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# INTERIOR FIELDS OF CHARGED FLUID SPHERE IN THE EINSTEIN-CARTAN THEORY

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We obtain various explicit solutions to the interior Einstein-Maxwell field equations corresponding to charged fluid spheres in the Einstein-Cartan theory. The physical 3-space $t =$ constant of the solution is spheroidal. The physical features of one of these solutions are also discussed.

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## 1. Introduction

Following the work of Trautman (1972, 1973), the Einstein-Cartan equations with different contexts have been studied by different authors.

Kuchowicz (1975) gave a detailed description of methods of deriving exact solutions of spherical symmetry in the Einstein-Cartan theory (EC theory in brief) for a perfect fluid with a classical description of spin. The predictions of the EC theory differ from those of general relativity only for non-empty regions. Therefore besides cosmology, an important application field for the EC theory is relativistic astrophysics dealing with neutron stars with some alignment of spins of the constituent particles and under conditions when the torsion may give rise to some observable effects. Prasanna (1975) and Singh and Yadav (1978) have obtained some analytic solutions of the Einstein-Cartan field equations for the interior of uncharged fluid spheres.

Nduka (1977) generalized Prasanna's problem by considering a static charged fluid sphere in the EC theory. He has found that the pressure is discontinuous at the boundary of the fluid sphere. Some exact solutions of charged fluid sphere in the EC theory have also been discussed by Singh and Yadav (1978). In this paper we obtain some new exact solutions of charged fluid spheres in the EC theory in which the physical 3-space $t =$ constant is spheroidal. Vaidya and Tikekar (1982) have discussed in great detail the spacetimes with spheroidal physical 3-space. They have expressed the line element of such space-times

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in the form

$$ds^2 = e^{2\nu}dt^2 - \left(1 - \frac{ar^2}{R^2}\right)\left(1 - \frac{r^2}{R^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1.1}$$

where $\nu$ is a function of $r$ only. Here $R$ and $a$ are constants, $a < 1$. The line element (1.1) is regular at all points where $r^2 < R^2$. We denote the coordinates as $x^1 = r, x^2 = \theta, x^3 = \varphi, x^4 = t$. When $a = 0$, the physical 3 space $t =$ constant is spherical.

## 2. *The field equations*

The Einstein-Cartan-Maxwell field equations are

$$R^i_k - \tfrac{1}{2}\delta^i_k R = -8\pi t^i_k, \tag{2.1}$$

$$Q^i_{jk} - \delta^i_j Q^l_{lk} - \delta^i_k Q^l_{jl} = -8\pi S^i_{jk}, \tag{2.2}$$

$$(\sqrt{-g}\, F^{ik})_{,k} = 4\pi\sqrt{-g}\, J^i, \tag{2.3}$$

$$F_{[ik,j]} = 0, \tag{2.4}$$

where $Q^i_{jk}$, $t^i_k$, $S^i_{jk}$, $F_{ik}$ and $J^i$ are respectively the torsion tensor, the canonical asymmetric energy momentum tensor corresponding to charged perfect fluid, the spin tensor, the electromagnetic field tensor and the current four vector. The rest of notation is standard.

It is well known that if we assume classical description of spin, we have

$$S^k_{ij} = S_{ij}v^k \text{ with } S_{ij}v^j = 0, \tag{2.5}$$

where $S_{ij}$ is the antisymmetric tensor of the density of spin, $v^i$ are the components of the flow vector of the fluid. For the line element (1.1) we have

$$v_i = (0, 0, 0, e^\nu). \tag{2.6}$$

In the case of static spherical symmetry, the only non vanishing component of $S^{ij}$ is $S_{23}$. From (2.5) and (2.6) it is clear that the non-zero components of $S^i_{jk}$ are

$$S^4_{23} = -S^4_{32} = K(r), \tag{2.7}$$

where $K$ is a function of $r$ only. Hence from the equation (2.2) the surviving components of $Q^i_{jk}$ are

$$Q^4_{23} = -Q^4_{32} = -8\pi K. \tag{2.8}$$

The assumption of classical description of spin implies that $t^i_k = T^i_k$, where $T^i_k$ is the usual energy momentum tensor of the charged perfect fluid given by

$$T^i_k = (P+\varrho)v^i v_k - P\delta^i_k - F^{in}F_{kn} + \tfrac{1}{4}\delta^i_k F_{mn}F^{mn}. \tag{2.9}$$

Here $P$ and $\varrho$ are respectively the pressure and the density of the fluid. The fluid has been assumed to have null conductivity, so that

$$J^i = \sigma v^i, \tag{2.10}$$

where $\sigma$ denotes the charge density. We now assume that the metric tensor $g_{ik}$ is given by (1.1). Since there is only a radial electric field, the only surviving component of $F_{ik}$ is $F_{14}$. The Maxwell equations (2.3) and (2.4) lead to

$$F_{14} = \frac{e^{\nu}}{r^2}\left(1-\frac{ar^2}{R^2}\right)^{1/2}\left(1-\frac{r^2}{R^2}\right)^{-1/2}\int_0^r 4\pi\sigma r^2\left(1-\frac{ar^2}{R^2}\right)^{1/2}\left(1-\frac{r^2}{R^2}\right)^{-1/2} dr \quad (2.11)$$

and

$$4\pi\sigma = \frac{1}{r^2}\left[\frac{d}{dr}(r^2E)\right]\left(1-\frac{r^2}{R^2}\right)\left(1-\frac{ar^2}{R^2}\right)^{-1/2}, \quad (2.12)$$

where the function $E^2(r)$ is defined by

$$-F_{14}F^{14} = E^2(r)/4; \quad (2.13)$$

the function $E(r)$ can be interpreted as the field intensity. It is easy to see that the quantity

$$Q(r) = 4\pi\int_0^r\left(1-\frac{ar^2}{R^2}\right)^{1/2}\left(1-\frac{r^2}{R^2}\right)^{-1/2}\sigma r^2 dr \quad (2.14)$$

represents the total charge contained within the sphere of radius $r$. If we define $\bar{P}$ and $\bar{\varrho}$ by the relations

$$\bar{P} = P-2\pi K^2, \qquad \bar{\varrho} = \varrho-2\pi K^2 \quad (2.15)$$

the Einstein-Cartan-Maxwell equations (2.1) reduce to

$$-8\pi\bar{P}+E^2 = \left[\frac{1-a}{R^2}-\frac{2}{r}\nu'\left(1-\frac{r^2}{R^2}\right)\right]\left(1-\frac{ar^2}{R^2}\right)^{-1}, \quad (2.16)$$

$$8\pi\bar{P}+E^2 = \left(\nu'+\nu'^2-\frac{1}{r}\right)\left(1-\frac{r^2}{R^2}\right)\left(1-\frac{ar^2}{R^2}\right)^{-1} \quad (2.17)$$
$$-\frac{r}{R^2}(1-a)\left(\nu'+\frac{1}{r}\right)\left(1-\frac{ar^2}{R^2}\right)^{-2},$$

$$8\pi\bar{\varrho} = E^2+\frac{3}{R^2}(1-a)\left(1-\frac{ar^2}{3R^2}\right)\left(1-\frac{ar^2}{R^2}\right)^{-2}. \quad (2.18)$$

Here and in what follows an overhead prim indicates differentiation with respect to $r$. The equation (2.16), (2.17) and (2.18).constitute the general set of relevant equations.

The conservation laws give us the relation

$$P'+\tfrac{1}{2}(P+\varrho)\nu'-\frac{E^2}{8\pi}\left\{(E^2)'+\frac{4E^2}{r}\right\}$$
$$= -4\pi K(K'+K\nu'). \quad (2.19)$$

If we assume the usual equation of hydrostatic equilibrium

$$P'+\tfrac{1}{2}(P+\varrho)\nu'-\frac{E^2}{8\pi}\left\{(E^2)'+\frac{4E^2}{r}\right\}=0 \tag{2.20}$$

we get

$$K'+K\nu' = 0. \tag{2.21}$$

The equation (2.21) can be easily integrated.
We have

$$K = A_1 e^{-} \tag{2.22}$$

where $A_1$ is a constant of integration.

In principle we have a completely determined system if an equation of state is specified. However, as is well known, in practice the above general set of equations is formidable to solve using a preassigned equation of state.

Therefore, we shall apply other methods to solve the above set of equations in the next section.

## 3. *The solutions of the field equations*

We have three equations (2.16), (2.17) and (2.18) for four unknown functions $\overline{P}$, $\overline{\varrho}$, $\nu$ and $E^2$. Therefore, the choice of any one of them is at our disposal. In the Case I we shall choose $\nu$ and in the Cases II and III we shall choose $E^2$.

### Case I

Let us make the simplifying mathematical assumption

$$\nu' = \frac{\beta r}{R^2}\left(1-\frac{ar^2}{R^2}\right)^{1/2}\left(1-\frac{r^2}{R^2}\right)^{-1/2}, \tag{3.1}$$

where $\beta$ is an arbitrary constant. The solution of (3.1) can be expressed in the form

$$\nu = \beta(-a)^{1/2}\left[\tfrac{1}{2}z\left(\frac{a-1}{a}-z^2\right)^{1/2} + \left(\frac{a-1}{2a}\right)\sin^{-1}\left\{\left(\frac{a}{a-1}\right)^{1/2}z\right\}\right]+A, \tag{3.2}$$

where $A$ is a constant of integration and $a$ is assumed to be negative. Also $z^2 = 1-\dfrac{r^2}{R^2}$.

The equations (2.16), (2.17) and (2.18) determine $E^2$, $\overline{P}$ and $\varrho$ as

$$2E^2 = \frac{r^2}{R^4}\left[\beta^2+a(a-1)\left(1-\frac{ar^2}{R^2}\right)^{-2}\right], \tag{3.3}$$

$$8\pi\bar{P} = \frac{r^2}{2R^4}\left[\beta^2 + a(a-1)\left(1-\frac{ar^2}{R^2}\right)^{-2}\right]$$

$$+\frac{(a-1)}{R^2}\left(1-\frac{ar^2}{R^2}\right)^{-1} + \frac{2\beta}{R^2}\left(1-\frac{r^2}{R^2}\right)^{1/2}\left(1-\frac{ar^2}{R^2}\right)^{-1/2}, \tag{3.4}$$

$$8\pi\bar{\varrho} = \frac{(1-a)}{R^2}\left[3-\frac{a}{2}\frac{r^2}{R^2}\right]\left(1-\frac{ar^2}{R^2}\right)^{-2} - \frac{\beta^2 r^2}{2R^4}. \tag{3.5}$$

From (3.3) it is easy to see that $E^2 > 0$ and when $r = 0, E^2 = 0$. Thus the electric field vanishes at the origin. This is a physically reasonable property of the solution.

With the aid of the result (2.12) we can find the charge density $\sigma$. It is given by

$$4\pi\sigma = \frac{1}{E}\left[\frac{3E^2}{r} - \frac{a^2(a-1)r^3}{2R^6\left(1-\frac{ar^2}{R^2}\right)^3}\right]\left(1-\frac{r^2}{R^2}\right)^{1/2}\left(1-\frac{ar^2}{R^2}\right)^{-1/2},$$

where $E^2$ is given by (3.3). Using the fact that $a < 0$ it is not hard to see that $\sigma$ is positive.

We consider a situation where the spherical charged distribution extends to a finite radius $b < R$. The appropriate boundary conditions are (Singh and Yadav 1978):

(*i*) The pressure $\bar{P}$ must vanish at the boundary of the sphere $r = b$.

(*ii*) The metric functions of the interior metric (2.1) with $\nu$ given by (3.2) must be continuous across the boundary, i.e. must be matched with the metric functions of the Reissner-Nördstrom line element

$$ds^2 = \left(1-\frac{2m}{r}+\frac{q^2}{r^2}\right)dt^2 - \left(1-\frac{2m}{r}+\frac{q^2}{r^2}\right)^{-1/2}dr^2 \tag{3.7}$$

$$- r^2(d\theta^2 + \sin^2\theta d\varphi^2).$$

Here $m$ and $q$ denote the total mass and the total charge of the sphere respectively. The relations (2.14) and $Q(b) = q$ determine $q$ as

$$q^2 = \frac{b^6}{2R^4}\left[\beta^2 + a(a-1)\left(1-\frac{ab^2}{R^2}\right)^{-2}\right], \tag{3.8}$$

which is clearly positive.

The boundary conditions (*i*) and (*ii*) give

$$\frac{\beta^2 b^2}{2R^2} + 2\beta\left(1-\frac{b^2}{R^2}\right)^{1/2}\left(1-\frac{ab^2}{R^2}\right)^{-1/2}$$

$$+(a-1)\left(1-\frac{ab^2}{2R^2}\right)\left(1-\frac{ab^2}{R^2}\right)^{-2} = 0, \tag{3.9}$$

$$\frac{2m}{b} = 1+\frac{\beta^2 b^4}{2R^4}+\left(1+\frac{b^2}{R^2}\right)\left(1-\frac{ab^2}{R^2}\right)^{-1}$$

$$+\frac{a(a-1)b^4}{2R^4}\left(1-\frac{ab^2}{R^2}\right)^{-2}, \tag{3.10}$$

$$A = \log\left(1-\frac{b^2}{R^2}\right)-\log\left(1-\frac{ab^2}{R^2}\right)$$

$$+\beta(-a)^{1/2}\left[\left(1-\frac{b^2}{R^2}\right)^{1/2}\right]\left(\frac{b}{R^2}-\frac{1}{a}\right)^{1/2}$$

$$+\frac{(a-1)}{2a}\sin^{-1}\left\{\left(\frac{a}{a-1}\right)^{1/2}\left(1-\frac{b^2}{R^2}\right)^{1/2}\right\}. \tag{3.11}$$

Equation (3.9) determines $\beta$. Substituting this $\beta$ into (3.10) and (3.11) we can find $m$ and $A$.

Case II. Let us assume the form of the function $E^2$ as

$$E^2 = \frac{l^2 r^2 e^{-\nu}}{2R^4\left(1-\frac{ar^2}{R^2}\right)^2}\left\{8-\frac{4r^2}{R^2}(a+3)+a(7+a)\frac{r^4}{R^4}\right\}, \tag{3.12}$$

where $l$ is a constant. This peculiar choice of $E^2$ makes the differential equation for $e^\nu$ integrable. Substituting this value of $E^2$ into (2.16) and (2.17) and equating the two values of $\overline{P}$ we get the differential equation

$$(1-a+az^2)\frac{d^2F}{dz^2}-az\frac{dF}{dz}-a(1-a)F = 0, \tag{3.13}$$

where $F = e^\nu-\frac{l^2 r^4}{R^4}$ and $z^2 = 1-\frac{r^2}{R^2}$.

We have checked that $E^2$ given by (3.12) is positive for $a = -2$ provided $0 < \frac{r}{R} < 0.717$. If we take $a = -3$ then $E^2 > 0$ gives the result $0 < \frac{r}{R} < 0.816$. We have verified that if $a \leqslant -4$, then $E^2$ remains positive throughout the range $0 < \frac{r}{R} < 1$.

To obtain a closed form solution of (3.13) we choose the value $a = -7$. For this value of $a$, $E^2$ is always positive. The equation (3.13) has been discussed in detail by Vaidya and Tikekar (1982) for $l = 0$. Following the same method the closed form solution of (3.13) for $a = -7$ can be expressed as

$$e^\nu = C\left(\frac{49r^4}{R^4}-\frac{14r^2}{R^2}-11\right)+B\left(1-\frac{r^2}{R^2}\right)^{1/2}\left(1-\frac{7r^2}{R^2}\right)^{3/2}+\frac{l^2 r^4}{R^4}, \tag{3.14}$$

where $C$ and $B$ are arbitrary constants. The quantities $\bar{P}$ and $\bar{\varrho}$ in this case are found to be

$$8\pi\bar{P}\left[e^{\nu}-l^2\frac{r^2}{R^4}\right]\left(1+\frac{7r^2}{R^2}\right)^2 = 4\frac{l^2r^2}{R^4}\left(3+\frac{14r^2}{R^2}-\frac{14r^4}{R^4}\right)+h\left(\frac{r}{R}\right), \tag{3.15}$$

$$8\pi\bar{\varrho}R^2\left(1+\frac{7r^2}{R^2}\right)^2 = 8\left(3+\frac{7r^2}{R^2}\right)-4l^2e^{-\nu}\frac{r^2}{R^2}\left(1+\frac{2r^2}{R^2}\right), \tag{3.16}$$

where the function $h(r/R)$ is defined by

$$h(r/R)\left(1+7\frac{r^2}{R^2}\right)^{-1} = -\frac{8}{R^2}e^{-\nu}+2C\left(1-\frac{r^2}{R^2}\right)\left(\frac{196r^2}{R^4}-\frac{28}{R^2}\right)$$
$$+\frac{42B}{R^2}\left(1-\frac{r^2}{R^2}\right)^{3/2}\left(1+\frac{7r^2}{R^2}\right)^{1/2}-\frac{2B}{R^2}\left(1-\frac{r^2}{R^2}\right)^{1/2}\left(1+\frac{7r^2}{R^2}\right)^{3/2} \tag{3.17}$$

and $e^{\nu}$ is given by (3.14). Following the procedure adopted for case I, we can determine the constants $m, q, C$ and $B$. We can also determine the charge density $\sigma$. For the sake of brevity we shall not report expressions for $m, q, C, B$ and $\sigma$ here.

Case III. Let us assume the form of the function $E^2$ as

$$E^2 = [\alpha^2+\mu^2e^{-\nu}]\frac{r^2}{R^4}\left(1-\frac{ar^2}{R^2}\right)^{-2}, \tag{3.18}$$

where $\alpha$ and $\mu$ are arbitrary constants.

Substituting $E^2$ from (3.18) in (2.16) and (2.17) and equating the two values of $\bar{P}$ we get the differential equation

$$(1-a+az^2)\frac{d^2F}{dz^2}-az\frac{dF}{dz}-[a(1-a)+2\alpha^2]F = 0, \tag{3.19}$$

where $z^2 = 1-\dfrac{r^2}{R^2}$ and

$$F = e^{\nu}+\frac{2\mu^2}{a(1-a)+2\alpha^2}.$$

The equation (3.19) has been solved by Patel and Pandya (1986) in the case $\mu = 0$ and for $a = -1+\sqrt{1+2\alpha^2}$. Following the same method the closed form solution of (3.19) for $a = -1+\sqrt{1+2\alpha^2}$, in the present case can be expressed in the form

$$e^{\nu} = -\frac{2\mu^2}{a(1-a)+2\alpha^2}+Mz\left[1-\frac{2a}{3(a-1)}z^2\right]+N\left[1-\frac{a}{a-1}z^2\right]^{3/2}, \tag{3.20}$$

where $M$ and $N$ are constants of integration. From (3.18) is it clear that $E^2 > 0$ and $E^2 = 0$ at the origin.

Following the procedure adopted for Case I, we can evaluate the constants $m, q, M$ and $N$. For the sake of brevity we shall not give these expressions here.

The quantities $\bar{P}$ and $\bar{\varrho}$ in this case are found to be

$$8\pi\bar{P} = \mu^2\left[\frac{1-z^2}{1-a+az^2} - \frac{2(a-1)}{3a}\right] + Mz\left\{a-3+\frac{2a(7-a)}{3(a-1)}z^2\right\} + N\left(1-\frac{a}{a-1}z\right)^{1/2}\left\{a-1+\frac{a(7-a)}{a-1}z^2\right\} \tag{3.21}$$

and

$$8\pi\bar{\varrho} = \frac{(a^2-10a+6)+a(4-a)z^2}{2R^2(1-a+az^2)^2} - \frac{\mu^2(1-z^2)e^{-\nu}}{r^2(1-a+az^2)^2}, \tag{3.22}$$

where $e^{\nu}$ is given by (3.20). In this case we can also find the charge density $\sigma$. But for the sake of brevity we shall not give the expression for $\sigma$ here.

Here it should be noted that when $\mu = 0$ our solution reduces to the EC analogue of the solution discussed by Patel and Pandya (1986) and when $\alpha = 0$ our solution gives the EC analogue of the solution discussed by Patel and Koppar (1987).

## 4. *Some physical features of the solution discussed in Case I*

Let $\bar{P}_0$ and $\bar{\varrho}_0$ be the values of $\bar{P}$ and $\bar{\varrho}$ at $r = 0$. Clearly we obtain

$$8\pi\bar{P}_0 = \frac{1}{R^2}(a-1+2\beta), \tag{4.1}$$

$$8\pi\bar{\varrho}_0 = \frac{3}{R^2}(1-a). \tag{4.2}$$

When $K = 0$, the spin disappears and $\bar{P}$ and $\bar{\varrho}$ become the usual pressure and density respectively. Therefore the central values $P_0$ and $\varrho_0$ of $P$ and $\varrho$ are given by

$$8\pi P_0 = \frac{1}{R^2}(a-1+2\beta)+16\pi A_1^2 e^{-2\nu_0} \tag{4.3}$$

and

$$8\pi\varrho_0 = \frac{3}{R^2}(1-a)+16\pi A_1^2 e^{-2\nu_0}, \tag{4.4}$$

where $\nu_0$ is the value of $\nu$ at $r = 0$ and it is given by

$$\nu_0 = -\beta(-a)^{1/2}\left[\tfrac{1}{2}\left(-\frac{1}{a}\right)^{1/2} + \left(\frac{a-1}{2a}\right)\sin^{-1}\left\{\left(\frac{a}{a-1}\right)^{1/2}\right\}\right] + A. \tag{4.5}$$

If $\beta$ is negative, then (4.3) implies $P_0 < 0$ when $A_1 = 0$. This is not a desirable feature of the solution. Therefore we assume that $\beta$ is positive. The physical requirements are

$$P_0 \geqslant 0, \ \varrho_0 - 3P_0 \geqslant 0. \tag{4.6}$$

The conditions (4.6) imply that the constant $A_1$ has to satisfy the inequality,

$$\frac{(1-a-2\beta)e^{2\nu_0}}{16\pi^2 R^2} \leqslant A_1^2 \leqslant \frac{3(1-a-\beta)e^{2\nu_0}}{16\pi^2 R^2}. \tag{4.7}$$

This inequality puts a restriction on the constant $\beta$. It is given by

$$0 \leqslant \beta \leqslant 2(1-a). \tag{4.8}$$

From the result (3.5) it is clear that the density $\bar{\varrho}_b$ at the boundary $r = b$ is positive provided

$$\beta^2 < G, \tag{4.9}$$

where $G$ is defined by

$$G = 2(1-a)\left(3 - \frac{ab^2}{2R^2}\right)\left(\frac{b^2}{R^2}\right)^{-1}\left(1 - \frac{ab^2}{R^2}\right)^{-2}. \tag{4.10}$$

The ratio $\lambda = \dfrac{\bar{\varrho}_b}{\varrho_0}$ can be easily obtained. It is given by

$$\lambda = \tfrac{1}{3}\left(3 - \frac{ab^2}{2R^2}\right)\left(1 - \frac{ab^2}{R^2}\right)^{-2} - \frac{\beta^2\left(\dfrac{b^2}{R^2}\right)}{6(1-a)}. \tag{4.11}$$

The positive $\beta$ satisfying the equation (3.9) is given by

$$\tfrac{1}{2}\beta\left(\frac{b^2}{R^2}\right) = \left[\left(1 - \frac{b^2}{R^2}\right)\left(1 - \frac{ab^2}{R^2}\right)^{-1} - \frac{(a-1)b^2}{2R^2}\left(1 - \frac{ab^2}{2R^2}\right)\left(1 - \frac{ab^2}{2R^2}\right)^{-2}\right]^{1/2}$$
$$- \left(1 - \frac{b^2}{R^2}\right)^{1/2}\left(1 - \frac{ab^2}{R^2}\right)^{-1/2}.$$

If $a$ and $\left(\dfrac{b}{R}\right)$ are given, we can determine $\beta$, $\lambda$ and $G$ from (4.12), (4.11) and (4.10) respectively. For numerical study we have selected the value $a = -2$. For this value of $a$ and for various values of $\dfrac{b}{R}$ satisfying $0 < \dfrac{b}{R} < 1$, the values of $\beta$, $\lambda$ and $G$ are tabulated in the following Table.

From the Table it is clear that $\beta$, $\lambda$ and $G$ are decreasing functions of $\dfrac{b}{R}$. Also $\lambda$ is always less than 1. This in turn may mean that the density at the boundary is less than the

TABLE

| b/R | $\beta$ | $\lambda$ | G |
|---|---|---|---|
| 0.02500 | 1.49823 | 0.99763 | 28734.12036 |
| 0.05000 | 1.49301 | 0.99059 | 7134.47705 |
| 0.07500 | 1.48440 | 0.97902 | 3135.06421 |
| 0.10000 | 1.47257 | 0.96317 | 1735.87085 |
| 0.12500 | 1.45772 | 0.94337 | 1088.88155 |
| 0.15000 | 1.44012 | 0.92001 | 738.07833 |
| 0.17500 | 1.42006 | 0.89353 | 527.19579 |
| 0.20000 | 1.39785 | 0.86443 | 390.94651 |
| 0.22500 | 1.37383 | 0.83318 | 298.12830 |
| 0.25000 | 1.34833 | 0.80027 | 232.29630 |
| 0.27500 | 1.32169 | 0.76618 | 184.11110 |
| 0.30000 | 1.29422 | 0.73135 | 147.94599 |
| 0.32500 | 1.26622 | 0.69619 | 120.24470 |
| 0.35000 | 1.23799 | 0.66106 | 98.66859 |
| 0.37500 | 1.20977 | 0.62628 | 81.62760 |
| 0.40000 | 1.18181 | 0.59212 | 68.00964 |
| 0.42500 | 1.15429 | 0.55879 | 57.01775 |
| 0.45000 | 1.12741 | 0.52647 | 48.06874 |
| 0.47500 | 1.10132 | 0.49531 | 40.72803 |
| 0.50000 | 1.07615 | 0.46540 | 34.66667 |
| 0.52500 | 1.05202 | 0.43680 | 29.63215 |
| 0.55000 | 1.02902 | 0.40954 | 25.42838 |
| 0.57500 | 1.00725 | 0.38365 | 21.90137 |
| 0.60000 | 0.98677 | 0.35911 | 18.92915 |
| 0.62500 | 0.96767 | 0.33589 | 16.41426 |
| 0.65000 | 0.95001 | 0.31396 | 14.27826 |
| 0.67500 | 0.93385 | 0.29326 | 12.45763 |
| 0.70000 | 0.91931 | 0.27373 | 10.90059 |
| 0.72500 | 0.90645 | 0.25531 | 9.56476 |
| 0.75000 | 0.89541 | 0.23792 | 8.41522 |
| 0.77500 | 0.88634 | 0.22148 | 7.42313 |
| 0.80000 | 0.87947 | 0.20590 | 6.56452 |
| 0.82500 | 0.87510 | 0.19109 | 5.81944 |
| 0.85000 | 0.87368 | 0.17693 | 5.17119 |
| 0.87500 | 0.87586 | 0.16328 | 4.60577 |
| 0.90000 | 0.88275 | 0.14995 | 4.11139 |
| 0.92500 | 0.89627 | 0.13665 | 3.67810 |
| 0.95000 | 0.92035 | 0.12286 | 3.29747 |

central density. This is a desirable feature of our solution. As $a = -2$, we have $2(1-a) = 6$. From the Table it is clear that the inequalities (4.8) and (4.9) are satisfied. Thus we have a physically viable model of a charged fluid sphere in the EC theory.

We have done the numerical study of various parameters for $a = -2$. But the method is quite general and it can be applied to any negative value of $a$.

The physical features and the numerical estimates of various parameters of the solutions discussed in cases II and III can also be obtained on the similar lines. We shall not give these details here.

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