A NON-PERTURBATIVE CONTRIBUTION TO THE VACUUM ENERGY IN SUPERSYMMETRIC QCD

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It is shown that instanton-anti-instanton induce a negative infrared finite vacuum energy in massless supersymmetric QCD. In the massive theory the same field configuration induces no vacuum energy because its classical action diverges due to the contribution of the mass term. Only if the scalar field is classically zero (in a background of an instanton-anti-instanton) a vacuum energy is found in the massive theory. However, it is negative and infrared divergent.

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Supersymmetry breaking was studied extensively in the past few years [1]. Perturbatively such a breaking is not possible due to the cancellation between bosons' and fermions' contributions to the vacuum energy. Non-perturbative effects were also studied [2]. In particular instanton's contribution to the vacuum energy is found to be zero in SYM and in SQCD due to the fermionic zero modes of the relevant Dirac operator in the topologically non-trivial background [2]. Instanton-anti-instanton configuration, however, has a zero topological charge and there are no exact zero modes. As a result instanton-anti-instanton contribution to the vacuum energy may not vanish. Indeed, it was shown in previous publications [3, 4] that quantum fluctuations in a background of an instanton-anti-instanton induce negative vacuum energy, which may signify an explicit breaking of supersymmetry if it is not cancelled by other non-perturbative effects. The induced vacuum energy, though, is infrared divergent and a cutoff of the instanton (anti-instanton) size was introduced to define the integrals. This by itself may be the source of supersymmetry breaking found, because the bosonic and fermionic zero modes form a supermultiplet [5]. This structure is spoiled by cutting off the instanton size, thus supersymmetry breaking might have been introduced by hand.

In the following we will analyse a theory where such a cutoff is not needed. We study the contribution of an instanton-anti-instanton to the path integral in massless SQCD.

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In this theory the matter supermultiplet contains a scalar field which is classically non--vanishing. Its contribution to the classical action in a background of an instanton (or anti-instanton) introduces a Gaussian factor which makes the integration over the instanton (or anti-instanton) size finite. Quantum fluctuations around this configuration have a positive contribution to the path integral if functional integration over Weyl spinors in Euclidean space is taken to be the square root of the determinant of the associated Dirac operator. The induced vacuum energy is then infrared finite and negative. In the massive theory, on the other hand, the same field configuration induces no vacuum energy because its classical action diverges due to the contribution of the mass term. The configuration $\Phi_{\text{cl}} = 0$ has a finite action; however, its contribution to the vacuum energy is infrared divergent. Thus only if a finite action configuration, whose contribution to the path integral is infrared finite is found, would it be possible to analyse unambiguously the breaking of supersymmetry in the massive theory.

To be more specific we work with an SU(2) supersymmetric model, which contains one matter and one anti-matter supermultiplets transforming under the fundamental representation of the gauge group. The Lagrangian in Euclidean space-time can be written as

$$\mathscr{L}_{\text{E}} = \mathscr{L}_{\text{SYM}} + \mathscr{L}_{\text{matter}}, \tag{1}$$

where $\mathscr{L}_{\text{SYM}}$ is the super-Yang-Mills Lagrangian given in the Wess-Zumino gauge by

$$\mathscr{L}_{\text{SYM}} = \tfrac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\lambda}^a i D_\mu \bar{\Sigma}_\mu \lambda^a, \tag{1a}$$

with $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc} A^b_\mu A^c_\nu$, $D^{ac}_\mu = \delta^{ac} + g\varepsilon^{abc} A^b_\mu$. A^a_μ are the vector potentials and λ^a are Weyl spinors. They are expressed in Euclideanised Weyl basis with Dirac matrices being

$$\gamma_\mu = \begin{pmatrix} 0 & \Sigma_\mu \\ \bar{\Sigma}_\mu & 0 \end{pmatrix}, \quad \Sigma_\mu = \bar{\Sigma}^\dagger_\mu = (i\sigma_i, 1) \quad \text{and} \quad \operatorname{Tr} \Sigma_\mu \bar{\Sigma}_\nu = 2\delta_{\mu\nu}.$$

$\mathscr{L}_{\text{matter}}$ is the matter field Lagrangian

$$\begin{aligned} \mathscr{L}_{\text{matter}} = {} & (\mathscr{D}_\mu \Phi_1)^\dagger \mathscr{D}_\mu \Phi_1 + (\mathscr{D}^*_\mu \Phi_2)^\dagger \mathscr{D}^*_\mu \Phi_2 + \bar{\psi}^{\text{T}}_1 i \mathscr{D}_\mu \bar{\Sigma}_\mu \psi_1 \\ & + \psi^{\text{T}}_2 i \mathscr{D}^*_\mu \Sigma_\mu \bar{\psi}_2 + \frac{ig}{\sqrt{2}} (\Phi^\dagger_1 \tau^a \lambda^a \psi_1 + \Phi^{\text{T}}_2 \tau^a \lambda^a \psi_2) \\ & + \frac{g^2}{32} (\Phi^\dagger_1 \tau^a \Phi_1 - \Phi^{\text{T}}_2 \tau^a \Phi^*_2)^2 + \text{h.c.} \end{aligned} \tag{1b}$$

such that $\{\psi_i, \Phi_i\}$ $(i = 1, 2)$ form the matter supermultiplet, $\mathscr{D}_\mu = \partial_\mu + igA^a_\mu \dfrac{\tau^a}{2}$ and $\dfrac{\tau^a}{2}$ are the SU(2) generators in the fundamental representation.

Classically, the vacuum in Minkowski space is given by $\psi_1 = \psi_2 = \lambda = F_{\mu\nu} = 0$ Φ_1 parallel to Φ^*_2 in group space and $\mathscr{D}_\mu \Phi_1 = 0$ $(\mathscr{D}^*_\mu \Phi_2 = 0)$. This last equation has an integrability condition $F_{\mu\nu}\Phi_1 = 0$ which is trivially satisfied in the vacuum. The solution is then $\Phi_1 = P \exp(-ig \int A_\mu dx_\mu) \begin{pmatrix} v \\ 0 \end{pmatrix}$, where the integral is along a path from $-\infty$ to

x and v is a constant. The integrability condition guarantees the path independence of the solution. We compactify the Euclidean 3-space into S_3 in which case the vacuum configurations are given by $A_\mu = \frac{i}{g} U^{-1}(x)\partial_\mu U(x)$ where $U(x) \in \mathrm{SU}(2)$ and $U(x) \xrightarrow[|\vec{x}|\to\infty]{} 1$, and $\Phi_1(x) = \Phi_2^*(x) = U(x)\begin{pmatrix} v \\ 0 \end{pmatrix}$. $U(x)$ defines the set of maps $S_3 \to \mathrm{SU}(2)$ classified by $\Pi_3(\mathrm{SU}(2)) = \boldsymbol{Z}$. The same maps $(\Pi_3(S_3))$ classify also the vacuum configuration of the scalar fields. Thus the scalar fields and the gauge potential are classified by the Pontryagin index, n [6]. The vacuum is given by $|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{in\theta}|n\rangle$ and quantum mechanical tunnelling between vacua differing by one unit of topological charge is provided by the single instanton or anti-instanton

$$A_\mu^{\mathrm{I}a} = \frac{2}{g}\,\frac{\eta_{a\mu\nu}(x-x_1)_\nu}{(x-x_1)^2+\varrho_1^2}; \qquad A_\mu^{\bar{\mathrm{I}}a} = \frac{2}{g}\,\frac{\bar{\eta}_{a\mu\nu}(x-x_2)_\nu}{(x-x_2)^2+\varrho_2^2} \tag{2}$$

and the scalar field configurations [2]

$$\Phi_{1\mathrm{I}} = \Phi_{2\mathrm{I}}^* = \frac{(x-x_1)_\mu\bar{\tau}_\mu}{[(x-x_1)^2+\varrho_1^2]^{1/2}}\begin{pmatrix} v \\ 0 \end{pmatrix}; \qquad \Phi_{1\bar{\mathrm{I}}} = \Phi_{2\bar{\mathrm{I}}}^* = \frac{(x_1-x_2)_\mu\tau_\mu}{[(x-x_2)^2+\varrho_2^2]^{1/2}}\begin{pmatrix} v \\ 0 \end{pmatrix} \tag{3}$$

which satisfy the field equation $\mathscr{D}^2\Phi_1 = 0$ ($\mathscr{D}^2\Phi_2^* = 0$) in the appropriate background. In the above $\eta_{a\mu\nu}$, $\bar{\eta}_{a\mu\nu}$ are the 't Hooft symbols [2], x_1, x_2, ϱ_1, ϱ_2 are the locations and sizes of the instanton and anti-instanton respectively and $\tau_\mu^\dagger = \bar{\tau}_\mu(-i\tau_i, 1)$.

As it was mentioned previously quantum mechanical tunnelling by these field configurations in a supersymmetric model is completely suppressed due to the zero modes of the Dirac operator, $i\gamma_\mu D_\mu$, in the above background. In the absence of the Yukawa coupling in (1b) we have six left-handed zero modes in a background of an instanton: 4 for the gluino and 2 for the matter field

$$(\lambda_{\mathrm{ss}}^{(+)})_\alpha^a = \frac{\sqrt{2}}{\pi}\,\frac{\varrho_1^{5/2}}{[(x-x_1)^2+\varrho_1^2]^2}\,(\sigma^a)_\alpha{}^\beta u_\beta^{(+)},$$

$$(\lambda_{\mathrm{sc}}^{(+)})_\alpha^a = \frac{1}{\pi}\,\frac{\varrho_1^{3/2}}{[(x-x_1)^2+\varrho_1^2]}\,(\sigma^a)_\alpha{}^\beta(\Sigma_\mu(x-x_1)_\mu)_{\beta\dot{\delta}}\bar{v}^{(+)\dot{\delta}},$$

$$(\psi_1^{(+)})_{\kappa\xi} = (i\tau_2\psi_2^{(+)})_{\kappa\xi} = \frac{1}{\pi\sqrt{2}}\,\frac{\varrho_1^{3/2}}{[(x_1-x_2)^2+\varrho_1^2]^{3/2}}\,\varepsilon_{\kappa\xi}, \tag{4}$$

where $\kappa = 1, 2$, $\xi = 1, 2$ are spinor and color indices respectively. For the anti-instanton we have six right-handed zero modes

$$(\bar{\lambda}_{\mathrm{ss}}^{(-)})^{a\dot{\alpha}} = \frac{\sqrt{2}}{\pi}\,\frac{\varrho_2^{5/2}}{[(x-x_2)^2+\varrho_2^2]^2}\,(\sigma^a)^{\dot{\alpha}}{}_{\dot{\beta}}\bar{v}^{(-)\dot{\beta}},$$

$$(\bar{\lambda}^{(-)}_{\mathrm{sc}})^{a\dot{\alpha}} = \frac{1}{\pi} \frac{\varrho_2^{3/2}}{[(x-x_2)^2+\varrho_2^2]^2} (\bar{\Sigma}_\mu(x-x_2)_\mu)^{\dot{\alpha}\beta}(\sigma^a)_\beta{}^\delta u_\delta^{(-)},$$

$$(\bar{\psi}_1^{(-)})^{\kappa\dot{\xi}} = (i\tau_2\bar{\psi}_2^{(-)})^{\kappa\dot{\xi}} = \frac{1}{\pi\sqrt{2}} \frac{\varrho_2^{3/2}}{[(x-x_2)^2+\varrho_2^2]^{3/2}} \varepsilon^{\kappa\dot{\xi}}. \tag{5}$$

Here $u^{(\pm)}$, $v^{(\pm)}$ are unit vectors given by either (1, 0) or (0, 1).

When the Yukawa coupling is turned on λ_{sc}, ψ_1, $i\tau_2\psi_2$ are no longer zero modes and their contribution to the action becomes

$$\int d^4x \psi_2^{\mathrm{T}}\tau^a\lambda^a_{\mathrm{sc}_1}\Phi_2^* = \int d^4x \Phi_1^\dagger\tau^a\lambda_{\mathrm{sc}_2}\psi_1 = \frac{\varrho_1 v}{\sqrt{2}} \tag{6}$$

for the instanton and a similar contribution for the anti-instanton. Thus $\{\lambda_{\mathrm{sc}}, \bar{\psi}\}$ combine together to form a massive Dirac spinor. (We denote the pair $\{\psi_1, i\tau_2\psi_2\}$ by ψ. Together they have only two spin components). λ_{ss}, on the other hand, stays as a zero mode because $\int d^4x\Phi^\dagger\tau^a\lambda^a_{\mathrm{ss}}\psi = 0$, since the integrand is odd under space-time reflections. As a result quantum mechanical tunnelling is suppressed even when the Yukawa coupling is turned on.

In the absence of tunnelling in a background of one instanton or one anti-instanton we are led to consider the tunnelling in a background of an instanton-anti-instanton. Without loss of generality we take the distance between the instanton and the anti-instanton in the time-like direction, $\Delta_\mu = (x_2-x_1)_\mu = \Delta\delta_{\mu 4}$. Later we will integrate over its direction. Then the instanton-anti-instanton configuration is given by

$$A_\mu^{\mathrm{I\bar{I}}} = A_\mu^{\mathrm{I}}\theta(R-t)+A_\mu^{\bar{\mathrm{I}}}\theta(t-R), \tag{7}$$

where $R_\mu = \frac{1}{2}(x_1+x_2)_\mu = R\delta_{\mu 4}$ and generality is not lost by taking the locations to be such that $\vec{x}_1 = \vec{x}_2 = 0$. We later integrate over R_μ as well. In a similar way we can write the expression for the classical scalar fields associated with the instanton-anti-instanton configuration in (7).

Quantum fluctuations around this background yield one over the square root of each bosonic determinant. For the Weyl fermions we first double the number of degrees of freedom to generate Dirac fermions. We then define the functional integral over the Weyl spinors as the square root of the functional integral over the Dirac fermions, thus getting the square root of the determinant of the Dirac operator. The fermionic determinant has 12 approximate zero modes as listed in (4) and (5). In the limit of infinite separation 4 of them ($\lambda_{\mathrm{ss}}^{(+)}$, $\bar{\lambda}_{\mathrm{ss}}^{(-)}$) become exact zero modes. The bosonic determinant has 16 approximate zero modes associated with the invariance under translations, dilatations and group orientations of the instanton and the anti-instanton. These are factored out and treated by the collective coordinate method. Factoring out also the approximate fermionic zero modes we finally get the square root of the ratio of the non-zero modes fermionic over the bosonic determinants in the background of an instanton-anti-instanton. In the approximation of far separation this determinantal factor can be approximated by the product of the determinantal factors of the instanton and the anti-instanton, and each is equal to one

[2]. As a result we approximate the path integral by

$$\langle\theta|e^{-HT}|\theta'\rangle_{I\bar{I}} \simeq \delta(\theta-\theta')\frac{1}{(8\pi^4)^2}\int d^4x_1\frac{d\varrho_1}{\varrho_1^5}d^4x_2\frac{d\varrho_2}{\varrho_2^5}d\Omega d\Omega_R$$

$$\times\left(\frac{8\pi^2}{g^2(\varrho_1)}\right)^4\left(\frac{8\pi^2}{g^2(\varrho_2)}\right)^4 K(x_1-x_2,\varrho_1,\varrho_2,\Omega_R)e^{-S_E}. \tag{8}$$

In Eq. (8) we integrate over the locations x_1, x_2 the sizes ϱ_1, ϱ_2 and the group orientations Ω,Ω_R of the instanton and anti-instanton respectively. Ω_R is the relative orientation of the anti-instanton compared to the instanton and $K(x_1-x_2, \varrho_1, \varrho_2, \Omega_R)$ is the square root of the fermionic determinant evaluated in the subspace of the fermionic zero modes listed in (4) and (5) with σ^a in Eq. (5) replaced by $\sigma^a_R = R^a_b\sigma^b$ and with $\varepsilon^{\kappa\dot{\xi}}$ being replaced by $(U(R)\varepsilon)^{\kappa\dot{\xi}}$. R is the rotation matrix of the relative orientation and $U(R)$ is the associated SU(2) representation. S_E is the Euclidean action

$$S_E = \frac{4\pi^2}{g^2(\varrho_1)}+\frac{4\pi^2}{g^2(\varrho_2)}+S_{int}+4\pi^2v^2(\varrho_1^2+\varrho_2^2), \tag{9}$$

where the last term is the contribution of the scalar fields and it is this contribution which eventually makes the ϱ integration infrared finite as was pointed out in Ref. [2]. S_{int} is the interaction action between the instanton and the anti-instanton.

To calculate K we double the number of fermions to get Dirac fermions and calculate the determinant of the Dirac operator (including the Yukawa couplings) in the subspace of zero modes. Therefore

$$K^2 = \det\begin{bmatrix} 0 & 0 & 0 & A & B & 0 \\ 0 & 0 & K_2^\dagger & C & D & 0 \\ 0 & K_2^\dagger & 0 & 0 & 0 & 0 \\ A^\dagger & C^\dagger & 0 & 0 & 0 & 0 \\ B^\dagger & D^\dagger & 0 & 0 & 0 & K_1 \\ 0 & 0 & 0 & 0 & K_1 & 0 \end{bmatrix}, \tag{10}$$

where the entries are all 2×2 matrices. In (10) we have picked up the dominant contribution to the determinant for large Δ. Thus the contribution of some of the matrix elements to the determinant is zero only up to the leading order in Δ^{-1}. Using the fact that $\lambda^{(+)}_{ss}$, $\lambda^{(+)}_{sc}$ ($\bar{\lambda}^{(-)}_{ss}$, $\bar{\lambda}^{(-)}_{sc}$) are exact zero modes in a background of an instanton (anti-instanton) we find that

$$\int d^4x\bar{\lambda}^{(-)}iD_\mu\bar{\Sigma}_\mu\lambda^{(+)} = i\int d^3x\bar{\lambda}^{(-)}\left(-\frac{\Delta}{2},\vec{x}\right)\lambda^{(+)}\left(\frac{\Delta}{2},\vec{x}\right), \tag{11}$$

where D_μ is the covariant derivative in the background (7). Then

$$A = \int d^4x\bar{\lambda}^{(-)}_{ss}iD_\mu\bar{\Sigma}_\mu\lambda^{(+)}_{ss} = 2ia\varrho_1^{5/2}\varrho_2^{5/2}\sigma_c\sigma^c_R, \tag{12a}$$

$$B = \int d^4x \bar{\lambda}_{ss}^{(-)} iD_\mu \bar{\Sigma}_\mu \lambda_{sc}^{(+)} = \frac{\sqrt{2}}{2} ia\Delta \varrho_1^{3/2} \varrho_2^{5/2} \sigma_c \sigma_R^c, \tag{12b}$$

$$C = \int d^4x \bar{\lambda}_{sc}^{(-)} iD_\mu \bar{\Sigma}_\mu \lambda_{ss}^{(+)} = -\frac{\sqrt{2}}{2} ia\Delta \varrho_1^{5/2} \varrho_2^{3/2} \sigma_c \sigma_R^c, \tag{12c}$$

$$D = \int d^4x \bar{\lambda}_{sc}^{(-)} iD_\mu \bar{\Sigma}_\mu \lambda_{sc}^{(+)} = i\left(b - a\frac{\Delta^2}{4}\right) \varrho_1^{3/2} \varrho_2^{3/2} \sigma_c \sigma_R^c, \tag{12d}$$

$$K_i = \frac{g\varrho_i v}{2}, \qquad i = 1, 2, \tag{12e}$$

with

$$a; b = \frac{1}{\pi} \int_0^\infty dx \frac{x^2; x^4}{\left(x^2 + \frac{\Delta^2}{4} + \varrho_1^2\right)^2 \left(x^2 + \frac{\Delta^2}{4} + \varrho_2^2\right)^2} \tag{13}$$

and only the leading order terms in Δ^{-1} should be kept for widely separated instanton--anti-instanton.

Expressing the relative orientation in terms of a unimodular four vector u_μ we find

$$\det(\sigma_a \sigma_R^a) = 1 + 8u_4^2$$

then

$$\int d\Omega_R K = 384\pi^2 (\varrho_1 \varrho_2)^5 a^2 \left(\frac{g\varrho_1 v}{2}\right)^2 \left(\frac{g\varrho_2 v}{2}\right)^2. \tag{14}$$

Substituting (14) in (8) and integrating over $d\Omega d^4R$ we get a factor $2\pi^2 VT$ (space-time volume). We are then left with the integrations over ϱ_1, ϱ_2, and Δ. To account for the Δ integration we have to know the interaction action. This was calculated in Ref. [7] to yield

$$S_{int} = \begin{cases} 4 \ln \dfrac{\varrho_1 \varrho_2}{\Delta^2} & \text{for} \quad \Delta \ll \varrho_1, \varrho_2 \\ \dfrac{32\pi^2}{g^2} \left(\dfrac{\varrho_1 \varrho_2}{\Delta^2 + \varrho_1^2 + \varrho_2^2}\right)^2 (3 - 4u_4^2) & \text{for} \quad \Delta \gg \varrho_1, \varrho_2. \end{cases}$$

The effect of S_{int} is to suppress the contribution of configurations which are not widely separated. Thus we may ignore the interaction action and integrate from a minimal distance Δ_0 up to ∞. We take $\Delta_0^2 = \chi(\varrho_1^2 + \varrho_2^2)$ where χ is some number which we estimate in the following way:

$$\exp\left(-\frac{24\pi^2}{g^2(\chi+1)^2}\right) \leqslant \exp(-S_{int}) \leqslant \exp\left(\frac{8\pi^2}{g^2(\chi+1)^2}\right).$$

Thus if we take $\frac{8\pi^2}{g^2(\chi+1)^2} \simeq 1 \ll \frac{8\pi^2}{g^2}$, the interaction action can be ignored compared to the total action $\left(\text{which is } \frac{16\pi^2}{g^2}\right)$. Using this we choose χ to be

$$(\chi+1)^2 = \frac{8\pi^2}{g^2}. \tag{15}$$

Indeed, with this choice, if the interaction action is ignored, the integrals over $\varrho_1, \varrho_2, \Delta$ produce a result which is very close to the bound found in Ref. [4] for the vacuum energy in SYM theory. In view of the fact that the integrals we have here are very similar to the ones appearing there, the choice (15) for the minimal distance should be reasonably good.

From now on we have fairly simple integrals to perform. We only need to use the renormalization group equation

$$\frac{8\pi^2}{g^2(\varrho)} = \frac{8\pi^2}{g^2(\mu)} - 5 \ln \varrho\mu. \tag{16}$$

Substituting (14)–(16) in (8) and keeping only the leading order terms in the coupling constant we get

$$\langle\theta|e^{-HT}|\theta\rangle_{\text{I}\bar{\text{I}}} \gtrsim \frac{6VT}{35(4\pi^2)^4}\left(\frac{8\pi^2}{g^2(\mu)}\right)^{9/2} \frac{\mu^{10}}{v^6} \exp\left(-\frac{16\pi^2}{g^2(\mu)}\right). \tag{17}$$

Or using the renormalization group invariant scale

$$\Lambda_{\text{QCD}}^{10} = \mu^{10} \exp\left(\frac{-16\pi^2}{g^2(\mu)}\right), \tag{18}$$

we get for the leading order contribution to the vacuum energy

$$\frac{E(\theta)}{V} \lesssim -\frac{6}{35(4\pi^2)^4}\left(\frac{8\pi^2}{g^2(v)}\right)^{9/2}\left(\frac{\Lambda_{\text{QCD}}}{v}\right)^6 \Lambda_{\text{QCD}}^4. \tag{19}$$

We would like to comment now on the mass dependence of the result. If we start with a massive theory, we have the following mass term in the Lagrangian

$$\mathcal{L}_{\text{mass}} = \Phi_1^\dagger m^2 \Phi_1 + \Phi_2^\dagger m^2 \Phi_2 + \psi_1^T m \psi_2 + \bar{\psi}_1^T m \bar{\psi}_2. \tag{20}$$

For this theory the vacuum state in Minkowski space is uniquely determined by $\Phi_1 = \Phi_2 = \psi_1 = \psi_2 = \lambda = 0$, $A_\mu = \frac{i}{g} U^{-1}(x)\partial_\mu U(x)$. Thus the Pontryagin index labels the gauge potentials only. In Euclidean space the solutions (3) in a background of an instanton or anti-instanton do not have a finite action as the mass term of the classical configuration diverges. As a result tunnelling with this configuration is suppressed even in the background of an instanton-anti-instanton, and the vacuum energy stays at zero. One may use instead the configuration $\Phi_{cl} = 0$ in the background of an instanton-anti-instanton. The determinant to be calculated is then similar to the one in (10) with mass terms appropriately inserted. Since the contribution of the scalar fields to the classical action is zero, an infrared cutoff, ϱ_c, over the ϱ_1, ϱ_2 integrations is needed. The resulting vacuum energy is then similar to (19) with $m^2\varrho_c^8$ replacing v^{-6}. Otherwise one may seek a solution to the Euclidean equation $\mathscr{D}^2\Phi_1 = m^2\Phi_1$ ($\Phi_1 = \Phi_2^*$) which has a finite action. If such a solution is found and its contribution to the action is as in (9), then it would be possible to find whether instanton--anti-instantons induce a vacuum energy in this massive theory. It is expected that for such a solution the result (if non-zero) will be infrared finite.

We have thus demonstrated that instanton-anti-instanton induce vacuum energy in massless SQCD. It is negative and infrared finite. The sign of the vacuum energy is fixed by the fact that the functional integration over Weyl spinors is given by the square root of the determinant of the Dirac operator. Dividing by the functional integral in a background of the vacuum (which is 1 in a supersymmetric theory), we get a positive contribution to the functional integral, thus making the vacuum energy negative. It is infrared finite due to the contribution of the scalar fields to the classical action, which is $4\pi^2v^2(\varrho_1^2+\varrho_2^2)$, and which makes the integration over ϱ_1, ϱ_2 finite. The density of the vacuum energy is proportional to Λ_{QCD}^4, and $\frac{\Lambda_{QCD}}{v}$ could be used as an expansion parameter if v is large enough.

In the massive theory the same field configuration induces no vacuum energy as the classical action is not finite due to the divergence of the mass term. Only if a finite action field configuration whose contribution to the path integral is infrared finite is found, would it be possible to check unambiguously whether supersymmetry is broken or genuinely preserved at the quantum level. In the massless theory, on the other hand, instanton-anti--instanton induce an explicit breaking of supersymmetry.

REFERENCES

[1] E. Witten, *Nucl. Phys.* **B188**, 513 (1981); **B202**, 253 (1982); I. Affleck, M. Dine, N. Seiberg, *Nucl. Phys.* **B241**, 493 (1984) and references therein; S. Kalara, S. Raby, *Phys. Lett.* **158B**, 131 (1985); D. Amati, G. C. Rossi, G. Veneziano, *Nucl. Phys.* **B249**, 1 (1985) and references therein; A. I. Vainshtein, V. I. Zakharov, *Pis'ma Zh. Tekh. Fiz.* **35**, 258 (1982); V. A. Novikov, M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, *Nucl. Phys.* **B223**, 445 (1983); **B229**, 381, 407 (1983).

[2] G. 't Hooft, *Phys. Rev.* **D14**, 3432 (1976).

[3] R. K. Kaul, L. Mizrachi, CERN preprint TH-3816 (1984).

[4] L. Mizrachi, *Phys. Lett.* **175B**, 325 (1986).

[5] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, V. B. Voloshin, V. I. Zakharov, *Nucl. Phys.* **B229**, 394 (1983).

[6] G. Woo, *Phys. Rev.* **D16**, 1014 (1977); *J. Math. Phys.* **18**, 1756 (1977).

[7] D. I. Dyakonov, V. Yu. Petrov, *Nucl. Phys.* **B245**, 259 (1984).