NUCLEUS-NUCLEUS INTERACTIONS IN THE GLAUBER APPROACH*

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A short review of the method for calculation of elastic scattering and multiple production cross section in nucleus-nucleus interactions in Glauber approach is presented.

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Here we present a short summary of the method for calculation of nucleus-nucleus scattering amplitude in the Glauber approach (see Ref. [1] for details). The method developed allows one to perform summing contributions of all graphs. This problem for the case of elastic nucleus-nucleus scattering has been formulated in 1969 in papers [2, 3]. In Ref. [4] graph classification in loop numbers was proposed. The examples of graphs of various classes are given in Fig. 1. It was noticed that loop graphs contain a parameter

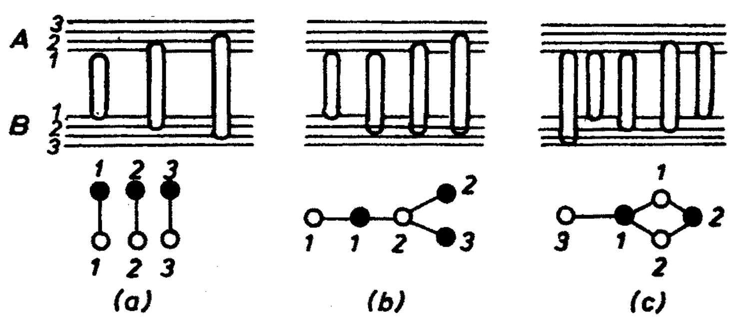

Fig. 1. Examples of Glauber diagrams: a) non-connected graphs corresponding to Czyż-Maximon approximation; b) tree graph; c) graph with a loop. Black dots correspond to nucleons of nucleus A and white dots to nucleons of B

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$\sigma^{el}_{NN}/\sigma^{\pm}_{NN} \approx 1/4$ and therefore a special class of tree graphs (without loops) was analyzed [4–6].

We have obtained the solution of the problem of summing all graphs including ones with loops as a power expansion in nuclear densities. Particularly, the answer is obtained for sum of all tree graphs (see Fig. 1b), which coincides with the optical approximation formula. Earlier this approximation was related to summation of a more limited class of graphs (so called Czyż-Maximon approximation [2], see Fig. 1a). It is shown also that the value of loop expansion parameter is not small, as it was suggested before [4, 5] but large.

We will assume, as it is usually supposed, that nucleons in nuclei are independent. We will suppose also that the nucleon numbers A and B in colliding nuclei are large enough, so that nuclear sizes are much larger than NN interaction range.

We will discuss the method for the simplified problem with constant two-dimensional nuclear densities:

$$n_A(a_i) = (A/\pi R_A^2)\theta(R_A - a_i), \quad n_B(b_j) = (B/\pi R_B^2)\theta(R_B - b_j).$$

This problem has all the typical combinatorial difficulties of the diagram summation and the results obtained can be generalized to the realistic case of nonuniform distributions [1]. We assume that only $\tilde{A}$ and $\tilde{B}$ nucleons which are in the nucleus-nucleus overlap region interact

$$\tilde{A} = n_A V(b), \quad \tilde{B} = n_B V(b),$$

where $V(b)$ is two-dimensional overlap region (see Fig. 2).

The method uses the Generating Function (GF) technique considering systems with arbitrary numbers of nucleons N, M in the overlap region. The GF for the whole set of graphs is expressed in terms of the S-matrix elements for interactions of these systems as follows

$$\Xi(z_A, z_B; V) = \sum_{M,N=0} S_{MN}(V) \frac{z_A^M}{M!} \frac{z_B^N}{N!} V^{M+N}, \tag{1}$$

where $S_{00} = S_{01} = S_{10} = 1$.

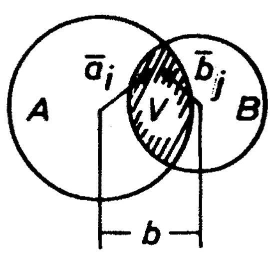

Fig. 2. Geometrical picture of nucleus-nucleus collision

S-matrix $S_{\tilde{A}\tilde{B}}$ is obtained from the GF $\Xi(z_A, z_B)$ by contour integrations

$$S_{\tilde{A}\tilde{B}}(V) = \frac{\tilde{A}!\tilde{B}!}{V^{\tilde{A}+\tilde{B}}} \oint \frac{dz_A}{2\pi i} \oint \frac{dz_B}{2\pi i} \frac{\Xi(z_A, z_B; V)}{z_A^{\tilde{A}+1} z_B^{\tilde{B}+1}}, \tag{2}$$

where the contours in the complex planes z_A and z_B enclose the points $z_A = 0$ and $z_B = 0$.

As usual, the GF for all graphs, $\Xi(z_A, z_B; V)$ is expressed through the GF for the connected graphs only, $V\Phi(z_A, z_B)$. In this form V dependence in the limit of heavy nuclei is separated out explicitly

$$\Xi(z_A, z_B; V) = \exp\left[V\Phi(z_A, z_B)\right]. \tag{3}$$

The connected graphs are constructed from the so-called block graphs, which are exemplified in Fig. 3. The tree graphs are built from the simplest blocks (Fig. 3a), and for the construction of general graph more complicated blocks with loops are needed.

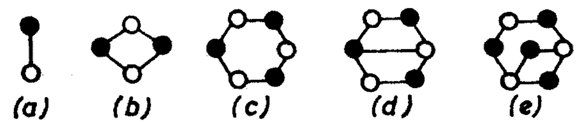

Fig. 3. Examples of block graphs

We have obtained [1] a system of equations which relate the GF for connected graphs to the GF for block graphs $a(z_A, z_B)$. These equations contain the auxiliary GF $u(z_A, z_B)$ and $v(z_A, z_B)$ for the so called rooted graphs, i.e. graphs with one marked vertex (root) for nucleons of nuclei A and B respectively. This system of equations has the following form

$$u = z_A \exp\left[\frac{\partial}{\partial u} a(u, v)\right], \tag{4}$$

$$v = z_B \exp\left[\frac{\partial}{\partial v} a(u, v)\right], \tag{5}$$

$$\Phi(z_A, z_B) = u + v - \left(u\frac{\partial}{\partial u} + v\frac{\partial}{\partial v} - 1\right) a(u, v). \tag{6}$$

The estimation of the integral (2) by the saddle-point method allows one to express the S-matrix through the GF of block graphs $a(n_A, n_B)$, where n_A, n_B are nuclear densities. It is remarkable that it is not necessary to know explicit solutions of Eqs (4)–(6). The answer is rather simple:

$$S_{\tilde{A}\tilde{B}} \approx C(n_A, n_B) \exp\left[Va(n_A, n_B)\right], \tag{7}$$

where $C(n_A, n_B)$ is the preexponential factor.

In the free approximation, which is equivalent to keeping only the first term in the series for $a(n_A, n_B)$ (see Fig. 3a), the formula of the optical approximation is recovered

$$S_{\tilde{A}\tilde{B}}^{(\mathrm{tree})} \approx C \exp\left[-\frac{\sigma_{NN}^{t}}{2} n_A n_B V\right]. \tag{8}$$

It is possible to analyse the convergence of the series for the function $a(n_A, n_B)$ in the Gaussian approximation for NN-scattering amplitude

$$f_{NN}(k) = i\frac{\sigma_{NN}^{t}}{2} \exp(-k^2 r_{NN}^2/4). \tag{9}$$

In this case

$$Va(n_A, n_B) = -\frac{2V}{\sigma_{NN}^{t}} \sum_{\{A_{MN}\}} \left(-\frac{n_A \sigma_{NN}^{t}}{2}\right)^M \left(-\frac{n_B \sigma_{NN}^{t}}{2}\right)^N \left(\frac{\sigma_{NN}^{t}}{2\pi r_{NN}^2}\right)^L \frac{1}{d(A_{MN}) s(A_{MN})}. \tag{10}$$

The summation is performed here over all block graphs A_{MN}, where M, N are numbers of interacting nucleons from A and B correspondingly, L is a loop number, $d(A_{MN})$ is the complexity degree of the graph A_{MN}, and $s(A_{MN})$ is its symmetry index.

It follows from Eq. (10) that the loop parameter is equal to $\sigma_{NN}^{t}/2\pi r_{NN}^2 = 4\sigma_{NN}^{el}/\sigma_{NN}^{t} \gtrsim 1$ and, besides this, the expansion in nuclear densities contains dimensionless parameters $n_A \sigma_{NN}^{t}/2 \sim 0.4A^{1/3}$ and $n_B \sigma_{NN}^{t}/2 \sim 0.4B^{1/3}$, which are large for heavy nuclei.

Therefore, though the solution of the problem of compound object scattering is given formally by Eq. (7), the convergence of the series for the function $a(n_A, n_B)$ needs further investigation.

Another interpretation of the results obtained above is given by the thermodynamical analogy noticed in Ref. [1]. There is a complete equivalence between calculation of nucleus-nucleus scattering amplitude at fixed impact parameter b and the calculation of partition function for thermodynamical system which is a mixture of two 2-dimensional liquids with non-diagonal interactions within volume $V(b)$. Our method corresponds to the virial expansion method in statistical mechanics.

A similar technique can be applied to the inelastic nucleus-nucleus collisions. Important additional elements in this case are the cutting rules of Abramovsky, Gribov, Kancheli [6] (AGK rules), which give the possibility to calculate different characteristics of inelastic processes.

It is useful to express contributions of different inelastic cuttings of Glauber diagrams in a diagrammatic way. If for a contribution of an arbitrary diagram to elastic scattering amplitude each line is associated with NN-scattering amplitude $if_{NN} \approx -\sigma_{NN}^{t}/2$, then according to AGK rules a total imaginary part of the diagram is equal to the sum of contributions of the same diagram but with two types of lines. The lines of the first type correspond to the discontinuity of NN amplitude $2\,\mathrm{Im}\, f_{NN} = \sigma_{NN}^{t}$ and the lines of the second type $if_{NN} + (if_{NN})^* = -2\,\mathrm{Im}\, f_{NN} = -\sigma_{NN}^{t}$ correspond to uncut NN amplitudes (absorption). Fig. 4 illustrates one of these cuttings. The contributions of the discontinuities with no NN amplitudes cut are expressed in a slightly modified way [7].

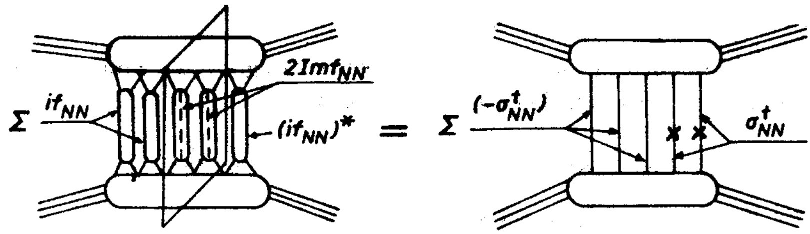

Fig. 4. Example of cutting of Glauber diagram and AGK-rule prescription

Diagram with different number of cut NN amplitudes correspond to processes with different number of inelastic NN collisions. It is possible to give a more detailed classification of contributions to imaginary part, by extracting from the total discontinuity of NN amplitude some process, satisfying a definite criterion, $\sigma^t_{NN} = \sigma_1 + \sigma_2$, where $\sigma_1 = \sigma^c_{NN}$ is the cross section of this selected process in NN interactions and $\sigma_2 = \sigma^t_{NN} - \sigma^c_{NN}$ is the cross section of all other processes. In this case it is necessary to consider graphs with three types of lines σ_1, σ_2 and $(-\sigma^t_{NN}) = \sigma_0$.

Let us denote contributions of the graph G with one (x_1), two $(x_1; x_2)$ or three $(x_1; x_2; x_3)$ types of lines by $T_G(x_1)$, $T_G(x_1; x_2)$ and $T_G(x_1; x_2; x_3)$ (the contributions, for which one of the types of lines is absent, are also included). As arguments of these functions the NN cross sections if_{NN}, $(-\sigma^t_{NN})$, σ^t_{NN}, σ_1, σ_2, etc. The contribution of a diagram G to elastic S-matrix of nucleus-nucleus scattering as a function of the NN elastic scattering amplitude is expressed as $T_G(if_{NN})$ and its contribution to, for example, the production cross section being a sum of contributions of all inelastic cuttings is equal to

$$T_G(-\sigma^t_{NN}, \sigma^{in}_{NN}; \sigma^{el}_{NN}) - T_G(-\sigma^t_{NN}; \sigma^{el}_{NN}).$$

(The second term is necessary for compensation of the contribution of graphs without inelastic cuttings.)

As the number of distributions of different type lines is determined by binomial coefficients, we have (by binomial formula)

$$T_G(x_1; x_2; x_3) = T_G(x_1 + x_2; x_3) = T_G(x_1 + x_2 + x_3).$$

The same equation is valid also for a sum of all graphs $T_{AB} = \sum_{\{G\}} T_G$:

$$T_{AB}(x_1; x_2; x_3) = T_{AB}(x_1 + x_2; x_3) = T_{AB}(x_1 + x_2 + x_3). \quad (11)$$

The function of a single argument $T_{AB}(x)$ plays an important role because it determines different inelastic cross sections in nucleus-nucleus collisions. The total cross section of AB interaction is connected to it as follows

$$\sigma^t_{AB}/2 \approx -T(-\sigma^t_{NN}/2).$$

(For hadron-nucleus interactions similar function has the form $T_{hA}(x) = \exp(n_A x) - 1$.)

Eq. (11) allows one to obtain in a simple way numerous consequences of AGK rules for nucleus-nucleus interactions. Here we present some of them.

1) Selfabsorption theorem

Consider the cross section for some selected process (e.g. events with particle production or events with at least one lepton pair, etc.). Then

$$\sigma^{c}_{AB} = T_{AB}(-\sigma^{t}_{NN};\, \sigma^{c}_{NN};\, \sigma^{t}_{NN}-\sigma^{c}_{NN}) - T_{AB}(-\sigma^{t}_{NN};\, \sigma^{t}_{NN}-\sigma^{c}_{NN}).$$

Using the fact that $T_{AB}(0) = 0$ and Eq. (11) we obtain

$$\sigma^{c}_{AB} = -T_{AB}(-\sigma^{c}_{NN}). \tag{12}$$

Thus, in order to calculate such a cross section it is necessary to calculate a contribution of the Glauber diagrams with a substitution $\sigma^{t}_{NN}/2 \rightarrow \sigma^{c}_{NN}$ and, consequently, the contribution of multiple rescatterings becomes less important as the corresponding NN cross section σ^{c}_{NN} decreases. This is the selfabsorption theorem which has been formulated for hadron-nucleus collisions in Ref. [8] and was discussed for nucleus-nucleus collisions in Ref. [9]. For a particular case of particle production cross section in nucleus-nucleus collisions we have

$$\sigma^{prod}_{AB} = -T_{AB}(-\sigma^{in}_{NN}),$$

where σ^{in}_{NN} is the total inelastic cross section of NN interactions.

It is important that for selected processes cross sections have the same form as the elastic amplitude, but, because they depend on different arguments, contributions of loop graphs for them are different. In particular, for processes, which have small cross sections in NN interactions, the virial expansion considered above is valid and it is possible to determine the contribution of loop graph from experimental data. It may be convenient, for example, to study the high p_T particle production. In this case the argument of the function T_{AB} can be varied continuously changing the considered region of transverse momenta.

2) Distribution of the number of inelastic interactions

Let us consider, as another application, a distribution of the number of inelastically cut NN amplitudes, $\sigma_{AB,k}$. Introduce the corresponding generating function

$$G^{prod}_{AB}(\beta) = \sum_{k=1}^{\infty} \sigma_{AB,k}\beta^{k}.$$

As it was mentioned above, this function can be obtained from the basic function T_{AB} by multiplying each inelastically cut line contribution by β, i.e. changing the argument:

$$G^{prod}_{AB}(\beta) = T_{AB}(-\sigma^{t}_{NN};\, \sigma^{el}_{NN};\, \beta\sigma^{in}_{NN}) - T_{AB}(-\sigma^{t}_{NN};\, \sigma^{el}_{NN})$$

$$= T_{AB}((\beta-1)\sigma^{in}_{NN}). \tag{13}$$

This generating function has an especially simple form for the case of tree approximation

$$G^{prod}_{(tree)}(\beta) \approx \exp\left[(\beta-1)\sigma^{in}_{NN} n_A n_B V\right] - 1 \tag{14}$$

and thus for fixed value of an impact parameter b

$$\sigma_{AB,k}^{(tree)}(b) = \frac{(\sigma_{NN}^{in} n_A n_B V)^k}{k!} \exp(-\sigma_{NN}^{in} n_A n_B V), \tag{15}$$

i.e. in this approximation this distribution is a Poisson one at fixed b.

3) Inclusive spectra

Another important consequence of AGK rules is a theorem on the cancellation of multiple interaction contributions to inclusive processes.

In terms of graphs a contribution to inclusive spectrum is determined by the graph with a "marked line" (an analog of the rooted graphs containing marked vertex). The marked line corresponds to the inelastically cut NN amplitude from which the detected particle is taken out, i.e. $d\sigma_{NN}/dy = F_{NN}(y)$, while all other lines in the graph correspond to either inelastic cut σ_{NN}^{in} or to absorption $(-\sigma_{NN}^{in})$. Thus from all the graphs only the simplest one (see Fig. 5) does not cancel and its contribution at fixed b is equal to

$$F_{AB}(y; b) = \int n_A(a) n_B(\bar{b}-\bar{a}) d^2a \cdot F_{NN}(y).$$

Integration of this expression over b gives a generalization of AGK results for nucleus-nucleus collisions

$$F_{AB}(y) = ABF_{NN}(y). \tag{16}$$

(Strictly speaking, this result is valid only in a central rapidity region at asymptotically large energies. Situation in fragmentation regions and at finite energies is discussed below).

The loop graphs do not contribute to inclusive spectra in the central region. For many-particle inclusive spectra and correlation functions it is easy to see using the AGK rules that the loop graphs contribute to correlations only in the forth and higher orders (in the forth order the simplest loop graph of Fig. 6 does not cancel).

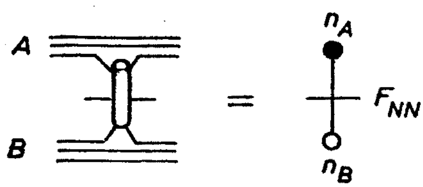

Fig. 5. Graph contributing to inclusive spectrum in central region

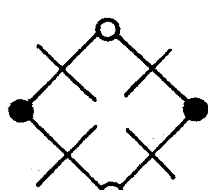

Fig. 6. Loop graph contributing to the forth order correlation

Let us note that Eq. (15) was obtained under the assumption that the diagrams of non--Glauber type are small. In particular, the diagrams with interaction between Pomerons (Fig. 7) result in a different A, B dependence of inclusive spectra in central region $F_{AB}(y) \sim A^{2/3}B^{2/3}$ (for strongly interacting Pomerons). It is important to check whether the relation (16) is satisfied experimentally. For hadron-nucleus collisions the contribution of interactions between Pomerons is not very essential.

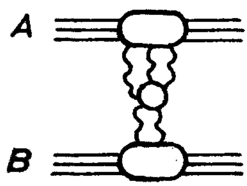

Fig. 7. Graph with interaction between Pomerons

4) Mean multiplicity and mean transverse energy

The relations between mean multiplicity and mean transverse energy (for fixed rapidity interval in central region) in AB and NN collisions follow directly from Eq. (16)

$$\int_{\Delta} F_{AB}(y)dy = \sigma_{AB}^{prod}\langle n_{AB}\rangle_{\Delta y} = AB\sigma_{NN}^{in}\langle n_{NN}\rangle_{\Delta y}.$$

Taking into account that each particle produced in NN and AB collision has approximately the same transverse energy, we obtain

$$\sigma_{AB}^{prod}\langle E_{T,AB}\rangle_{\Delta y} = AB\sigma_{NN}^{in}\langle E_{T,NN}\rangle_{\Delta y}.$$

If σ_{AB}^{prod} is approximated by $\sigma_{AB}^{prod} \approx (A^{1/3}+B^{1/3})^2$, then the quantity $\langle E_{T,AB}\rangle (A^{1/3}+B^{1/3})^2/AB$ should not depend on A and B. This prediction does not contradict experimental data [10].

5) Energy-momentum conservation for inclusive spectra

The AGK results were obtained for an arbitrary diagram with fixed number of Reggeons at asymptotically high energy. In reality it is necessary to know contributions of different cuts at finite energies. The AGK rules for diagrams with large number of cut Pomerons cannot be valid at fixed energy due to energy-momentum conservation effects [11] — the total energy is shared between many cut Pomerons.

Thus only those nucleons which have a small number of inelastic interactions can contribute to the hard part of inclusive spectra ($x_F \sim 1$). These effects strongly influence A, B dependence of inclusive spectra in fragmentation regions. It is possible to prove, however, that the contribution of loop graphs to inclusive spectra do cancel even in this case, though with an account of energy conservation the cancellations take place within a more limited class of graphs with a fixed number of inelastic interactions in vertices of the marked line. In fact, for the graphs where nucleons of A and B nuclei connected to the marked line interact inelastically i and k times, respectively, only non-loop graphs of Fig. 8 do not cancel.

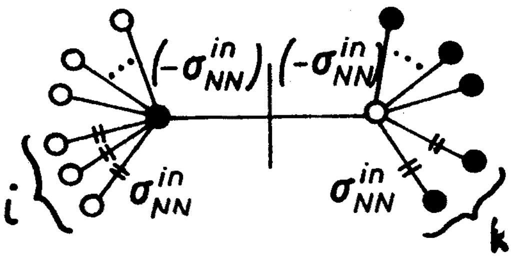

Fig. 8. Grapsh contributing to inclusive spectrum when energy conservation is taken into account

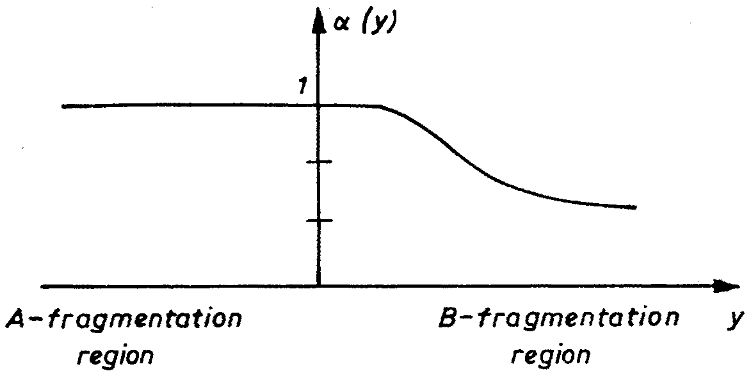

Fig. 9. Atomic number dependence for different regions of spectrum

Energy conservation leads to a modification of Eq. (16) in the fragmentation regions. It is possible to show that, if for nucleon-nucleus interactions $F_{NA}(y) \sim A^{\alpha_{NA}(y)}$ and for nucleus-nucleus interactions $F_{AB}(y) \sim A^{\alpha(y)}B^{\alpha(-y)}$, then (see Fig. 9)

$$\alpha(y) = \begin{cases} \alpha_{NA}(y) & \text{in the fragmentation region of nucleus B,} \\ 1 & \text{in the fragmentation region of nucleus A.} \end{cases}$$

The method outlined above allows one to calculate also multiplicity and transverse energy distributions, as well as the properties of distributions of the number of wounded nucleons. Detailed description of the corresponding technique and discussion of the results will be published elsewhere.

The results discussed in this paper could be useful for analysis of high energy nucleus-nucleus collisions where the conventional mechanism gives a substantial background for quark-gluon plasma searches. We formulated some general and model independent predictions of the traditional Glauber approach, based on assumption of independent interactions of nucleons of colliding nuclei. Experimental observation of deviations from these predictions would be an evidence for existence of extra mechanisms in nucleus-nucleus interactions.

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