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ROLE OF THE RECOIL AND CENTRE-OF-MASS CORRECTIONS TO THE STATIC APPROXIMATION IN THE BAG MODEL*

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The internal consistency of various methods of improving the static spherical cavity approximation to the MIT Bag Model is investigated by studying electromagnetic current matrix elements between boosted bag and the boosted hadron states. In the case when the bag state is identified with the hadron bound in an external potential the consistency requirement fixes parameters of this potential allowing to calculate the improved values of the electromagnetic properties of nucleons.

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1. *Introduction*

It is a common belief now that QCD is the correct theory of strong interactions. Unfortunately, this theory is quantitatively effective only in the domain of asymptotic freedom, and even the simplest questions about static properties of hadrons are still beyond its predictive power. For this reason various models of hadron structure do not loose their significance as it would undoubtfully happen if QCD could cope with the bound state problem. Models of composite hadrons are at best "QCD inspired", but essentially they are just phenomenological. One of the most popular among them, especially in application to the light hadrons, is the MIT Bag Model [1, 2, 3].

Although formulated as a relativistically covariant classical field theory, it was reduced to something much less ambitious during the approximate quantization procedure, and under the name of Static Spherical Cavity Approximation (SSCA) is nothing more than a specific phenomenological model of quark confinement. In this approximation only the quark degrees of freedom are quantized. Quarks, described by the free Dirac equation, move

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independently in a certain region of space, or cavity, being confined to this region by an appropriate boundary conditions. The positive volume energy stabilizes the system. In the ground state, the cavity takes a spherical shape. These are the most essential ingredients of the model. It may be enriched by various additional terms in the expression for the energy of the bag, but for the purpose of our investigations it will be enough to keep only the volume energy and the kinetic energy terms.

2. *The wave packet formalism*

The bag state in the SSCA distinguishes the origin of the reference frame as the centre of the bag and is obviously not translationally invariant. As such, it can correspond to the hadron at rest, only as a rough approximation. Trying to improve this approximation one has to answer the question what hadron state corresponds to the bag state, if it is not hadron state of zero momentum.

In one of the first papers devoted to this problem, Donoghue and Johnson [4] proposed to identify the bag state $|B\rangle$ localized at the origin with a wave packet of the appropriate physical hadron:

$$|B\rangle = \int d^3p\phi(\vec{p})\,|\vec{p}\rangle. \tag{1}$$

They determine unknown physical form factors by equating matrix elements of the relevant quark current calculated for the bag state to the matrix element of the current calculated for the wave packet (1). For the magnetic moment they obtain for example,

$$\mu_B^{\text{static}} \cong \mu_{\text{proton}}\left(1-\tfrac{1}{2}\,\frac{\langle\vec{p}^2\rangle}{m^2}\right). \tag{2}$$

Solving this equation for μ_{proton}, they obtain the "CM corrected" result

$$\mu_{\text{proton}} \cong \mu_B^{\text{static}}\left(1+\tfrac{1}{2}\,\frac{\langle\vec{p}^2\rangle}{m^2}\right). \tag{3}$$

The wave function $\phi(\vec{p})$ enters through the expectation value of $\langle\vec{p}^2\rangle$ only. Independent of the function ϕ, the correction is obviously positive and its order of magnitude is determined by the size of the bag. From the point of view of phenomenology, the positive correction is just what is needed because, as it is well known, in the static approximation the MIT predictions for magnetic moments are below the experimental values.

This clear picture was called in question in the paper of Carlson and Chachkunashvili [5]. Making the same assumption that Donoghue and Johnson did, they took the formula (1) and inverted it to get

$$|\vec{p}\rangle = \frac{2E}{\phi(\vec{p})}\int d^3R e^{-i\vec{p}\vec{R}}|B(\vec{R})\rangle, \tag{4}$$

where $|B(\vec{R})\rangle$ is the bag state centered at $\vec{R}$ (see also Ref. [6]).

With the help of (4) the authors expressed directly the physical form factors as integrals of matrix elements calculated between shifted bag states. For the magnetic formfactor

G_M, for example, they obtained:

$$G_M(\vec{q}^{\,2})\left(\frac{i\vec{\sigma}\times\vec{q}}{2m}\right)_{\lambda'\lambda} = \frac{2E^2}{m\phi^2(\frac{1}{2}\vec{q})}\int d^3R\,d^3r\,e^{i\vec{q}\vec{R}}\langle B(\vec{R}-\tfrac{1}{2}\vec{r})\lambda'|\vec{J}(0)\,|B(\vec{R}+\tfrac{1}{2}\vec{r})\lambda\rangle. \tag{5}$$

Integrating numerically they found for the magnetic moment of the proton

$$\mu_{\text{proton}} \cong \mu_B^{\text{static}}(1-0.15), \tag{6}$$

so their correction is negative! In view of the discrepancy between (3) and (6), it seems that the basic assumption contained in (1) is not accurate enough. Actually, there are following physical arguments supporting this statement: (*i*) the bag state on the left hand side of (1) is an eigenstate of energy of quarks, while the right hand side is not, (*ii*) the bag state is localized in space all the time while the wave packet spreads out indefinitely, (*iii*) in a relativistic theory, the localized state in general can not be expanded into the positive energy one-particle free states only. In other words, the bag state in SSCA is not on the mass shell, while the state on the right hand side of (1) is. Having in mind the above limitations of the wave packet method of restoring translational invariance, some authors [7–11] tried to improve SSCA taking another point of view. Representative here are papers of Guichon [7] and Goldflam and Betz [8]. They take a generally correct relation:

$$\langle\vec{p}'|J_\mu(0)\,|\vec{p}\rangle = \frac{1}{(2\pi)^3\delta^3(0)}[\varrho(\vec{p}')\varrho(\vec{p})]^{1/2}\int d^3x\,e^{i\vec{q}\vec{x}}\langle m|U^\dagger(\vec{v}')J_\mu(\vec{x},0)U(\vec{v})\,|m\rangle, \tag{7}$$

where $\varrho(\vec{p})$ depends on the normalization of state $|\vec{p}\rangle$, and $|m\rangle$ is the physical hadron state of the four momentum $(m,\vec{0})$. The unitary operator $U(\vec{v})$ boosts hadron to the velocity $\vec{v} = \dfrac{\vec{p}}{m}$. In this formula they substitute for $|m\rangle$ the SSCA bag state

$$|m\rangle \to [(2\pi)^3\delta^3(0)]^{1/2}|B\rangle \tag{8}$$

getting the result which differs from the SSCA formula for the formfactors by the presence of $U(\vec{v})$ and $U^\dagger(\vec{v}')$. Corrections which follow are called "the recoil corrections". Numerically they are very small, in practice negligible. This led some authors [12] to the conclusion, that within the accuracy of the MIT bag model there is no need at all to correct the SSCA. However, the smallness of the "recoil corrections" has nothing to do with the question of how big are the CM corrections, and does not solve the puzzle of the discrepancy between the results of papers [4, 6] and [5].

3. The bound state approach to the CM motion and to the recoil problem

Another approach to the problem of correcting the SSCA was presented in our earlier papers [13] and [14]. We proposed there to replace the hadron wave packet in (1) by a physical hadron bound in some external suitably chosen potential:

$$|B\rangle \cong |h, \text{bound state}\rangle. \tag{9}$$

The purpose of the present paper is twofold. First of all we want to present quantitative arguments in favour of (9) in comparison with (1). Second, we would like to have a reliable criterion allowing to select from a given class the best external potential in which motion of the whole hadron is as similar as possible to the motion of CM occuring in the static bag state. The method of achieving these two goals is suggested by results of Guichon [7]. For his limited purpose of investigating recoil corrections he developed a method of boosting bag states, which is exact up to the terms linear in velocity.

Taking (9) and boosting it to an arbitrary (but small) velocity we have an equation

$$|B(\vec{v})\rangle = |h(\vec{v}), \text{bound}\rangle, \tag{10}$$

which is the identity with respect to $\vec{v}$.

Taking Fourier transform of matrix elements of currents we obtain

$$\int d^3x e^{i\vec{q}\vec{x}} \langle B(\vec{v}_1)|\, J(x)\, |B(\vec{v}_2)\rangle = \int d^3x e^{i\vec{q}\vec{x}} \langle h(\vec{v}_1), \text{bound}\, |J(x)|\, h(\vec{v}_2), \text{bound}\rangle \tag{11}$$

which is now an identity with respect to $\vec{v}_1$, $\vec{v}_2$ and $\vec{q}$. If the state $|h$, bound$\rangle$ is replaced by $|m, \vec{0}\rangle$ and if $\vec{q} = m\vec{v}_2 - m\vec{v}_1$ than (11) is exactly the same as used by Guichon to calculate his recoil corrections. If we put $\vec{v}_1 = \vec{v}_2 = 0$ and replace $|h$, bound$\rangle$ by the wave packet of Donoghue and Johnson, we essentialy reproduce their results. If we finally take $\vec{v}_1 = \vec{v}_2 = 0$, $\vec{q}$ arbitrary and keep $|h$, bound$\rangle$ we obtain equation used in our earlier papers [13, 14] to calculate the CM corrections.

Eq. (11) with arbitrary $\vec{v}_1$, $\vec{v}_2$ and $\vec{q}$, treated as an identity with respect to these variables provides a consistency check for either ansatz (1) or (9). It also defines corrected values of physical hadron parameters and, in addition, allows to fix arbitrary parameters of the external potential.

Expanding both sides of (11) up to the terms linear in $\vec{v}_1$, $\vec{v}_2$ and $\vec{q}$, we obtain for either proton or neutron, a set of three equations for coefficients multiplying linearly independent terms

$$\vec{\sigma} \times \vec{q}, \quad \vec{\sigma} \times (\vec{v}_1 - \vec{v}_2) \text{ and } \vec{v}_1 + \vec{v}_2,$$

which are present on both sides of the Eq. (11). The expansion of the left hand side is universal, but the expansion of the right hand side depends whether we take $|h$, bound$\rangle$ or $|h$, wave packet$\rangle$. Let us begin with an arbitrary wave packet defined by the profile $\phi(\vec{p})$. In this case we obtain the following equations

$$\begin{pmatrix} 1 \\ -2/3 \end{pmatrix} \mu_{\mathrm{B}}^{\text{static}} = \tfrac{1}{3} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \left[2 \left\langle \frac{m}{E} \right\rangle + \left\langle \left(\frac{m}{E} \right)^2 \right\rangle \right] + \tfrac{1}{3} \begin{pmatrix} \kappa_{\mathrm{p}} \\ \kappa_{\mathrm{n}} \end{pmatrix} \left[1 + \left\langle \frac{m}{E} \right\rangle + \left\langle \left(\frac{m}{E} \right)^2 \right\rangle \right], \tag{12, 12'}$$

$$\begin{pmatrix} 1 \\ -2/3 \end{pmatrix} \frac{2x_0 - 3}{6(x_0 - 1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tfrac{1}{3} \left[1 - \left\langle \frac{m}{E} \right\rangle \right], \tag{13, 13'}$$

$$1 = \tfrac{1}{3} + \tfrac{2}{3} \left\langle \left(\frac{m}{E} \right)^2 \right\rangle, \tag{14}$$

where $x_0 = 2.04\ldots$. For definitness we fix the relation between energy and radius of the bag with volume energy and kinetic energy of massless quarks only.

The bag energy is then identified with the average energy of the wave packet.

$$m\left\langle\frac{E}{m}\right\rangle = \frac{4x_0}{R_B}. \tag{15}$$

The (12) and (12′) are identical with those obtained in [15][1]. They are only slightly more general than found by Donoghue and Johnson as they do not rely on the expansion of $\left\langle\frac{M}{E}\right\rangle$ for small $\langle\vec{p}^2\rangle$. Corrections to μ which follow from these equations are definitely positive. Eq. (14) can only be satisfied, if the wave packet has zero width in which case we are back at the static approximation. Eq. (13′) is totaly wrong! The failure may be interpreted as the proof that some transformation properties of the bag state and of an arbitrary wave packet, are completely different.

Now let us examine (11) for a particular state $|h, \text{bound}\rangle$, namely the hadron bound in an infinite square well of the scalar potential. This is the ansatz we used before in [13] and [14]. Equations corresponding to (12), (13) and (14) are

$$\begin{pmatrix}1\\-2/3\end{pmatrix}\frac{4x_0-3}{6x_0(x_0-1)}(mR_B) = \begin{pmatrix}1\\0\end{pmatrix}\frac{Y(4\Omega+2Y-3)}{3[2\Omega(\Omega-1)+Y]} + \begin{pmatrix}\kappa_p\\\kappa_n\end{pmatrix}\frac{\Omega(4\Omega+2Y-3)}{3[2\Omega(\Omega-1)+Y]}, \tag{16, 16′}$$

$$\begin{pmatrix}1\\-2/3\end{pmatrix}\frac{2x_0-3}{6(x_0-1)} = \begin{pmatrix}1\\0\end{pmatrix}\frac{(\Omega-Y)(2\Omega-3)}{3[2\Omega(\Omega-1)+Y)]} + \begin{pmatrix}\kappa_p\\\kappa_n\end{pmatrix}\frac{2x^2(\Omega-1)}{3Y[2\Omega(\Omega-1)+Y]}, \tag{17, 17′}$$

$$1 = 1, \tag{18}$$

where $Y = mR_{\text{well}}$, $\Omega = \sqrt{x^2+Y^2}$ and x is the solution of the eigenvalue equation for the massive spin 1/2 nucleon in the infinite square well

$$\operatorname{tg} x = \frac{x}{1-\Omega-Y}. \tag{19}$$

In addition, there is an equation saying that the energy of the bag is equal to the energy of the nucleon in the infinite well

$$\frac{4x_0}{mR_B} = \frac{\Omega}{Y}. \tag{20}$$

From the formal point of view (15) and (16) form a set of 4 equations for 3 unknowns: κ_p, κ_n and $Y = mR_{\text{well}}$. This is the overdetermined set, so strictly speaking there might be no exact solution of these equations. We will look for the values of κ_p, κ_n and mR_{well} for which (16) and (17) are satisfied approximately. For numerical reasons we determine κ_p and κ_n from (16) and calculate right hand side of Eq. (17) for various R_{well}. For a given

[1] In this paper we have argued that for the relevant wave packet profiles it is allowed th substitute $\langle E/m\rangle \cong (\langle m/E\rangle)^{-1}$, etc.

TABLE I

Y	RHS of Eq. (17) LHS = 0.17	RHS of Eq. (17) LHS = −0.12	κ_p	κ_n	$m\sqrt{\langle r_p^2\rangle}$	mR_B
4.5	0.25	−0.20	2.07	−1.94	4.21	6.92
5	0.22	−0.17	2.11	−1.99	4.17	7.09
5.5	0.19	−0.15	2.14	−2.02	4.09	7.24
6	0.17	−0.13	2.16	−2.04	3.96	7.36
6.5	0.15	−0.12	2.18	−2.06	3.80	7.46
7	0.13	−0.10	2.20	−2.08	3.60	7.54

R_{well} or Y, values of κ_p and κ_n are determined through Eq. (16). Results are summarized in Table I, where κ_p and κ_n are in nuclear magnetons and their experimental values are: $\kappa_p = 1.793$, $\kappa_n = -1.913$, whereas $m\sqrt{\langle r_p^2\rangle} = 4.10 \pm 0.06$ [16]. For Y in the range $6 \div 6.5$ (17) and (17′) are quite well satisfied. The bag radius and the radius of the external well seems a little bit too high, but the physical electromagnetic radius of the proton calculated from the formula (21)

$$m^2\langle r_p^2\rangle = (mR_B)^2\,\frac{2x_0^3-2x_0^2+4x_0-3}{6x_0^2(x_0-1)}$$
$$-\frac{Y^2[4\Omega^4-4\Omega^3+2\Omega^2(4+3Y-2Y^2)-2\Omega(3+4Y-2Y^2)+3Y(3-2Y-2Y^2)]}{6x^2[2\Omega(\Omega-1)+Y]} \tag{21}$$
$$+\tfrac{3}{2}\kappa_p-\tfrac{1}{2}\,\frac{Y}{\Omega}\,(mR_B)\,\frac{4x_0-3}{2x_0(x_0-1)}+\tfrac{1}{2}\,\frac{4\Omega-3+2Y}{\Omega[2\Omega(\Omega-1)+Y]}$$

(see Ref. [14]) is not too large. In view of the simplicity of our assumptions and of the fact that we do not fit any of the parameters (except the proton mass which fixes the scale) to the experimental values, the overall agreement is quite resonable.

The assumption that the hadron moves in an infinite square well of the scalar potential is of course a drastic simplification. Even if it is true for quarks in the bag, the centre of mass of those quarks will feel constraints turning on gradually when successive constituents are reaching the boundary. Motivated by this intuition we studied the behaviour of the Eq. (11) for the hadron state on the right hand side being a bound state in the harmonic oscillator potential:

$$V = \gamma_0\lambda m^3 r^2. \tag{22}$$

For simplicity reasons we did not attempt to solve the Dirac equation in the harmonic oscillator scalar potential, but used a very convenient and quite precise variational method investigated by Franklin and Intemann [17]. The postulated wave function of the ground state is

$$\Psi = \begin{pmatrix} u \\ v \end{pmatrix}, \quad u = \left(\frac{b}{\sqrt{\pi}}\right)^{\frac{3}{2}}(1+\tfrac{3}{2}\gamma^2)^{-\frac{1}{2}}e^{-\frac{1}{2}b^2r^2}\chi, \quad v = i\gamma b\vec{\sigma}\cdot\vec{r}u. \tag{23}$$

The variational principle fixes relation between energy parameters of the wave function b, γ and that of the potential λ

$$\frac{m}{b} = \frac{1}{2\gamma}\,\frac{1-8\gamma^2+\frac{15}{4}\gamma^4}{1-\frac{5}{2}\gamma^2},$$

$$\frac{m}{E} = \frac{(1+\frac{3}{2}\gamma^2)(1-8\gamma^2+\frac{15}{4}\gamma^4)}{9\gamma^2(1-\frac{5}{2}\gamma^2)+(1-\frac{3}{2}\gamma^2)(1-8\gamma^2+\frac{15}{4}\gamma^4)}, \quad \lambda = \left(\frac{b}{m}\right)^3 \frac{\gamma}{1-\frac{5}{2}\gamma^2} \tag{24}$$

Eqs. (16) and (17) take now the form

$$\begin{pmatrix}1\\-2/3\end{pmatrix}\frac{4x_0-3}{6x_0(x_0-1)}(mR_B) = \begin{pmatrix}1\\0\end{pmatrix}\frac{1-8\gamma^2+\frac{15}{4}\gamma^4}{(1+\frac{3}{2}\gamma^2)(1-\frac{5}{2}\gamma^2)} + \begin{pmatrix}\kappa_p\\ \kappa_n\end{pmatrix}\frac{1+\frac{1}{2}\gamma^2}{1+\frac{3}{2}\gamma^2}, \qquad (25), (25')$$

$$\begin{pmatrix}1\\-2/3\end{pmatrix}\frac{2x_0-3}{6(x_0-1)} = \begin{pmatrix}1\\0\end{pmatrix}\left[\frac{2\gamma}{1+\frac{3}{2}\gamma^2}\,\frac{E}{b} - \frac{1-\frac{1}{2}\gamma^2}{1+\frac{3}{2}\gamma^2}\right] + \begin{pmatrix}\kappa_p\\ \kappa_n\end{pmatrix}\frac{\gamma}{1+\frac{3}{2}\gamma^2}\,\frac{b}{m}, \qquad (26), (26')$$

while the Eq. (18) remains the same.

In complete analogy with the previous case we have 3 unknown quantities and four equations. In Table II, which is analogous to the Table I one reads right-hand sides of (26) and (26′) for various values of γ^2. Again there exists a narrow range of γ^2 (0.028÷0.031) for which all four equations are satisfied with a good accuracy. The physical predictions for magnetic moments and electromagnetic radius of the proton in this range are closer to the experimental values than in the case of the square well, in accord with our intuition.

TABLE II

γ^2	λ	RHS of Eq. (26) LHS = 0.17	RHS of Eq. (26′) LSH = −0.12	κ_p	κ_n	$m\sqrt{\langle r_p^2\rangle}$	mR_B
0.015	0.002	0.08	−0.07	2.14	−2.03	2.84	7.43
0.02	0.005	0.11	−0.09	2.08	−1.97	3.51	7.17
0.025	0.008	0.14	−0.11	2.02	−1.91	3.77	6.92
0.03	0.014	0.16	−0.13	1.96	−1.85	3.85	6.66
0.035	0.021	0.19	−0.15	1.90	−1.79	3.84	6.40
0.04	0.032	0.22	−0.17	1.83	−1.72	3.79	6.13

4. *Conclusions*

The hypothesis that SSCA bag state corresponds to the hadron moving in the external suitable potential was tested by examining the dependence of the current matrix elements on arbitrary (although small) velocities of the states under consideration. The hypothesis passed the test quite satisfactoryly which is not the case when the bag corresponds to a wave packet of free hadron states. Once the test is completed, the free parameter of the external potential is fixed what allows to calculate electromagnetic properties of the proton and neutron corrected for the centre of mass motion. The numerical values are already satisfactory, but we believe that our method of improving SSCA may be applied satisfactorily for more sophisticated and more realistic models than our simplified bag model.

REFERENCES

[1] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, V. F. Weisskopf, *Phys. Rev.* **D9**, 3471 (1974)
[2] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, *Phys. Rev.* **D10**, 2599 (1974).
[3] T. De Grand, R. L. Jaffe, K. Johnson, J. Kiskis, *Phys. Rev.* **D12**, 2060 (1975).
[4] J. F. Donoghue, K. Johnson, *Phys. Rev.* **D21**, 1975 (1980).
[5] C. E. Carlson, M. Chachkunashvili, Nordita preprint ISSN 0106-2646 (unpublished).
[6] C. W. Wong, *Phys. Rev.* **D24**, 1416 (1981).
[7] P. A. M. Guichon, *Phys. Lett.* **129B**, 108 (1983).
[8] M. Betz, R. Goldflam, *Phys. Rev.* **D28**, 2848 (1983).
[9] L. Picek, D. Tadić, *Phys. Rev.* **D27**, 665 (1983).
[10] A. O. Gattone, W. Y. Hwang, *Phys. Rev.* **D31**, 2874 (1985).
[11] W. Y. Hwang D. J. Ernst, *Phys. Rev.* **D31** 2884 (1985).
[12] L. R. Dodd, A. G. Williams, A. W. Thomas, *Phys. Rev.* **D35**, 1040 (1987).
[13] A. Szymacha, *Phys. Lett.* **146B**, 350 (1984).
[14] A. Szymacha, S. Tatur, *Phys. Lett.* **146B**, 353 (1984).
[15] J. Bartelski, L. Mankiewicz, A. Szymacha, S. Tatur, *Phys. Rev.* **D29**, 1035 (1984).
[16] G. Simon et al., *Z. Naturforsch.* **35a**, 1 (1980).
[17] J. Franklin, R. L. Inteman, *Phys. Rev. Lett.* **54**, 2068 (1985).