1}(x, p_1)$$

$$= -\tfrac{1}{2} \sum_{\alpha_2\alpha_3\alpha_4} \int d\Gamma_2 d\Gamma_3 d\Gamma_4 |M_{\alpha_1\alpha_2,\alpha_3\alpha_4}(p_1 p_2, p_3 p_4)|^2$$

$$\times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \{ f_{\alpha_1}(x, p_1) f_{\alpha_2}(x, p_2) [1 - f_{\alpha_3}(x, p_3)] [1 - f_{\alpha_4}(x, p_4)]$$

$$- f_{\alpha_3}(x, p_3) f_{\alpha_4}(x, p_4) [1 - f_{\alpha_1}(x, p_1)] [1 - f_{\alpha_2}(x, p_2)] \}, \tag{2.2}$$

where f_α is the distribution function for quarks, $F^{\mu\nu}$ is the external electromagnetic field tensor, q_α is the electric charge of a quark with internal quantum numbers α. The quantity $M_{\alpha_1\alpha_2,\alpha_3\alpha_4}$ is the transition amplitude for the two-body process $p_1\alpha_1 + p_2\alpha_2 \to p_3\alpha_3 + p_4\alpha_4$, calculated to lowest order in the screened quark-quark interaction. For the sake of simplicity we put $\hbar = c = 1$. In what follows we shall usually drop the space-time and momentum arguments.

The entropy four-flow, relevant for the calculation of the transport coefficients, is given by

$$S^\mu = -k_\mathrm{B} \sum_\alpha \int d\Gamma [f_\alpha \ln f_\alpha - (1 - f_\alpha) \ln (1 - f_\alpha)] p^\mu. \tag{2.3}$$

The energy-momentum tensor, $T^{\mu\nu}$, the electric current, j^μ, and the quark number current, $\mathcal{N}^\mu$, are given by the formulae

$$T^{\mu\nu} = \sum_\alpha \int d\Gamma f_\alpha p^\mu p^\nu, \tag{2.4}$$

$$j^\mu = \sum_\alpha \int d\Gamma f_\alpha q_\alpha p^\mu, \tag{2.5}$$

$$\mathcal{N}^\mu = \sum_\alpha \int d\Gamma f_\alpha p^\mu. \tag{2.6}$$

In the hydrodynamic regime the distribution function for quarks deviates slightly from the local equilibrium one, f_α^0,

$$f_\alpha = f_\alpha^0 + f_\alpha^0(1-f_\alpha^0)\Phi_\alpha, \quad f_\alpha^0 = \left[1+\exp\left(\frac{p^\mu V_\mu - \mu}{k_B T}\right)\right]^{-1}. \tag{2.7}$$

Here, μ is the local chemical potential of quarks assumed to be flavor, color and spin independent, T is the local temperature and V_μ is the hydrodynamic velocity. The energy-momentum tensor can be split into equilibrium and dissipative components,

$$T^{\mu\nu} = T^{\mu\nu}_{\text{eq}} + \delta T^{\mu\nu}, \quad T^{\mu\nu}_{\text{eq}} = (P+\varepsilon)V^\mu V^\nu - g^{\mu\nu}P, \tag{2.8}$$

where P and ε are pressure and energy density, respectively. We adopt the Eckart definition of V^μ. Thus, μ, T, V^μ are implicitly defined by the formulae

$$\delta T^{\mu\nu}V_\mu V_\nu = 0, \quad \mathcal{N}^\mu = nV^\mu, \tag{2.9}$$

where n is the particle (quark) number density in a comoving reference frame and P, ε, n are assumed to be the same functions of T and μ as in equilibrium. The quantities $\delta T^{\mu\nu}$ and j^μ are functionals of Φ,

$$\delta T^{\mu\nu} = \sum_\alpha \int d\Gamma f_\alpha^0(1-f_\alpha^0)p^\mu p^\nu \Phi_\alpha, \quad j^\mu = \sum_\alpha \int d\Gamma f_\alpha^0(1-f_\alpha^0)q_\alpha p^\mu \Phi_\alpha. \tag{2.10}$$

2.3. Entropy production and transport coefficients

Our calculation of the transport coefficients will be based on the Chapman-Enskog method, which will be applied to the relativistic Boltzmann equation, Eq. (2.2). In this approach we obtain a relation between (small) gradients of hydrodynamic quantities, and the unknown function $\Phi_\alpha(x,p)$, which describes the deviation of the quark distribution function from the local equilibrium one. After tedious calculations we get following relation, linear in unknown functions Φ_α,

$$\frac{f_{\alpha_1}^0(1-f_{\alpha_1}^0)}{k_B T}\left\{\tfrac{1}{2}A_{\mu\nu}p_1^\mu p_1^\nu + \left[\frac{1}{\varepsilon+P}P_{,\nu} - \frac{1}{T}T_{,\nu}\right]\Delta^\nu_{\nu'}p_1^{\nu'}p_1^{\mu'}V_{\mu'} - \left[\Delta^\mu_\nu T\left(\frac{\mu}{k_B T}\right)_{,\mu} - F^\mu_\nu q_{\alpha_1}V_\mu\right]p_1^\nu\right\}$$

$$= \tfrac{1}{2}\sum_{\alpha_2\alpha_3\alpha_4}\int d\Gamma_2 d\Gamma_3 d\Gamma_4 |M|^2(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)$$

$$\times f_{\alpha_1}^0 f_{\alpha_2}^0(1-f_{\alpha_3}^0)(1-f_{\alpha_4}^0)(\Phi_{\alpha_1}+\Phi_{\alpha_2}-\Phi_{\alpha_3}-\Phi_{\alpha_4}). \tag{2.11}$$

Here,

$$\Delta^{\mu\nu} = g^{\mu\nu} - V^\mu V^\nu, \quad A_{\mu\nu} = \Delta^{\mu'}_\mu \Delta^{\nu'}_\nu (V_{\mu',\nu'} + V_{\nu',\mu'}) - \tfrac{2}{3}\Delta_{\mu\nu}V^{\mu'}_{,\mu'}. \tag{2.12}$$

We use the notation $F_{,\mu} = \partial F/\partial x^\mu$. In our approximation, the dissipative part of the energy-momentum tensor and the electric current take the form [8]

$$\delta T^{\mu\nu} = \eta A^{\mu\nu} + \kappa V^{\{\nu}\Delta^{\mu\}\mu'}\left(T_{,\mu'} - \frac{T}{\varepsilon+P} P_{,\mu'}\right), \tag{2.13}$$

$$j^\mu = \sigma F^{\mu\nu} V_\nu, \tag{2.14}$$

where η, σ and κ are the shear viscosity, the electric conductivity and thermal conductivity, respectively. Script brackets denote symmetrization in the corresponding indices. Notice, that in our approximation the bulk viscosity coefficient vanishes (ultrarelativistic gas, [8, 10]). A very fortunate decoupling of heat conduction from electric conduction results from the assumed flavor symmetry (only electric charges of quarks are not flavor symmetric). The entropy production in a unit volume, $\dot{S} = S^\mu{}_{,\mu}$, can be calculated in three ways. Microscopically, the entropy production results from the two-body scattering and, in lowest order perturbation theory, can be expressed as a functional of Φ,

$$\dot{S}_S(\Phi) = \tfrac{1}{8} k_B \sum_{\substack{\alpha_1\alpha_2 \\ \alpha_3\alpha_4}} \int d\Gamma_1 d\Gamma_2 d\Gamma_3 d\Gamma_4 |M|^2 (2\pi)^4 \delta^4(p_1+p_2-p_3-p_4)$$
$$\times f^0_{\alpha_1} f^0_{\alpha_2}(1-f^0_{\alpha_3})(1-f^0_{\alpha_4})(\Phi_{\alpha_1}+\Phi_{\alpha_2}-\Phi_{\alpha_3}-\Phi_{\alpha_4})^2. \tag{2.15}$$

On the other hand, entropy production can be expressed in terms of the dissipative components of the energy-momentum tensor, calculated as a functional of Φ,

$$\dot{S}_D(\Phi) = \frac{V_\mu}{T} F^{\nu\mu} j_\nu(\Phi) + \frac{1}{2T} A_{\mu\nu}\delta T^{\mu\nu}(\Phi)$$
$$- \frac{1}{T^2}\left(T_{,\mu} - T\frac{P_{,\mu}}{\varepsilon+P}\right)\Delta^{\mu\nu}\delta T^{\nu\nu'}(\Phi) V_{\nu'}. \tag{2.16}$$

Finally, the entropy production can be calculated from the macroscopic expression for the dissipative part of the energy-momentum tensor, involving transport coefficients κ, σ and η,

$$\dot{S}_M = \frac{\eta}{2T} A_{\mu\nu}A^{\mu\nu} + \frac{\sigma}{T} V_\mu V_\nu F^{\mu'\mu} F^\nu{}_{\mu'}$$
$$- \frac{\kappa}{T^2}\left(T_{,\mu} - T\frac{P_{,\mu}}{\varepsilon+P}\right)\Delta^{\mu\nu}\left(T_{,\nu} - T\frac{P_{,\nu}}{\varepsilon+P}\right). \tag{2.17}$$

In the case when Φ *is the exact* solution of the Boltzmann equation, the values of $\dot{S}_S$, $\dot{S}_D$ and $\dot{S}_M$ are strictly equal.

Our model of the quark-quark interaction, derived in Section 3, will be a very simplified one. In view of this, it does not seem reasonable to put a great effort in an exact solution of Boltzmann equation, Eq. (2.2). Much more practical seems to us to obtain an estimate of transport coefficients using the Ritz variational principle for the Boltzmann equation.

This variational principle can be written in the form [8, 11]

$$\frac{\dot{S}_S(\tilde{\Phi})}{|\dot{S}_D(\tilde{\Phi})|^2} \geqslant \frac{\dot{S}_S(\Phi)}{|\dot{S}_D(\Phi)|^2} = \frac{1}{\dot{S}_M}, \tag{2.18}$$

where $\tilde{\Phi}$ is a trial function, and Φ is a solution to Boltzmann equation, Eq. (2.2). Notice that $\tilde{\Phi}$ is determined up to an irrelevant normalization constant.

Introducing appropriate gradients of hydrodynamic quantities T, μ and V^ν and an external electromagnetic field, $F^{\mu\nu}$, corresponding to a transport process under consideration, and guessing reasonable trial functions, $\tilde{\Phi}_\alpha$, consistent with symmetry requirements, we can obtain approximate values (actually: lower bounds) of transport coefficients.

2.4. Variational expressions for κ, σ and η

We shall fix the local reference frame by requiring that the local three-velocity vanishes. In order to estimate the heat conductivity of strange matter let us introduce a small temperature gradient along the x axis. The natural choice of $\tilde{\Phi}_\alpha$ is then

$$\tilde{\Phi}_\alpha(p) = -p^1(p^0-\mu). \tag{2.19}$$

Using variational principle, Eq. (2.18), we get

$$\frac{1}{\kappa} = T^2 \frac{\dot{S}_S(\tilde{\Phi})}{[\delta T^{01}(\tilde{\Phi})]^2}. \tag{2.20}$$

Inserting expression (2.19) into the above expression we obtain the formula

$$\frac{1}{\kappa} = \frac{k_F T}{90\pi}\left\langle W_\kappa \sin^2\frac{\theta}{2}\right\rangle. \tag{2.21}$$

Here, k_F is the Fermi wavenumber for quarks (in our approximation $k_F = \mu/\hbar c$) and W_κ is the transition rate, relevant for the heat conduction,

$$W_\kappa = \overline{W} = \sum_{\substack{\alpha_1\alpha_2\\ \alpha_3\alpha_4}} |M_{\alpha_1\alpha_2,\alpha_3\alpha_4}|^2. \tag{2.22}$$

The brackets $\langle\ldots\rangle$ denote an average over the Fermi surface, which is characteristic of degenerate systems,

$$\langle F(\theta,\varphi)\rangle = \int_0^\pi \frac{\sin\theta d\theta}{4\pi}\int_0^{2\pi} d\varphi \frac{F(\theta,\varphi)}{2\cos\frac{\theta}{2}}. \tag{2.23}$$

Here, θ is the angle between $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ and φ is the angle between the planes determined by the initial and final momenta of colliding particles.

In order to calculate the electric conductivity we consider a weak electric field directed along the x axis. The variational principle gives us the formula

$$\frac{1}{\sigma} = T\frac{\dot{S}_S(\tilde{\Phi})}{|j^1|^2}. \tag{2.24}$$

A natural choice for $\tilde{\Phi}_\alpha$ seems to be

$$\tilde{\Phi}_\alpha = \gamma_f p^1. \tag{2.25}$$

We have $\gamma_s = \gamma_d$. The condition that the total quark number flow vanish leads to $\gamma_u = -2\gamma_d$. The variational principle yields then

$$\frac{1}{\sigma} = \frac{k_F(k_B T)^2 \pi}{32 e^2} \left\langle W_\sigma \sin^2 \frac{\theta}{2} \sin^2 \frac{\varphi}{2} \right\rangle . \tag{2.26}$$

where the transition rate, W_σ, is given by

$$W_\sigma = \sum_{\substack{\alpha_1\alpha_2 \\ \alpha_3\alpha_4}} (1-\delta_{f_1 f_2})(\delta_{f_1 u} + \delta_{f_2 u}) \, |M_{\alpha_1\alpha_2,\alpha_3\alpha_4}|^2 . \tag{2.27}$$

For the calculation of the shear viscosity it is sufficient to consider the case of a slow flow of strange matter in the direction of the x axis, the local value of the hydrodynamic velocity depending on the y coordinate only. The variational expression for η is then given by

$$\frac{1}{\eta} = T \frac{\dot{S}_S(\tilde{\Phi})}{(\delta T^{12})^2} . \tag{2.28}$$

The appropriate Ansatz for the spin, color and flavor independent $\tilde{\Phi}$ reads

$$\tilde{\Phi}_\alpha = -p_1 p_2 . \tag{2.29}$$

Final formula for the variational estimate of η is

$$\frac{1}{\eta} = \frac{5(k_B)^2 \pi}{216 k_F} \left\langle \overline{W} \sin^4 \frac{\theta}{2} \sin^2 \varphi \right\rangle . \tag{2.30}$$

In order to obtain explicit formulae for κ, σ and η we need to calculate corresponding transition rates $\overline{W}$ and W_σ.

3. *Scattering rates in strange matter*

The constituents of strange matter interact by the exchange of photons and gluons. These interactions are described using QED and QCD, respectively. In the case of two quarks in vacuum both interactions are long range ones. However, the interaction of two quarks embedded in strange matter becomes finite range because of screening effects. In what follows we discuss separately the cases of the QED and QCD interactions in strange matter. We assume that quarks are massless.

3.1. QCD scattering rates

In our calculation of the quark-quark scattering rates due to gluon exchange we shall use a very simplified version of the QCD. We shall assume a small, *constant* value of the QCD coupling constant $\alpha_c = g^2/4\pi$. The calculation of the transition rates is standard, similar to that in QED, with some complications being caused by the color structure of the quark-gluon vertices (see, eg. [12]). The effect of the *color screening* will be calculated to lowest order in α_c. We shall use a very simple prescription, in which screening is essentially the same as Coulomb screening in QED plasma. In this picture, the interaction with the quark gas gives the gluons an effective mass [13]. Color screening will be then described by a single parameter, which is the inverse color screening length [13],

$$k_{sc} = (6\alpha_c/\pi)^{1/2} k_F. \tag{3.1}$$

Typical values of α_c for the standard models of strange matter are $\alpha_c = 0.1$–0.2, so that $k_{sc}/k_F = 0.4$–0.6. Color screening is thus very strong and at high baryon density the resulting cut-off of momentum transfer from below is sufficiently large to justify (qualitatively) the use of a constant, small α_c in the expression for the transport coefficients.

It should be stressed, that the use of only one screening parameter is a very rough approximation, in view of a complicated structure of the color interaction. Another simplification consists in using the *static limit* of the screening. One may argue, that the use of the static limit for the ultrarelativistic quarks is justified by the smallness of the parameter $\hbar\omega/\hbar k_{sc}c$, where typical excitation energy is $\hbar\omega \sim k_B T$ and thus less than 1 MeV for $T < 10^{10}$ K (cf. [14]).

Apart from the quark-quark scattering, one should consider in principle other microscopic QCD processes leading to irreversible entropy production. An example of such a process involves one quark emitting or absorbing a QCD plasmon. However, the color plasmon energy in strange matter is very high, $\hbar\omega_{pc} \cong (5\alpha_c/2\pi)^{1/2}\hbar k_F c$ [13], on the order of hundreds of MeV. Hence, color plasmon absorption process can be neglected because of the Boltzmann factor $\exp(-\hbar\omega_{pc}/k_B T) < 10^{-18}$. On the other hand, the Pauli exclusion principle for quarks inhibits the color plasmon emission process in degenerate strange matter. This justifies our approximation consisting in taking into account only quark-quark scattering.

Using the Born approximation for the color screened QCD interaction we obtain, after lengthy but otherwise standard calculation, the following expression for the two-body QCD scattering rates in strange matter:

$$W_\sigma^{QCD} = 32(2\pi)^2 \frac{\alpha_c^2 \hbar c^2}{k_F^4} F_1(\theta, \varphi), \tag{3.2}$$

$$\overline{W}^{QCD} = 12(2\pi)^2 \frac{\alpha_c^2 \hbar c^2}{k_F^4} \left[5F_1(\theta, \varphi) - \tfrac{2}{3} F_2(\theta, \varphi) + F_3(\theta, \varphi)\right], \tag{3.3}$$

where functions F_i are defined by

$$F_1 = \frac{\sin^4 \dfrac{\theta}{2}\left(1+\cos^4 \dfrac{\varphi}{2}\right)}{\left(\sin^2 \dfrac{\theta}{2} \sin^2 \dfrac{\varphi}{2} + x_{sc}^2\right)^2}, \quad F_3(\theta, \varphi) = F_1(\theta, \pi-\varphi), \tag{3.4}$$

$$F_2 = \frac{\sin^4 \frac{\theta}{2}}{\left(\sin^2 \frac{\theta}{2} \sin^2 \frac{\varphi}{2} + x_{sc}^2\right)\left(\sin^2 \frac{\theta}{2} \cos^2 \frac{\varphi}{2} + x_{sc}^2\right)} \tag{3.5}$$

and $x_{sc} = k_{sc}/2k_F = (3\alpha_c/2\pi)^{1/2}$.

3.2. QED scattering rates

The QCD scattering rates have been calculated in the preceding section with QED interaction being switched off. In this approximation, the interaction is flavor symmetric and this enabled us to introduce significant simplifications in our calculation of the transport coefficients. The QED interaction violates the flavor symmetry. However, as we shall show below, the effects of the QED interaction on the scattering rates can be safely neglected.

The QED interaction of quarks is modified by the charge screening. This effect will be calculated in the *static limit* by applying the Debye-Hueckel theory to the flavor symmetric strange matter. For the Coulomb screening length we get

$$r_s = 1/k_s = (\pi/8\alpha)^{1/2} 1/k_F, \tag{3.6}$$

where α is the fine structure constant of QED. Notice that r_s is flavor independent. The use of the static limit for the ultrarelativistic quarks is justified by the smallness of the parameter $\hbar\omega/\hbar k_s c$, where typical excitation energy is $\hbar\omega \sim k_B T$ and thus less then 1 MeV for $T < 10^{10}$ K [14].

Processes involving one quark emitting or absorbing a QED plasmon can be neglected. Plasmon energy in strange matter is $\hbar\omega_p = 2(2\alpha/\pi)^{1/2}\hbar k_F c \sim 40$ MeV [4]. Hence, plasmon absorption process can be neglected because of the Boltzmann factor $\exp(-\hbar\omega_p/k_B T) < 10^{-17}$. On the other hand, emission of a plasmon by a quark embedded in degenerate strange matter is strongly inhibited by the Pauli exclusion principle.

In our calculation of the QED effects we shall separate the QED and the QCD interactions. This is (qualitatively) justified by the fundamental properties of the interactions in strange matter. QED interaction is color independent and hence does not affect the color distribution, while QCD interaction is flavor independent and hence does not affect the electric charge distribution.

In order to estimate the QED scattering rates we calculate the corrections to $\overline{W}$ and W_σ resulting from the switching on of the QED interaction. Using Born approximation for the screened QED interaction, we get, after lengthy but standard calculations, the following formulae for the corrections implied by the QED scattering,

$$\Delta W_\sigma^{\mathrm{QED}} = \tfrac{64}{9}(2\pi)^2 \frac{\alpha^2 \hbar c^2}{k_F^4} F_1(\theta, \varphi), \tag{3.7}$$

$$\Delta \overline{W}^{\mathrm{QED}} = \tfrac{4}{3}(2\pi)^2 \frac{\alpha^2 \hbar c^2}{k_F^4} [11F_1(\theta, \varphi) + 2F_2(\theta, \varphi) + F_3(\theta, \varphi)], \tag{3.8}$$

where F_i are defined in Eq. (3.4, 3.5), with x_{sc} replaced by $x_s = k_s/2k_F = (2\alpha/\pi)^{1/2} = 6.82 \cdot 10^{-2}$. We shall show in the next Section that these corrections give negligible contribution to transport coefficients.

4. Results in absence of magnetic field

Let us consider transport processes with the QED interaction being switched off. In order to obtain order-of-magnitude estimates of transport coefficients, we may use expansions of the angular averages in terms of the parameter $x_{sc} = 0.218(\alpha_c/0.1)^{1/2}$. The leading terms of the small-x_{sc} expansion are

$$\left.\frac{1}{\kappa}\right|_{QCD} = \frac{\pi^3}{3}\sqrt{\frac{2\pi}{3}}\,\frac{\alpha_c^{1/2}T}{\hbar k_F^3 c^2} + \dots, \tag{4.1}$$

$$\left.\frac{1}{\sigma}\right|_{QCD} = \frac{3\pi^4}{4}\sqrt{\frac{2\pi}{3}}\,\frac{\alpha_c^{3/2}k_B^2T^2}{\hbar k_F^3 c^2 e^2} + \dots, \tag{4.2}$$

$$\left.\frac{1}{\eta}\right|_{QCD} = \frac{25\pi^4}{6}\sqrt{\frac{2\pi}{3}}\,\frac{\alpha_c^{3/2}k_B^2T^2}{\hbar^3 k_F^5 c^2} + \dots. \tag{4.3}$$

Let us now estimate the QED contribution to transport coefficients. The charge screening is rather weak and we will take advantage of smallness of the screening parameter $x_s = 6.82 \cdot 10^{-2}$. The leading terms of the small x_s expansions of the transport coefficients are then

$$\Delta\left.\frac{1}{\kappa}\right|_{QED} = \frac{\pi}{18}\sqrt{\frac{\pi}{2}}\,\frac{T\alpha^{1/2}}{\hbar c^2 n_B} + \dots, \tag{4.4}$$

$$\Delta\left.\frac{1}{\sigma}\right|_{QED} = \frac{\pi^2}{6}\sqrt{\frac{\pi}{2}}\,\frac{T^2k_B^2\alpha^{3/2}}{\hbar e^2c^2 n_B} + \dots, \tag{4.5}$$

$$\Delta\left.\frac{1}{\eta}\right|_{QED} = \frac{25\pi}{27}\,\frac{\pi^{1/6}}{\sqrt{2}}\,\frac{k_B^2T^2\alpha^{3/2}}{\hbar^3c^2n_B^{5/3}} + \dots. \tag{4.6}$$

For $\alpha_c = 0.1$ the QED contributions are nearly two orders of magnitude smaller than the QCD values. Thus, transport of heat, charge and momentum in strange matter is limited by the QCD interaction between quarks.

Expressions suitable for a rapid order-of-magnitude estimates of the transports coefficients are

$$\kappa \cong 3.4 \cdot 10^{22}\left(\frac{\alpha_c}{0.1}\right)^{-1/2} T_{10}^{-1}\,\frac{n_B}{n_{B0}}\,\frac{\text{erg}}{\text{cm s K}}, \tag{4.7}$$

$$\sigma \cong 5.8 \cdot 10^{25} \left(\frac{\alpha_c}{0.1}\right)^{-3/2} T_{10}^{-2} \frac{n_B}{n_{B0}} \, s^{-1}, \tag{4.8}$$

$$\eta \cong 7.0 \cdot 10^{15} \left(\frac{\alpha_c}{0.1}\right)^{-3/2} T_{10}^{-2} \left(\frac{n_B}{n_{B0}}\right)^{5/3} \frac{g}{cm\, s}. \tag{4.9}$$

For $\alpha_c \lesssim 0.2$ the above expressions yield estimates which differ by at most a factor of two from those obtained from the full expressions, Eqs (2.21, 2.26, 2.30).

We performed our calculation of the full expressions for κ, σ and η using a specific model of strange matter, derived by Haensel et al. [3]. It has been obtained assuming $B = 60$ MeV fm^{-3} for the MIT bag constant, $m_s c^2 = 200$ MeV and $\alpha_c = 0.17$. For this model, strange matter of baryon density $n_{BS} = 0.2902$ fm^{-3} and mass density $\varrho_s = 4.816 \cdot 10^{14}$ g cm^{-3} is *self-bound* and energetically preferred even over the nucleon configuration in the form of ^{56}Fe crystal. This model of strange matter has been used by Haensel et al. [3] in the calculation of the models of strange stars.

Our results for transport coefficients at $T = 10^{10}$ K versus density of strange matter are given in Figs 1–3. In practical applications the dependence of the transport coefficients on the baryon density may be needed. In order to enable the transformation from the n_B to ϱ variable, we give in Fig. 4 the relation $\varrho = \varrho(n_B)$ for the considered model of strange matter. Notice, that the maximum density reached in stable strange stars built of such matter is $\varrho_{max} = 2.45 \cdot 10^{15}$ g cm^{-3} ($n_{Bmax} = 1.20$ fm^{-3}). Approximate expression, Eq. (4.7), gives a very good approximation to the value of κ. However, in the case of σ and η Eq. (4.8) and Eq. (4.9) give only about half of the full value of these transport coefficients. The values at other temperatures may be obtained by scaling the corresponding curve by a factor T_{10}^{-2} in the case of σ and η and a factor T_{10}^{-1} in the case of κ, respectively.

5. *Transport coefficients in the presence of strong magnetic field*

If strange stars are born in (some?) supernova explosions, or are products of a phase transition in a strongly magnetized massive neutron star, we should expect them to possess very intense magnetic field, $B \sim 10^{12}$–10^{13} Gs. At the origin of such a strong magnetic field would be, as in the case of ordinary neutron stars, very high electric conductivity of dense matter, which would enable a huge amplification of magnetic field during the shrinking of a collapsing star.

In order to study the influence of such a huge magnetic field on the transport properties of strange stars we should calculate the Larmor radius of a quark in the neighborhood of the Fermi surface (only such quarks contribute to transport in a degenerate system)

$$r_B \cong \frac{\mu_f}{|q_f| B} \cong 10^{-6} \mu_{300} B_{12}^{-1} \text{ cm}. \tag{5.1}$$

Here, μ_f is the quark chemical potential, $\mu_{300} = \mu_f/300$ MeV, $B_{12} = B/10^{12}$ Gs and $f =$ u, d, s. The motion of quarks in magnetic field is *nonquantized* as long as the distance between the Landau levels, $\hbar\omega_B = \hbar c/r_B$, is less than the energy of the thermal motion of quarks,

$$\frac{\hbar\omega_B}{k_B T} \cong 10^{-2} B_{12}\mu_{300}^{-1} T_8^{-1} < 1, \tag{5.2}$$

where $T_8 = T/10^8$ K. This condition is satisfied for the values of B and T relevant for not too old strange stars (say, of age $< 10^6$ y).

5.1. Weak and strong magnetic field

From the results for κ, σ and η, obtained in the preceding Section, one can derive typical values of the *relaxation times* characteristic of a specific transport process in absence of magnetic field. These relaxation times, τ_t ($t = \kappa, \sigma, \eta$), are related to the mean free path of quarks (at the Fermi surface) by $\lambda_t = \tau_t c$. From expressions, that will be derived in Sections 5.2, 5.3 and 5.4 we get,

$$\tau_\kappa = \frac{\hbar k_F}{n_B c} \frac{3\kappa}{\pi^2 k_B^2 T}, \tag{5.3}$$

$$\tau_\sigma = \frac{\hbar k_F}{2 n_B c} \frac{9\sigma}{e^2}, \tag{5.4}$$

$$\tau_\eta = \frac{15\eta}{n_B c \hbar k_F}. \tag{5.5}$$

Typical mean free path between the collisions, in the vicinity of the Fermi surface and in absence of magnetic field can be thus estimated as

$$\lambda_t \approx \beta_t 10^{-4} T_8^{-2}\ \mathrm{cm}, \tag{5.6}$$

where β_t depends on the transport process involved (but very roughly $\beta_t \sim 1$).

In the case when $r_B < \lambda_t$ transport processes in strange matter are strongly influenced by magnetic field. On the contrary, for $r_B \gg \lambda_t$ the effect of magnetic field on transport phenomena can be neglected. In terms of B and T both regimes can be determined by:

magnetic field important

$$r_B \lesssim \lambda_t \qquad B_{12} \gtrsim 10^{-2}\mu_{300} T_9^2/\beta_t \tag{5.7}$$

magnetic field can be neglected

$$r_B \gg \lambda_t \qquad B_{12} \ll 10^{-2}\mu_{300} T_9^2/\beta_t. \tag{5.8}$$

For $T_8 \sim 1$, corresponding to an expected *interior* temperature of a 10 y old strange star (such a value can be deduced from thermal evolution scenarios of a neutron star discussed in [15]), transport processes are strongly influenced by the magnetic field $B > 10^{10}$ Gs. So, magnetic field may start to play an important role at an early stage of thermal evolution of a strange star.

5.2. Thermal conductivity

In the presence of electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$, the general form of the Boltzmann equation for the quark distribution function f_α, in the relaxation time approximation for the collision integral, reads

$$\frac{\partial f_\alpha}{\partial t} + v^j \frac{\partial f_\alpha}{\partial x^j} + \mathscr{F}^j_\alpha \frac{\partial f_\alpha}{\partial p^j} = -\frac{f_\alpha - f^0_\alpha}{\tau_\alpha},$$
$$\mathscr{F}^j_\alpha = q_\alpha\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}\right)^j, \tag{5.9}$$

where $\boldsymbol{v} = \partial\varepsilon_p/\partial\boldsymbol{p}$ and we have used a convention of summing over the repeated indices j. Eq. (5.9) is a three dimensional reduction of the relativistic Boltzmann equation, Eq. (2.2), with collision integral expressed in the relaxation time approximation. Terms corresponding to coupling of magnetic field to the quark intrinsic magnetic moment have been neglected. In what follows we shall consider steady flows corresponding to $\partial f_\alpha/\partial t = 0$. In our calculation of transport coefficients we shall follow a general procedure sketched in [16] and explained to one of the authors (P. H.) by D. G. Yakovlev.

Let us consider the case of heat conduction in a constant magnetic field directed along the z-axis, $\boldsymbol{B} = (0, 0, B)$. We shall assume $\boldsymbol{E} = 0$. Using the notation $f^1_\alpha = f_\alpha - f^0_\alpha$, we reduce Eq. (5.9) to the form

$$v^j \frac{\partial f_\alpha}{\partial x^j} + \frac{q_\alpha}{c}(\boldsymbol{v}\times\boldsymbol{B})^j \frac{\partial f_\alpha}{\partial p^j} = -\frac{f^1_\alpha}{\tau^\kappa_\alpha}. \tag{5.10}$$

In our case

$$\frac{\partial f^0_\alpha}{\partial x^j} = -\frac{\partial f^0_\alpha}{\partial \varepsilon_p}\,\frac{\varepsilon_p - \mu}{T}\,\frac{\partial T}{\partial x^j}. \tag{5.11}$$

After linearization of Eq. (5.10) in the temperature gradient one can easily see, that the deviation of the distribution function from the local equilibrium one, f^1_α, has the form

$$f^1_\alpha = \mathscr{A}^j_\alpha(\varepsilon_p)p^j, \tag{5.12}$$

where $\mathscr{A}^j_\alpha$ are *unknown* functions of the quark energy, which are linear in $\partial T/\partial x^i$ ($i = 1, 2, 3$). Inserting expression (5.12) into Eq. (5.10), and solving the resulting system of linear equations for $\mathscr{A}^j_\alpha(\varepsilon)$, we find that the heat flux implied by a small temperature gradient is, in the presence of a strong magnetic field, given by

$$\boldsymbol{j}_\mathrm{h} = \kappa_\parallel \hat{\boldsymbol{B}}(\hat{\boldsymbol{B}}\cdot\nabla T) + \kappa_\perp[\nabla T - \hat{\boldsymbol{B}}(\hat{\boldsymbol{B}}\cdot\nabla T)] - \kappa_\wedge(\hat{\boldsymbol{B}}\times\nabla T), \tag{5.13}$$

where the thermal conductivity coefficients are

$$\kappa_{\parallel} = \kappa_0 = \frac{\pi^2 k_B^2}{3} T \frac{n_B c}{3\hbar k_F} \sum_f \tau_f^\kappa, \tag{5.14}$$

$$\kappa_{\perp} = \kappa_0 \sum_f \frac{\tau_f^\kappa}{1+(a_f^\kappa)^2} \Big/ \sum_f \tau_f^\kappa, \tag{5.15}$$

$$\kappa_{\wedge} = \kappa_0 \sum_f \frac{\tau_f^\kappa a_f^\kappa}{1+(a_f^\kappa)^2} \Big/ \sum_f \tau_f^\kappa. \tag{5.16}$$

Here, for the sake of generality we take into account possible dependence of the relaxation time relevant for the heat conduction (in absence of magnetic field!) on the quark flavor, f. Let us notice, that in our approximation τ_f^α is *independent* of f, $\tau_f^\kappa = \tau_\kappa$. The dimensionless parameters a_f^κ determine the importance of magnetic field. They are defined by

$$a_f^\kappa = \omega_B^f \tau_\kappa = q_f B \tau_\kappa / \hbar k_F \tag{5.17}$$

because in our approximation $\mu_f = \hbar k_F c$. It should be stressed that in deriving our expression (5.13) we have assumed that the pressure gradient vanishes (cf., Eq. (2.13)). Such an approximation is valid in the case of *degenerate* quark matter with constant baryon density.

5.3. Electric conductivity

In order to study the influence of a strong magnetic field on electric conductivity of strange matter, we consider the general case of $\mathscr{F}_\alpha = q_\alpha(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{B})$, assuming that $\boldsymbol{E}$ is *weak*. Equation (5.9) reads then

$$v^j \frac{\partial f_\alpha}{\partial x^j} + \mathscr{F}_\alpha^j \frac{\partial f_\alpha}{\partial p^j} = -\frac{f_\alpha^1}{\tau_\alpha^\sigma}. \tag{5.18}$$

The calculation of f_α^1 can be performed after linearizing the above equation in $\boldsymbol{E}$. Equation (5.18) implies then the following functional form of f_α^1,

$$f_\alpha^1 = \mathscr{C}_\alpha^j(\varepsilon_p) p^j, \tag{5.19}$$

where $\mathscr{C}_\alpha^j$ are unknown functions of ε_p, linear in $\boldsymbol{E}$. Inserting expression (5.19) into Eq. (5.18), we find a system of linear equations for $\mathscr{C}_\alpha^j$, which yields analytic expressions for these quantities. This enables us to calculate the electric current $\boldsymbol{j}_e$, implied by a weak electric field $\boldsymbol{E}$, in the presence of a *strong* magnetic field $\boldsymbol{B}$,

$$\boldsymbol{j}_e = \sigma_{\parallel} \hat{\boldsymbol{B}}(\hat{\boldsymbol{B}} \cdot \boldsymbol{E}) + \sigma_{\perp}[\boldsymbol{E} - \hat{\boldsymbol{B}}(\hat{\boldsymbol{B}} \cdot \boldsymbol{E})] - \sigma_{\wedge}(\hat{\boldsymbol{B}} \times \boldsymbol{E}). \tag{5.20}$$

Here, the electric conductivity coefficients are given by the formulae

$$\sigma_{\parallel} = \sigma_0 = \tfrac{1}{3}\frac{n_{\mathrm{B}}c}{\hbar k_{\mathrm{F}}}\sum_f q_f^2\tau_f^\sigma, \tag{5.21}$$

$$\sigma_{\perp}=\sigma_0 \sum_f \frac{q_f^2\tau_f^\sigma}{1+(a_f^\sigma)^2}\Big/\sum_f q_f^2\tau_f^\sigma, \tag{5.22}$$

$$\sigma_{\wedge} = \sigma_0 \sum_f \frac{q_f^2\tau_f^\sigma a_f^\sigma}{1+(a_f^\sigma)^2}\Big/\sum_f q_f^2\tau_f^\sigma. \tag{5.23}$$

In our approximation, in which all quarks are assumed to be massless, and flavor independent QCD interaction dominates over the QED one, $\tau_f^\sigma = \tau_\sigma$ and

$$a_f^\sigma = q_f B\tau_\sigma/p_{\mathrm{F}}. \tag{5.24}$$

5.4. Shear viscosity

The case of the shear viscosity is the most complicated one. This is due to the tensor character of the perturbation. Let us impose a *nonrelativistic* hydrodynamic velocity field $V(|V| \ll c)$. The local equilibrium distribution function reads then

$$f_\alpha^0 = \left[1+\exp\left(\frac{\tilde{\varepsilon}(p, V)-\mu}{k_{\mathrm{B}}T}\right)\right]^{-1}. \tag{5.25}$$

The above formula contains quark energy $\tilde{\varepsilon}$ calculated in the *local* rest frame of a moving fluid element. The bulk motion of strange matter is assumed to be *nonrelativistic* and therefore we may use the approximation

$$\tilde{\varepsilon}(\boldsymbol{p}, V) = \varepsilon_p - V\cdot \boldsymbol{p}. \tag{5.26}$$

Equation (5.10), after linearization in $\partial V^i/\partial x^j$, implies then the following form of f_α^1,

$$f_\alpha^1 = \mathscr{D}_\alpha^{jk}(\varepsilon_p)p^j p^k. \tag{5.27}$$

In what follows we shall assume that the fluid motion is described by a purely shear flow (density of each quark flavor n_f = const.). Equation (5.10) reduces then to a system of six linear equations for the quantities $\mathscr{D}_f^{jk}$, which are symmetric in j, k indices. Let us introduce the tensor

$$\bar{w}_{ij} = \frac{\partial V^i}{\partial x^j} + \frac{\partial V^j}{\partial x^i}. \tag{5.28}$$

We restrict ourselves to flavor symmetric flows without compression. After getting an analytic solution for $\mathscr{D}_f^{jk}$, we can express the dissipative components of the energy-momentum tensor in terms of the five viscosity coefficients and of the components of the $\bar{w}_{ij}$ tensor

(Baranov et al. [16]),

$$\delta T_{11} = -\tfrac{1}{2}\eta_0\bar{w}_{33}+\eta_1(\bar{w}_{11}+\tfrac{1}{2}\bar{w}_{33})+\eta_3\bar{w}_{12}, \tag{5.29}$$

$$\delta T_{22} = -\tfrac{1}{2}\eta_0\bar{w}_{33}+\eta_1(\bar{w}_{22}+\tfrac{1}{2}\bar{w}_{33})-\eta_3\bar{w}_{12}, \tag{5.30}$$

$$\delta T_{33} = \eta_0\bar{w}_{33}, \tag{5.31}$$

$$\delta T_{12} = \eta_1\bar{w}_{12}-\tfrac{1}{2}\eta_3(\bar{w}_{11}-\bar{w}_{22}), \tag{5.32}$$

$$\delta T_{13} = \eta_2\bar{w}_{13}+\eta_4\bar{w}_{23}, \tag{5.33}$$

$$\delta T_{23} = \eta_2\bar{w}_{23}-\eta_4\bar{w}_{13}. \tag{5.34}$$

Explicit formulae for the η_k coefficients read

$$\eta_0 = \tfrac{1}{5}\, n_B \hbar k_F c\, \tfrac{1}{3}\sum_f \tau^\eta_f, \tag{5.35}$$

$$n_1 = \eta_2 = \eta_0 \sum_f \frac{\tau^\eta_f}{1+4(a^\eta_f)^2} \Big/ \sum_f \tau^\eta_f, \tag{5.36}$$

$$\eta_3 = \eta_0 \sum_f \frac{\tau^\eta_f 2a^\eta_f}{1+4(a^\eta_f)^2} \Big/ \sum_f \tau^\eta_f, \tag{5.37}$$

$$\eta_4 = \eta_0 \sum_f \frac{\tau^\eta_f a^\eta_f}{1+(a^\eta_f)^2} \Big/ \sum_f \tau^\eta_f, \tag{5.38}$$

where the dimensionless coefficient a^η_f is defined by Eq. (5.17), with τ_κ replaced by τ_η. For the sake of generality, the formula for η_k has been written in the form valid for flavor dependent relaxation times.

5.5. Limiting cases of strong and weak magnetic field

In the case of a weak magnetic field $|a_f| \ll 1$ and strange matter can be treated as an isotropic medium with

$$\kappa_{\parallel} = \kappa_{\perp} = \kappa_0, \quad \kappa_{\wedge} = 0, \tag{5.39}$$

$$\sigma_{\parallel} = \sigma_{\perp} = \sigma_0, \quad \sigma_{\wedge} = 0, \tag{5.40}$$

$$\eta_1 = \eta_2 = \eta_0, \quad \eta_3 = \eta_4 = 0, \tag{5.41}$$

where κ_0, σ_0, and η_0 are transport coefficients calculated at $\boldsymbol{B} = 0$. Such a situation occurs for $B_{12} \ll 10^{-2}\, \mu_{300}\, T_8^2$. In an opposite case of a strong magnetic field ($B_{12} \gg 10^{-2}\, \mu_{300}\, T_8^2$) transport properties of strange matter become highly anisotropic. Transport processes taking place along the direction of the magnetic field are not affected by its presence, while those in the other directions are strongly suppressed. This property is visualized by the following approximate formulae, holding for $B_{12} \gg 10^{-2} T_8^2$ (notice, that $\mu_{300} \sim 1$),

$$\kappa_{\parallel} = \kappa_0, \quad \kappa_{\perp} \cong 6.75\kappa_0/a_\kappa^2 \ll \kappa_{\wedge} \cong 1.5\kappa_0/a_\kappa \ll \kappa_0, \tag{5.42}$$

$$\sigma_{\parallel} = \sigma_0, \quad \sigma_{\perp} \cong 4.5\sigma_0/a_\sigma^2 \ll \sigma_0, \quad \sigma_{\wedge} \cong 0, \tag{5.43}$$

$$\eta_1 = \eta_2 = \eta_0/a_\eta^2 \ll \eta_0, \tag{5.44}$$

$$\eta_1, \eta_2 \ll \eta_3 = 0.75\eta_0/a_\eta \ll \eta_0, \tag{5.45}$$

$$\eta_1, \eta_2 \ll \eta_4 = 1.5\eta_0/a_\eta \ll \eta_0. \tag{5.46}$$

The above formulae have been obtained in the approximation of the flavor independent relaxation times. Flavor independent parameters a are defined by $a_\kappa = eB\tau_\kappa/\hbar k_F$, etc. Equations (5.42–46) imply, e.g., that diffusive heat flow in a strange star with $B_{12} \gg 10^{-2} T_8^2$ takes place, to a very good approximation, along the direction of the stellar magnetic field.

6. *Discussion and conclusions*

Our results for thermal conductivity, electric conductivity and shear viscosity of strange star matter should be treated as very rough estimates of these quantities. Actually, using analytical techniques developed in the theory of degenerate Fermi liquids (see, eg. [17]) we might try to find *exact* solutions of relevant transport equations in absence of magnetic field. However, much larger uncertainties and errors, than those resulting from our variational calculation of transport coefficients, stem from our extremely rough treatment of the QCD interaction in quark plasma. First of all, we use an approximation which is valid only in the limiting case of a very dense quark matter. Secondly, our use of a small, constant value of α_c as well as our very simplified treatment of the color screening, are of course drastic approximations. Finally, we assumed flavor symmetry, which introduced great simplification in our calculation of transport coefficients from the relativistic Boltzmann equation.

Being aware of many simplifications and rough approximations, used in the present paper, we are convinced that going beyond our model would require a tremendous amount of work. For the time being, it does not seem to us reasonable to make such an effort for the description of the properties of such hypothetical and exotic objects as strange stars.

Despite simplicity of our model we think that our results are *qualitatively* correct. All transport processes in strange matter, considered in the present paper, including electric conductivity, are dominated by quarks. Electrons play a negligible rôle, because their fraction is very small, $Y_e = n_e/n_B \lesssim 10^{-4}$. On the other hand, transport of electrons in strange matter is very effectively limited by the Coulomb scattering off quarks. Let us recall that in the case of normal neutron star matter both κ and σ are determined by the transport of electrons.

The knowledge of the thermal conductivity of strange matter is required for studying the process of cooling of strange stars. Our results are shown in Fig. 1. At $\varrho \cong 2\varrho_0$ (ϱ_0 = normal nuclear density = $2.5 \cdot 10^{14}$ g cm^{-3}) the value of κ for strange matter is an

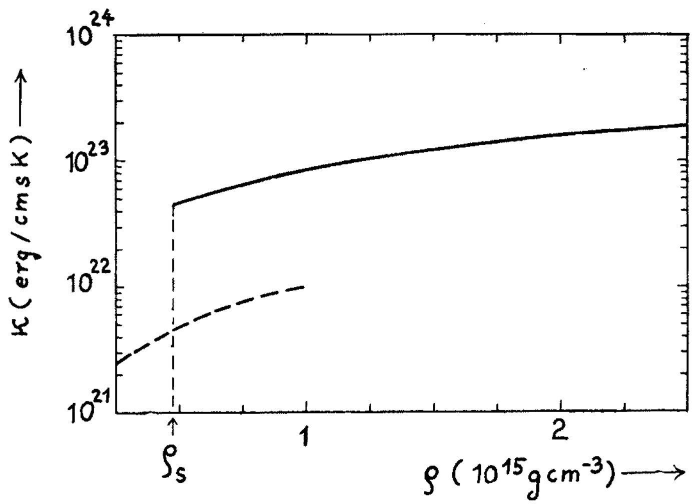

Fig. 1. Thermal conductivity of strange matter, in absence of magnetic field, at $T = 10^{10}$ K, versus density (solid line). Dashed line: thermal conductivity of normal neutron star matter at the same T (from Flowers and Itoh [17]). The curves scale with temperature as T_{10}^{-1}

order of magnitude larger than that of the normal neutron star matter at the same temperature [14, 18]. Results of Flowers and Itoh are shown only for $\varrho < 4\varrho_0$, because at higher ϱ the validity of their calculational scheme becomes questionable.

The electric conductivity of normal neutron star matter is determined by the motion of ultrarelativistic electrons. A simple expression for σ has been derived by Baym et al. [20]. Assuming a simple formula for the density dependence of electron fraction in normal neutron star matter, $Y_e = 0.02\,\varrho/\varrho_0$ [21], we transform the formula of Baym et al. [20] into a suitable form

$$\textit{neutron star matter:}\quad \sigma = 5.0 \cdot 10^{24} \left(\frac{\varrho}{\varrho_0}\right)^{3/2} T_{10}^{-2}\ \mathrm{s}^{-1}. \tag{6.1}$$

This formula yields the dashed line in Fig. 2. At $\varrho \cong 2\varrho_0$ the value of σ for strange matter is several times larger than that for normal neutron star matter. The validity of approximations used in deriving Eq. (6.1) breaks down for $\varrho \gtrsim 3\varrho_0$.

If the hypothesis of strange matter is correct, then one might contemplate possibility of existence of macroscopic, self-bound, stable nuggets built of strange matter. At room temperature the electric conductivity of such an object would be seventeen orders of magnitude larger than that of copper.

If strange stars are born in (some?) supernova explosions, then we expect them to possess a very intense magnetic field ($B \sim 10^{12}$–10^{13} Gs). The presence of such a strong magnetic field could strongly suppress transport processes in the direction parallel to magnetic field, resulting in a very strong anisotropy in the heat flow in the interior of hot strange star.

The shear viscosity of strange star matter is about one order of magnitude larger than

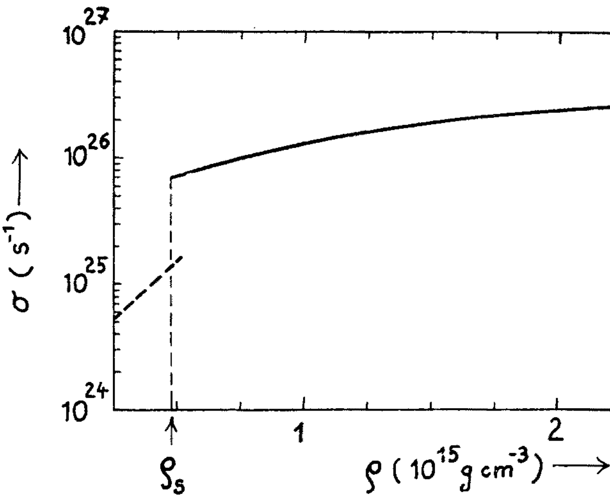

Fig. 2. Electric conductivity of strange matter (solid line). Dashed line: electric conductivity of normal neutron star matter calculated using Eq. (6.1). Results correspond to $T = 10^{10}$ K, in absence of magnetic field. The curves scale with temperature as T_{10}^{-2}

that of normal neutron star matter at the same temperature and density, Fig. 3. The values of η for normal neutron star matter are taken from Flowers and Itoh [18], who give them for $\varrho < 2\varrho_0$. An order-of-magnitude estimate of the characteristic timescale of the damping of shear motion in strange matter leads to conclusion, that for $T > 10^5$ K the effect of shear viscosity on, e.g., pulsations of strange star, can be safely neglected.

In the present paper we have restricted ourselves to transport processes which do not change the local density of matter. On the other hand, our model of quark plasma was flavor-symmetric. Consequently, we did not consider bulk (second) viscosity of strange matter, determined by the flavor changing weak interactions between quarks. Some information about this quantity can be extracted from the paper of Wang and Lu [22]. The importance of the bulk viscosity for the damping of the strange star pulsations results from the slowness of the flavor changing process $u+d \rightarrow s+u$, involving massive s quark. The results of Wang and Lu show, that bulk viscosity resulting from this process can lead to a damping of the radial pulsations of the strange star in less than a second. However, one should remind that this damping time is still three orders of magnitude longer than the period of strange star vibrations. So, bulk viscosity of strange star is negligible on the dynamic timescale, $\tau_d \sim (G\varrho)^{-1/2} \sim 10^{-4}$ s.

It should be stressed, that all results of the present paper have been obtained under assumption, that strange matter is a normal system. Actually, one cannot exclude a possibility that dense quark matter is a superconductor. Such a possibility has been considered by Bailin and Love [23, 24]. If strange matter was superconducting, than thermal and transport properties of strange stars would be dramatically different from those derived in the present paper. However, in view of the uncertainties, characteristic of the dense quark matter models, connected especially with the perturbative treatment of the QCD interac-

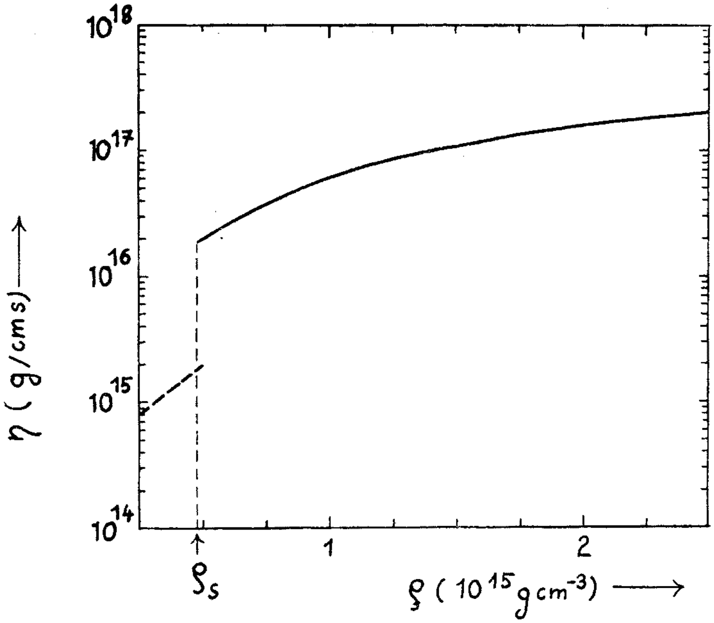

Fig. 3. Shear viscosity of strange matter versus density (solid line). Dashed line: shear viscosity of normal neutron star matter [13]. Results correspond to $T = 10^{10}$ K, in absence of magnetic field. The curves scale with temperature as T_{10}^{-2}

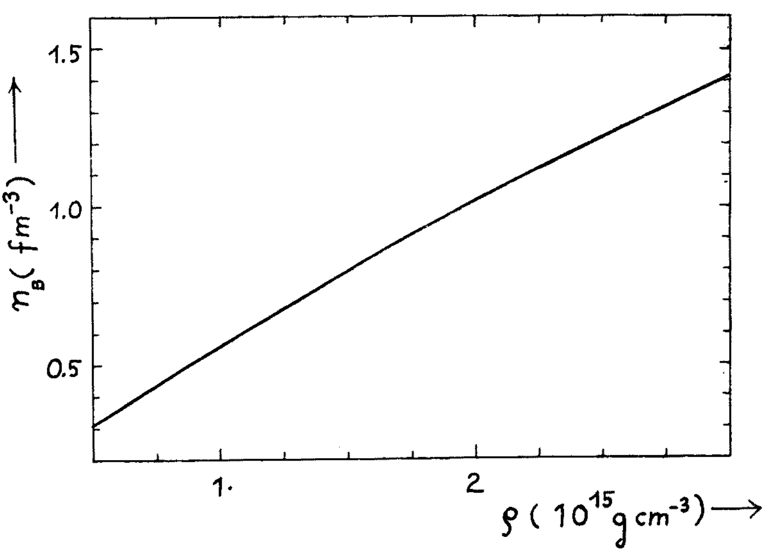

Fig. 4. Mass density of matter, defined as energy density divided by c^2, versus baryon density, n_B, for the strange matter model considered in the present paper

tions, it seems reasonable to treat the problem of the actual nature of the ground state of strange matter as an unsolved issue.

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