COVARIANT LIGHT-CONE ALGEBRA*

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By introducing a variable lightlike vector k^μ the light-cone gauge algebra of the bosonic string oscillators is rewritten in an apparently covariant form and it is shown that the apparent covariance becomes true covariance if, and only if, the parameters take the usual critical values. In particular it is shown that the k-independence of physical quantities is equivalent to the usual closure of the Lorentz group for fixed k^μ and also to the zero-curvature of the light-cone-gauge surface. The connection of the k-formalism with the BRST formalism, and with other aspects of physics, notably spontaneous symmetry breaking mechanisms, is discussed.

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1. *Introduction*

As is well-known, bosonic string theory is based on a Fock space representation of a combined oscillator and Virasoro algebra

$$[\alpha_n^\mu, \alpha_m^\nu] = n g^{\mu\nu}\delta_N, \quad [x^\mu, \alpha_n^\nu] = i g^{\mu\nu}\delta_n,$$

$$[L_n^c, \alpha_m^\nu] = -m\alpha_N^\nu, \quad [L_n^c, L_m^c] = (n-m)L_N^c + \frac{c}{12}n(n^2-1)\delta_N, \tag{1.1}$$

whery $g^{\mu\nu}$ is a D-dimensional Minkowskian metric, μ. ν range from 0 to $D-1$, m, n are any integers, $N = n+m$, and c is a central charge whose value is arbitrary in general, but is equal to D when the L_n are constructed as bilinears in the α_n^μ. The α_n^μ are actually the coefficients in the Laurent expansion of the string field

$$X^\mu(z) = x^\mu - i\alpha_0^\mu \ln z + i\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu z^{-n} \tag{1.2}$$

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which is a D-vector depending on the single variable z. The Fock space $\mathcal{F}_\alpha$ of the algebra (1.1), defined by the vacuum

$$\alpha_n^\mu|0\rangle = 0, \quad L_n^c|0\rangle = 0, \quad n > 0 \tag{1.3}$$

is not positive on account of the Minkowskian nature of the metric and the standard methods [1] to circumvent this difficulty is to use either the BRST formalism, in which the oscillators α_n^μ are supplemented by a set of ghost fields (b_n, c_n), or the light-cone formalism, in which the Fock space is restricted to a subspace on which, in some given reference frame, one sets

$$\alpha_n^+ = 0\,(n \neq 0), \quad x^+ = 0 \quad \text{and} \quad \alpha_n^- = \frac{1}{\alpha_0^+} L_n(\vec{\alpha}), \tag{1.4}$$

where $\pm$ denote components of the vector in lightlike directions. It is well-known that the BRST formalism is not manifestly positive, and the light-cone formalism is not manifestly covariant, and although both formalisms produce the same positive covariant theory in the end, it is not a priori clear that this should be the case. The consistency of assuming (1.4) in the light-cone formalism is also not a priori clear.

Accordingly, it would be desirable to have a formulation of the theory which is manifestly positive and covariant at all stages, which exhibits explicitly the equivalence of the BRST and light-cone formalism and which guarantees the self-consistency of the light--cone formalism. The purpose of the present paper is to present such a formulation. The idea is to introduce a variable lightlike Lorentz vector k^μ which is used to express the light--cone conditions (1.4) in the covariant form

$$\chi(z) \equiv k \cdot X(z) = -i \ln z \tag{1.5}$$

and to study the normalizer algebra of (1.5). The advantages of the k-vector formulation may be grouped under three headings.

(I) Insights into Lorentz-invariance

On account of the introduction of the external vector k^μ the recovery of Lorentz--invariance in (1.5) is only formal. However, it becomes real when physical quantities (in particular inner products) become k-independent, and it turns out that this happens when the parameters of the problem (c, D) take their critical values (e.g. $D = 26$). In fact, it is just the requirement of k-independence that forces the parameters to take their critical values. Further, k-independence for variable k^μ turns out to be equivalent to the closure of the Lorentz algebra for fixed k^μ and thus provides a new (and, to our mind, more convincing) argument for the closure of the Lorentz algebra in a given frame. Finally, it turns out that the condition for k-independence i.e. for the critical values of the parameters, can be formulated as a zero curvature condition as follows: in the subspace defined by the gauge condition (1.5) the ordinary derivative $\partial_\mu = \partial/\partial k^\mu$ is replaced by a covariant derivative D_μ and the condition for k-independence is that its components commute:

$$[D_\mu, D_\nu] = 0. \tag{1.6}$$

(2) **Insights into the structure of the light-cone gauge algebra and its relation to the BRST formalism**

The analysis of Section 2 shows that, for fixed k'', the normalizer algebra $\beta(k)$ of (1.5) can be written as a direct sum

$$\beta = \gamma \oplus \lambda^{(c-d)},$$

where γ is exactly the light-cone gauge algebra and $\lambda^{(c-d)}$ is a Virasoro algebra of centre $(c-d)$, where $d = D-2$. In particular for $c = D$ the centre is 2 and β generates a positive Fock space $\mathscr{F}_\beta$. Thus (modulo $\lambda^{(c-d)}$) the light-cone gauge algebra emerges as the normalizer algebra of $\chi(k)$, and has its self-consistency guaranteed. However, only for the critical values of the parameters (i.e. when (1.6) is satisfied) can one impose a condition

$$D_\mu \mathscr{F}_\gamma = 0 \tag{1.7}$$

on the Fock spaces generated by γ and thus transform the algebras $\mathscr{F}_\gamma$ into one another by parallel transport. The positive-norm states generated by $\lambda^{(c-d)}$, $c \geqslant d$ and $\lambda^{(2)}$ in particular, may be identified as the Brower physical states [3], and the relationship to the BRST formalism may be obtained by noting that they may be cancelled by using the Virasoro algebra of centre (-2) constructed from the BRST ghost-fields i.e. by treating the ghost--fields as conformal scalars.

(3) **Parallels with other branches of field theory**

The k-formalism is not special to string theory but parallels the approaches and situations in other areas. First the use of the normalizer, or little, algebra is the analogue of the use of the little group in the case of spontaneous symmetry breaking, with $\chi(z)$ playing the role of the Higgs field. In fact the string situation proceeds beyond the usual spontaneous symmetry breakdown to the more complicated case of colour symmetry breaking by monopoles [5]. It will be recalled that the latter phenomenon is characterized by the fact that, while the Higgs field $\phi(\Omega)$ is covariantly constant on the sphere at infinity its little algebra $G_\alpha(\Omega)$ is not i.e.

$$D_i \phi(\Omega) = 0, \quad [G_\alpha(\Omega), \phi(\Omega)] = 0 \tag{1.8a}$$

but (generically)

$$D_i G_\alpha(\Omega) \neq 0 \Leftrightarrow [B_{ij}, G_\alpha(\Omega)] \neq 0 \tag{1.8b}$$

and this leads to a further breakdown of symmetry by the gauge-field B_{ij} e.g. a further breakdown of SU(3) in SU(5) $\to$ S(U(3) $\times$ U(2)). Correspondingly in the string situation one has

$$[D_\mu, \chi(k)] \approx 0, \quad [\beta(k), \chi(k)] \approx 0 \tag{1.9a}$$

but (generically)

$$D_\mu \mathscr{F}_\beta \neq 0 \Leftrightarrow [D_\mu, D_\nu] \mathscr{F}_\beta \neq 0, \tag{1.9b}$$

where $\mathscr{F}_\beta$ is the Fock space created by the β's. From this point of view the condition (1.6) for the critical values of the parameters may also be interpreted as a condition for no further breaking of the 'colour' symmetry $\beta(k)$.

Finally there is a close analogy between the k-formalism and the use of a (time-like) n^μ vector normal to a spacelike surface in making the Hamiltonian formalism of QFT formally covariant. From this point of view the conditions (1.7) are the analogue of the Schrödinger-Lorentz (or Poincaré) conditions

$$(i\partial_\mu - P_\mu)\Psi = 0, \tag{1.10a}$$

$$\left(n_\mu \frac{\partial}{\partial n^\nu} - n_\nu \frac{\partial}{\partial n^\mu} - M_{\mu\nu}\right)\Psi = 0, \tag{1.10b}$$

for the wave-function in the interaction picture of quantum field theory, and the integrability conditions (1.6) are the analogue of the integrability conditions for (1.10), which are known [4] to be just the Tomonaga-Schwinger conditions for the case of flat spacelike surfaces (constant timelike n^μ).

2. *Formally covariant gauge condition and its little algebra*

The starting point for our considerations is the D-dimensional oscillator plus Virasoro algebra (1.1).

The property of this algebra that will be central to our procedure is the fact that for any lightlike vector k^μ the string coordinates $k \cdot X(z)$ commute for all values of z i.e.

$$[\chi(z), \chi(z')] = 0 \text{ all } z, z', \quad \text{where} \quad \chi(z) = k \cdot X(z). \tag{2.1}$$

This permits one to use the conformal invariance of the theory to fix the gauge such that

$$\chi(z) = -i \ln z \quad (e^{i\chi(z)} = z) \tag{2.2}$$

for the fields $X^\mu(z)$. This choice of gauge is not, of course, respected by the full algebra α_n^μ, and the maximal subalgebra within the linear algebra of $\{\alpha_n^\mu, L_n^c\}$ which does respect (2.2) i.e. the little or normalizer algebra of (2.2) in $\{\alpha_n^\mu, L_n^c\}$ will be the algebra of interest to us. This algebra will be called the β-algebra, and if the gauge condition (2.2) is decomposed into its Fourier components

$$k \cdot x = 0, \quad k \cdot \alpha_0 = 1, \quad k \cdot \alpha_n = 0\,(n \neq 0), \tag{2.3}$$

it is easy to see that the generators of the β-algebra are

$$\beta_n^\mu = \alpha_n^\mu - k^\mu L_n^c \quad \text{and} \quad x^\mu. \tag{2.4}$$

The β-algebra then takes the form

$$[\beta_n^\mu, \beta_m^\nu] = mk^\mu \beta_N^\nu - nk^\nu \beta_N^\mu + nG^{\mu\nu}(n)\delta_N, \tag{2.5}$$

$$[x^\mu, \beta_n^\nu] = i(g^{\mu\nu} - k^\nu \beta_n^\mu), \tag{2.6}$$

where

$$G^{\mu\nu}(n) = g^{\mu\nu}+\left[\frac{c}{12}(n^2-1)\right]k^{\mu}k^{\nu}. \tag{2.7}$$

Strictly speaking, since the algebra generated by $\{\alpha_n^{\mu}, L_n^c\}$ has a centre one should permit the little algebra to have a centre in which case the generators (2.4) should be generalized slightly to $\beta_n^{\mu}+\varepsilon k^{\mu}\delta_n$, where ε is an arbitrary c-number. However, since this would change the β-algebra only by adding a term $2\varepsilon k^{\mu}k^{\nu}$ to $G^{\nu\mu}$ it is convenient to omit ε for the moment and regard (2.5)–(2.7) as the canonical form of the β-algebra.

The existence of the normalizer algebra β of $\chi(z)$, is, of course, independent of whether the gauge condition (2.2) is adopted or not, but since the gauge condition is consistent with the β-algebra we shall assume throughout that it has been adopted. All equations which are true whenever the gauge condition is satisfied will then be denoted by a wavy equality sign e.g.

$$y = \chi(z)+i \ln z \Rightarrow y \approx 0. \tag{2.8}$$

Given the β-algebra (2.5) it is natural to construct the formal Sommerfield-Sugawara operators

$$L_n(\beta) = -\tfrac{1}{2}\sum_m :\beta_{n-m}\cdot\beta_m: \tag{2.9}$$

and one finds by computation that they satisfy a Virasoro algebra of centre $c-d$ (where $d=D-2$), the overall minus sign having been inserted so that the algebra takes the canonical Virasoro form

$$[L_n(\beta), L_m(\beta)] \approx \left\{(n-m)L_N(\beta)+n\delta_N\left[\frac{c-d}{12}(n^2-1)\right]\right\}. \tag{2.10}$$

On the other hand, one finds that the $[L_n, \beta]$ commutators are just the opposite of the $[L_n, \alpha]$ ones in the sense that they are linear in the L_n's rather than the β's. More precisely one finds from (2.9) that

$$[L_n(\beta), \beta_m^{\mu}] \approx -k^{\mu}\left\{(n-m)L_N(\beta)+n\delta_N\left[\frac{c-d}{12}(n^2-1)\right]\right\}. \tag{2.11}$$

The existence of the unconventional relation (2.11) between the L_n's and β's and the fact that the right hand sides of (2.10) and (2.11) are exactly the same (modulo k^{μ}), means that the $\{L_n(\beta), \beta\}$ algebra is actually a direct sum of two algebras, namely the Virasoro algebra $L_n(\beta)$ and the algebra generated by the quantities

$$\gamma_n^{\mu} = \beta_n^{\mu}+k^{\mu}L_n(\beta). \tag{2.12}$$

Thus, if one includes the Virasoro algebra $L(\beta)$, the little algebra of $\chi(z)$ may be written as a direct-sum algebra of the form

$$[\gamma_n^{\mu}, \gamma_m^{\nu}] \approx mk^{\mu}\gamma_N^{\nu}-nk^{\nu}\gamma_N^{\mu}+n\Gamma^{\mu\nu}(n)\delta_N,$$

$$[L_n(\beta), \gamma_m^\mu] \approx 0,$$

$$[L_n(\beta), L_m(\beta)] \approx (n-m)L_N(\beta) + \left(\frac{c-d}{12}\right) n(n^2-1)\delta_N, \tag{2.13}$$

where

$$\Gamma^{\mu\nu}(n) = g^{\mu\nu} + \left[\frac{d}{12}(n^2-1)\right] k^\mu k^\nu. \tag{2.14}$$

Note that the γ-algebra is just the special case of a β-algebra with $c = d$. One consequence of this is that the Virasoro algebra constructed from the γ_n^μ according to (2.9) must have centre zero and in fact a computation shows that (modulo χ_n) it is identically zero,

$$L_n(\gamma) \approx 0. \tag{2.15}$$

It will be seen in the next section that the β-algebras are positive and have a physical meaning if $c \geqslant d$, and that the γ-algebra, which corresponds to the minimal value of c for which this is true, is isomorphic to the string algebra.

3. *General β-algebras and their positivity properties*

For positivity it turns out to be convenient to consider the β-algebra in the original form (2.5) rather than the direct sum form (2.13). A Fock space for this algebra may be defined as usual by a vacuum $|h\rangle$ for which

$$\beta_n^\nu|h\rangle = 0 (n > 0), \qquad \beta_0^\nu|h\rangle = h^\nu|h\rangle, \tag{3.1}$$

where h^ν is some c-number vector. From the existence of the explicit representation (2.4) it is clear that the β-algebra is consistent for all values of k^2 (although for $k^2 \neq 0$ it is no longer a little algebra) and to understand the structure of the $k^2 = 0$ case it may be useful to consider first the simpler case $k^2 \neq 0$.

In this case one may project the β into components parallel and orthogonal to k^μ i.e. to write

$$k_n = -(k \cdot \beta_n)/k^2 \quad \text{and} \quad b_n^\mu = \beta_n^\mu + k^\mu k_n \tag{3.2}$$

and in terms of these projections the β-algebra takes the form

$$\begin{aligned} [k_n, k_m] &= (n-m)k_N + n\delta_N\left[\frac{c}{12}(n^2-1) + \frac{1}{k^2}\right], \\ [k_n, b_m^\mu] &= -mb_N^\mu, \\ [b_n^\mu, b_m^\nu] &= n\hat{g}^{\mu\nu}\delta_N, \end{aligned} \tag{3.3}$$

where $\hat{g}$ is the $(D-1)$-dimensional metric

$$\hat{g}^{\mu\nu} = g^{\mu\nu} - k^\mu k^\nu/k^2. \tag{3.4}$$

From (3.3) one sees that unless k^μ is lightlike the β-algebra is the sum of a $(D-1)$-dimensional oscillator algebra and an associated Virasoro algebra of centre c. It is well known [6] that such a semi-direct sum is actually a direct sum of the oscillator algebra and the Virasoro algebra generated by the differences $\Delta_n = k_n - L_n(b)$, where $L_n(b)$ are the Sommerfield-Sugawara Virasoro generators for the transverse algebra. Thus the algebra is positive iff the oscillator and Δ-algebras are positive i.e. iff $\hat{g}$ is a positive metric and the centre $c-(D-1)$ of Δ_n (and Δ_0) are positive. Thus the algebra (3.3) is positive iff k^μ is timelike, $D \leqslant (c+1)$ and $\Delta_0 \geqslant 0$.

After this digression we proceed to the case $k^2 = 0$, which is the relevant case for string theory. The most characteristic feature of the $k^2 = 0$ case is that the β-algebra admits the invariant abelian subalgebra corresponding to (1.4) with generators

$$\tilde{\chi}_n \equiv \chi_n - \delta_n = k \cdot \beta_n - \delta_n. \tag{3.5}$$

More precisely, one has from (2.5),

$$[\beta_n^\mu, \tilde{\chi}_m] = mk^\mu \tilde{\chi}_N \quad \text{and} \quad [\tilde{\chi}_n, \tilde{\chi}_m] = 0. \tag{3.6}$$

The fact that the $\tilde{\chi}_n$'s commute means that the Fock space norms of the states $\tilde{\chi}_{-n}|h\rangle$ are zero and that the Fock space norms of the mixed states $(w \cdot \beta_{-n} + x\tilde{\chi}_{-n})|h\rangle$, where w^μ and x are arbitrary but constant, are

$$|w \cdot \beta_{-n}|h\rangle|^2 - 2xn(w \cdot k)\tilde{\chi}_0. \tag{3.7}$$

But since $(w \cdot k)$ can be chosen non-zero and x is an arbitrary real number it is clear that the norms (3.7) can be positive only if $\tilde{\chi}_0$ is zero. Thus for k^μ lightlike the positivity of the Fock space requires that

$$\tilde{\chi}_0 = 0 \quad \text{or} \quad \chi_0 = k \cdot \beta_0 = 1. \tag{3.8}$$

Thus positivity forces the special value $\chi_0 = 1$ used in (2.3). In other words, although the vector k^μ was originally free, when $k^2 = 0$ the positivity condition fixes its scale (relative to β_0). A simple way to guarantee the condition (3.8) is to let

$$k^\mu = \frac{n^\mu}{n \cdot \beta_0}, \tag{3.9}$$

where the scale of n^μ is free. A second important feature of the lightlike β-algebra is that (whether or not (3.8) is satisfied) thé operator

$$L = -\beta_0^2 / 2(k \cdot \beta_0) \tag{3.10}$$

acts as the grade operator, and that the operators

$$\pi^\mu = \beta_0^\mu + k^\mu L \quad \text{with} \quad \pi^2 = 0, \quad \pi \cdot k = 1$$

form a central lightlike vector 'dual' to k^μ. Using the lightlike vectors k^μ and π^μ one may decompose the D-dimensional Minkowski space into a 2-dimensional Minkowski space and a Euclidean space with metric

$$\hat{g}^{\mu\nu} = g^{\mu\nu} - k^\mu \pi^\nu - k^\nu \pi^\mu \tag{3.11}$$

and, correspondingly, decompose the β-algebra into k, π and transverse components

$$\tilde{\chi}_n = k \cdot \beta_n - \delta_n, \quad w_n = -\pi \cdot \beta_n, \quad b_n^\mu = \beta_n^\mu + k^\mu \omega_n - \pi^\mu \chi_n. \tag{3.12}$$

In terms of these projections the β-algebra takes the form

$$\begin{aligned}
[\tilde{\chi}_n, \tilde{\chi}_m] &= 0, \\
[b_n^\mu, \tilde{\chi}_m] &= 0, \\
[b_n^\mu, b_m^\nu] &= n\delta_N \hat{g}^{\mu\nu}, \\
[w_n, \tilde{\chi}_m] &= -m\tilde{\chi}_N, \\
[w_n, b_m^\mu] &= -mb_N^\mu, \\
[w_n, w_m] &= (n-m)w_N + \frac{c}{12} n(n^2-1)\delta_N
\end{aligned} \tag{3.13}$$

from which one sees that, apart from the invariant abelian subalgebra $\tilde{\chi}_n$, the lightlike β-algebra is the semi-direct sum of a $d = (D-2)$ dimensional oscillator algebra and a Virasoro algebra of centre c. The oscillator part of the algebra is automatically positive, since $\hat{g}$ is positive, and by the same argument used for the timelike case, the necessary and sufficient conditions for the whole algebra to be positive are that the centre and generator Δ_0 of the difference Virasoro algebra $\Delta_n = w_n - L_n(b)$ should be positive, and it is easy to see that these conditions reduce to

$$(c-d) \geqslant 0 \quad \text{and} \quad h^2 \leqslant 0, \tag{3.14}$$

respectively. It is worth noting that in the minimal case of the γ-algebra both inequalities in (3.14) become equalities, since for the γ-algebra $c = d$ and from (3.8)

$$h_\gamma^2 = \left(h^\mu - k^\mu \frac{h^2}{2}\right)^2 = 0.$$

Thus the inequalities in (3.14) come exclusively from the Brower-Virasoro algebra $L(\beta)$. This can also be seen from the fact that, modulo $\tilde{\chi}$, the Brower-Virasoro algebra $L(\beta)$ and the difference Virasoro algebra Δ coincide, which is not surprising since both commute with the transverse oscillator algebra. Note that for the α-representation of the β's (with the SS Virasoro generators) $c = D$, the positivity conditions (3.14) are automatically satisfied, but with

$$(c-d) = 2 \quad \text{and} \quad h^2 = 0.$$

Thus for $c = D$ the centre of the Brower Virasoro algebra is two, and the Brower states have strictly positive norms.

4. *Necessary condition for Lorentz invariance*

It is clear that although formal Lorentz invariance has been maintained by using the external vector k^μ, true Lorentz invariance requires that all physical quantities be independent of k^μ. Furthermore, a natural requirement of string physics is that at each grade there should exist a single particle interpretation of the theory i.e. an interpretation in terms of k-independent Wignerian representations of the Poincaré group, and this we shall also require. In this section we wish to show that the k^μ-independence of the physical quantities and Wigner states requires that $c = 24$ where c is the centre of the β-algebra. In the following section we show that $c = 24$ is also a sufficient condition.

We first note that for the non-interacting string the important physical quantities are the total momentum P^μ and the inner products in Fock space, and since the inner products between different grades of L are zero the problem need be considered only at each given grade. At each such grade there are only a finite number of states for each value of k^μ and the question then is whether a generic linear subspace of the states can have k-independent inner products, and can be described by k-independent Wigner states i.e. by k-independent wave-functions of the form

$$u_{\mu_1\mu_2\ldots\mu_r}(p),$$

where $P^\mu = p^\mu$ at the grade in question.

To investigate these questions it is first necessary to give a precise definition of the total momentum P^μ. If we assume that P^μ is an element of the β-algebra and commutes with the grade-operator, then the only candidate would appear to be β_0^μ. However, one may recall from Section 2 that β_n^μ was actually only defined up to a transformation of the form

$$\beta_n^\mu \to \beta_n^\mu + \varepsilon k^\mu \delta_n,$$

a canonical β_n^μ being chosen only in order to simplify the algebra. Hence the most general Ansatz for P^μ in the little algebra is actually

$$P^\mu = \beta_0^\mu + \varepsilon k^\mu, \tag{4.1}$$

where ε is a constant to be determined.

Let us now consider the action of P^μ on the N^{th} grade Fock states, which are of the form

$$|\Phi_N\rangle = \beta^{\mu_1}_{-n_1}\beta^{\mu_2}_{-n_2} \ldots |h\rangle \quad (n_1+n_2+ \ldots = N). \tag{4.2}$$

From the special relation

$$[\beta_0^\mu, \beta_n^\nu] = nk^\mu\beta_n^\nu \tag{4.3}$$

of the β-algebra, one sees that with P^μ and $|\Phi_N\rangle$ so defined

$$P^\mu|\Phi_N\rangle = p_N^\mu|\Phi_N\rangle \quad \text{where} \quad p_N^\mu = h^\mu+(\varepsilon-N)k^\mu \tag{4.4}$$

and, since the operator P^μ is a physical quantity, the eigenvalue p_N^μ should be k-independent. (This implies of course that h^μ is k-dependent, in fact, is linear in k.) Since $k \cdot h = 1$ from

(3.8) one has

$$p_N^2 = h^2+2(\varepsilon-N) \tag{4.5}$$

so h^2 is actually k-independent. Furthermore, p_N^2 is linear in N (Regge-like).

Let us first consider the grade-one states

$$|u\rangle = u_\mu(p)\beta^\mu_{-1}|h\rangle, \quad p^\mu \equiv p_1^\mu = h^\mu+(\varepsilon-1)k^\mu. \tag{4.6}$$

From the β-algebra the norm of $|u\rangle$ is easily seen to be

$$\begin{aligned}\langle u|u\rangle &= u\cdot u-2(k\cdot u)(h\cdot u)\\ &= u\cdot u-2(k\cdot u)(p\cdot u)+2(\varepsilon-1)(k\cdot u)^2.\end{aligned} \tag{4.7}$$

Since p is k-independent, the norm (4.7) is evidently k-independent if and only if

$$\varepsilon = 1, \tag{4.8a}$$

$$p\cdot u = p^\mu u_\mu = 0. \tag{4.8b}$$

Note that with $\varepsilon = 1$ the mass spectrum (4.5) becomes

$$M^2 = -P^2 = 2(N-1)+m^2, \tag{4.9}$$

where

$$m^2 = -h^2 \geqslant 0, \tag{4.10}$$

the positivity coming from the positive-norm condition (3.14). If $m^2 > 0$, the k-independent states at $N = 1$ are massive and transform in the $(D-1)$-dimensional vector representation of the little group of p^μ. If, however, $m^2 = 0$ (as happens for the γ-algebra), the states become massless. Note that in this case the state $|p\rangle$ (with $u_\mu = p_\mu$) becomes one of the physical states, but because of its zero norm this particular state decouples from the rest leaving a $(D-2)$-dimensional transverse representation of the little group of the lightlike momentum vector p^μ.

Let us next consider the grade-two states. At grade 2 the states are massive and it is convenient to construct states which transform according to the traceless symmetric tensor, vector and scalar representations of the little group of p_2^μ i.e. to decompose the grade-two states in the following manner:

$$|T\rangle = T_{\mu\nu}(p)\{\beta^\mu_{-1}\beta^\nu_{-1}+\alpha_T k^\mu\beta^\nu_{-2}\}|h\rangle, \tag{4.11a}$$

$$|V\rangle = \{[p_\mu V_\nu(p)+p_\nu V_\mu(p)][\beta^\mu_{-1}\beta^\nu_{-1}+\alpha_V k^\mu\beta^\nu_{-2}]+\gamma_V V_\mu(p)\beta^\mu_{-2}\}|h\rangle, \tag{4.11b}$$

$$|S\rangle = \{[g_{\mu\nu}+xp_\mu p_\nu]\beta^\mu_{-1}\beta^\nu_{-1}+\gamma_S p_\mu\beta^\mu_{-2}\}|h\rangle, \tag{4.11c}$$

where

$$p^\mu T_{\mu\nu} = T^\mu_\mu = p^\mu V_\mu = 0, \quad p^\mu \equiv p_2^\mu = h^\mu-k^\mu,$$

and α_T, α_V, γ_V, γ_S and x are constants. Clearly the T-states are the most numerous and are thus the generic grade-two states.

The norm of the $|T\rangle$ states is given by

$$\langle T|T\rangle = 2\Big\{T_{\mu\nu}T^{\mu\nu}+(\alpha_T+1)(\alpha_T-3)k^\mu T_{\mu\nu}T^{\nu\sigma}k_\sigma + \left[\frac{c-8}{4}\alpha_T^2+6\alpha_T+2\right](k^\mu T_{\mu\nu}k^\nu)^2\Big\} \tag{4.12}$$

which is k-independent if

$$c = 24 \quad (\text{and } \alpha_T = -1) \tag{4.13a}$$

since the alternative possibility

$$c = -8/9 \quad (\text{and } \alpha_T = 3) \tag{4.13b}$$

is ruled out by the positive-norm condition $c \geqslant d \geqslant 0$.

Note that the condition (4.13a) does not in itself determine the dimension D, but, on account of the positive-norm condition (3.14), imposes the condition $D \leqslant 26$.

Once $c = 24$, the states $|T\rangle$, $|V\rangle$, $|S\rangle$ become orthogonal and have k-independent norms if

$$\alpha_V = -1 \qquad x = \frac{D-2p^2}{p^2(2p^2-1)}$$

$$\gamma_V = -p^2 \qquad \gamma_S = -1-x(1+p^2). \tag{4.14}$$

This fixes the remaining grade-two parameters in (4.11) and the norms of $|T\rangle$, $|V\rangle$ and $|S\rangle$ reduce to

$$\langle T|T\rangle = 2T^{\mu\nu}T_{\mu\nu},$$

$$\langle V|V\rangle = 2(V\cdot V)m^2(m^2+2),$$

$$\langle S|S\rangle = 2(D-1)\left\{\frac{(2+m^2)(13+4m^2)-D(1+m^2)}{(5+2m^2)^2}\right\}. \tag{4.15}$$

The states $|V\rangle$ and $|S\rangle$ which are in any case decoupled from the generic states $|T\rangle$ because of Lorentz invariance, become zero norm states in the critical case $m^2 = 0$, $D = 26$. This is exactly the case of the γ-algebra (i.e. the light-cone gauge algebra) for which $m^2 = 0$ and $c = d = 24$. Thus the states $|V\rangle$ and $|S\rangle$, which have positive norm for $m^2 > 0$ and/or $D < 26$, are actually combinations of grade-two Brower states, namely of the states

$$L^2_{-1}(\beta)\,|h\rangle, \quad L_{-2}(\beta)\,|h\rangle \quad \text{and} \quad \gamma^\mu_{-1}L_{-1}(\beta)\,|h\rangle.$$

Note that in any case the k-independent states at grade two can be summarized by defining

$$|A\rangle = A_{\mu\nu}(p)\{\beta^{\mu}_{-1}\beta^{\nu}_{-1}-(k^{\mu}+p^{\mu})\beta^{\nu}_{-2}\}\,|h\rangle, \tag{4.16}$$

where $A_{\mu\nu}$ satisfies

$$p^{\mu}A_{\mu\nu}p^{\nu} = \tfrac{1}{2}\,A^{\mu}_{\mu} \tag{4.17}$$

and the norm of $|A\rangle$ is

$$\langle A|A\rangle = 2A_{\mu\nu}A^{\mu\nu}+2p^{\mu}A_{\mu\nu}A^{\nu\sigma}p_{\sigma}. \tag{4.18}$$

5. *Sufficient condition for Lorentz invariance*

Having shown that the condition $c = 24$ (together with $\varepsilon = 1$) is necessary for the k-independence of the inner products, and hence for true Lorentz invariance, we now wish to show that it is also a sufficient condition. Clearly this will be the case if we can find a covariant derivative

$$D_{\mu} = d_{\mu}+iA_{\mu} \qquad A^{+}_{\mu} = A_{\mu}, \qquad d_{\mu} = \frac{\partial}{\partial n^{\mu}}, \tag{5.1}$$

where n^{μ} is the free lightlike vector introduced in (3.9), such that the states are covariantly constant, since for self-adjoint A_{μ}

$$D_{\mu}|f\rangle = 0 \Rightarrow \frac{\partial}{\partial n^{\mu}}\langle f|g\rangle = i[\langle A_{\mu}f|g\rangle-\langle f|A_{\mu}g\rangle] = 0. \tag{5.2}$$

However, the covariant derivative is not arbitrary, but must respect the gauge condition (2.2)

$$[D_{\mu}, \chi] \approx 0 \tag{5.3}$$

which means that A_{μ} is unique up to the addition of terms in the little algebra. It is gratifying to find that such a covariant derivative is provided by the Lorentz group itself, as follows. Let

$$M^{\mu\nu} = M^{\mu\nu}(n)+M^{\mu\nu}(\alpha),$$

where

$$M^{\mu\nu}(n) = -in^{[\mu}\tilde{\partial}^{\nu]} \tag{5.4}$$

and

$$M^{\mu\nu}(\alpha) = x^{[\mu}\alpha^{\nu]}_{0}-i\sum_{m\neq 0}\frac{1}{m}\,{:}\alpha^{\mu}_{-m}\alpha^{\nu}_{m}{:} \tag{5.5}$$

be the algebra of the Lorentz group that transforms all vectors n^{μ}, x^{μ}, α^{μ}_{n} i.e. the Lorentz algebra with respect to which we have formal covariance. By replacing α by β according to (2.4) one may re-write $M^{\mu\nu}$ in the form

$$M^{\mu\nu} = \Delta^{\mu\nu}(n) + M^{\mu\nu}(\beta), \tag{5.6}$$

where

$$M^{\mu\nu}(\beta) = \tfrac{1}{2}\{x^{[\mu}, P^{\nu]}\} - i \sum_{m\neq 0} \frac{1}{m} :\beta^{\mu}_{-m}\beta^{\nu}_{m}:, \tag{5.7}$$

the anticommutator being used for the x-term because x^{μ} and P^{μ} do not commute even for $\mu \neq \nu$, and where

$$\Delta^{\mu\nu}(n) = -in^{[\mu}D^{\nu]}, \tag{5.8}$$

with

$$D^{\mu} = d^{\mu} + iA^{\mu}, \qquad A^{\mu} = -\tfrac{1}{2}\left\{x^{\mu}, \frac{L_0^c - \varepsilon}{n \cdot \alpha_0}\right\} + \frac{i}{n \cdot \alpha_0} \sum_{m\neq 0} \frac{1}{m} :\alpha^{\mu}_{-m} L^c_m:. \tag{5.9}$$

Then, since $k \cdot X(z)$ is invariant with respect to $M^{\mu\nu}$ and $M^{\mu\nu}(\beta)$ by definition it is clear that it must be invariant with respect to $\Delta^{\mu\nu}$ and D^{μ} i.e.

$$[\Delta^{\mu\nu}, \chi(z)] \approx 0 \quad \text{and} \quad [D^{\mu}, \chi(z)] \approx 0, \tag{5.10}$$

so D^{μ} in (5.9) is the required covariant derivative up to terms in the little group. The geometrical interpretation of D^{μ} is that it is the derivative with respect to k (or n) which is tangent to the surfaces $k \cdot X = \text{const}$.

Equipped with the covariant derivative D^{μ} it is clear from (5.2) that one can guarantee k-independence and hence true Lorentz invariance if one can impose the covariant constancy condition

$$D^{\mu}|f\rangle \mathrel{\dot{\approx}} 0. \tag{5.11}$$

(The dot above the $\approx$ sign means that it is valid up to terms proportional to n^{μ}.) Note that when (5.11) can be imposed it determines the Fock spaces for all n^{μ} in terms of the Fock space for any given n^{μ}.

The consistency, or integrability condition for (5.11) is clearly

$$[D^{\mu}, D^{\nu}]\,|f\rangle \mathrel{\dot{\approx}} 0 \tag{5.12}$$

and if one now computes the commutator in (5.12) from (5.9) one finds

$$[D^{\mu}, D^{\nu}] \doteq \frac{1}{(n \cdot \alpha_0)^2} \sum_{m\neq 0} \left\{\left(2 - \frac{c}{12}\right) m + \left(\frac{c}{12} - 2\varepsilon\right)\frac{1}{m}\right\} :\alpha^{\mu}_{-m}\alpha^{\nu}_{m}:. \tag{5.13}$$

From (5.13) one sees at once that the conditions

$$c = 24, \qquad \varepsilon = 1 \tag{5.14}$$

are sufficient for the commutator of the D's to vanish (on any state) an hence is sufficient for (5.11) and true Lorentz invariance.

In the above form the condition for k-independence and true Lorentz invariance is very intuitive and has a geometrical significance since it relates to well-known results for covariant derivatives and curvature. Of course it must also be equivalent to the standard string condition on the Lorentz algebra and we shall now show that this is the case. In fact, if one cross-products (5.12) with n^α, n^β, one finds that (5.12) is equivalent to

$$[\Delta^{\alpha\mu}, \Delta^{\beta\nu}] \approx i\{g^{\alpha\beta}\Delta^{\mu\nu} - g^{\mu\beta}\Delta^{\alpha\nu} - g^{\alpha\nu}\Delta^{\mu\beta} + g^{\mu\nu}\Delta^{\alpha\beta}\} \tag{5.15}$$

which is just the condition that the Δ's close to form a Lorentz algebra. From (5.15) and the obvious tensor character of all operators with respect to $M^{\mu\nu}$ one then sees that (5.14) is the necessary and sufficient condition that the formal Lorentz algebra $M^{\mu\nu}$ decomposes into the direct sum of two separate Lorentz algebras i.e.

$$M^{\mu\nu} = \Delta^{\mu\nu} + M^{\mu\nu}(\beta) \Rightarrow \Delta^{\mu\nu} \oplus M^{\mu\nu}(\beta) \quad \text{if} \quad c = 24, \varepsilon = 1.$$

The fact that $M^{\mu\nu}(\beta)$ closes to form a Lorentz algebra is just the traditional condition [2] for the light-cone algebra to be Lorentz invariant, and the present formulation gives a new interpretation to that traditional condition, namely that the closure of $\Delta^{\mu\nu}(n)$ for fixed n is the necessary and sufficient condition for the algebra to be independent of the particular choice of n^μ.

The meaning of the covariantly constant condition (5.11) can be understood at each (finite-dimensional) level N by considering the explicit Fock space states,

$$H_{\mu_1\mu_2\cdots\mu_r}\beta^{\mu_1}_{-n_1}\,\beta^{\mu_2}_{-n_2}\cdots\beta^{\mu_r}_{-n_r}|0\rangle, \qquad n_1+n_2+\ldots+n_r = N. \tag{5.16}$$

We will use an α_0-diagonal basis in our Hilbert space and represent the x^μ operators in the usual way by

$$x^\mu = i\,\frac{\partial}{\partial\alpha_{0\mu}}. \tag{5.17}$$

In this representation the 'wave-functions' $H_{\mu_1\mu_2\ldots\mu_r}$ are functions of the variable α_0 and the external lightlike vector n^μ; while the partial derivative d_μ is for fixed α_0,

$$H_{\mu_1\mu_2\cdots\mu_r} = H_{\mu_1\mu_2\cdots\mu_r}(n, \alpha_0) \quad \text{and} \quad d_\mu = \left.\frac{\partial}{\partial n^\mu}\right|_{\alpha_0\ \text{fixed}} \tag{5.18}$$

Using (5.17), the covariant derivative D^μ takes the form

$$D^\mu \doteq d^\mu + \frac{\alpha_0^2 + M^2}{2(n\cdot\alpha_0)}\frac{\partial}{\partial\alpha_{0\mu}} + \frac{\alpha_0^\mu}{2(n\cdot\alpha_0)} - \tilde{A}^\mu, \tag{5.19}$$

where

$$\tilde{A}^\mu = \frac{1}{(n\cdot\alpha_0)}\sum_{m\neq 0}\frac{1}{m}:\alpha^\mu_{-m}L^c_m: \tag{5.20}$$

and we have used (4.9) to express L_0^c in terms of the mass-operator M^2 (which only depends on the grade of the state).

The expression (5.19) simplifies considerably if one uses the physical momentum

$$p^\mu = \alpha_0^\mu - \frac{n^\mu}{2(n \cdot \alpha_0)}(\alpha_0^2 + M^2) \tag{5.21}$$

instead of α_0. In fact with respect to the new partial derivative ∂_μ, where

$$\partial_\mu = \left.\frac{\partial}{\partial n^\mu}\right|_{p\,\text{fixed}}. \tag{5.22}$$

D^μ takes the simpler form

$$D^\mu = \frac{1}{\sqrt{\alpha_0^2 + M^2}}(\partial^\mu - \tilde{A}^\mu)\sqrt{\alpha_0^2 + M^2}, \tag{5.23}$$

and the square root factors in (5.23) are eliminated by the change of the integration variables in the transition

$$\int \frac{d^D\alpha_0}{(2\pi)^D}\,\frac{1}{\alpha_0^2 + M^2} f\left[\alpha_0 - \frac{n}{2(n \cdot \alpha_0)}(\alpha_0^2 + M^2)\right] \Rightarrow \int \frac{d^D p}{(2\pi)^D}\,\delta(p^2 + M^2) f(p).$$

Thus finally on physical states of the form

$$|\Phi_N\rangle = H_{\mu_1 \cdots \mu_r}(n, p)\beta^{\mu_1}_{-n_1} \cdots \beta^{\mu_r}_{-n_r}|0\rangle \tag{5.24}$$

the condition (5.11) reduces to

$$\partial^\mu |\Phi_N\rangle \doteq \tilde{A}^\mu |\Phi_N\rangle, \tag{5.25}$$

where $\tilde{A}^\mu$ is given by the normal ordered expression (5.20).

Let us now work out the solution of the physical state condition (5.25) for the first few grades. On the vacuum grade ($N = 0$) the single state is

$$|\Phi_0\rangle = H(n, p)\,|0\rangle \tag{5.26}$$

and (5.25) is equivalent to

$$\partial^\mu H = 0 \tag{5.27}$$

from which one sees that, as expected, the wave-function of the physical ground state of the theory must not depend on n^μ and is thus a function of the physical momentum p^μ only.

For the grade-one states

$$|\Phi_1\rangle = H_\mu(n, p)\beta^\mu_{-1}|0\rangle \tag{5.28}$$

(5.25) gives

$$(\partial^\mu - \tilde{A}^\mu)\,|\Phi_1\rangle \doteq \left\{\left(\partial^\mu H_\nu - \frac{(p\cdot H)}{(n\cdot p)}\,\delta^\mu_\nu\right)\alpha^\nu_{-1} - \frac{n^\nu}{(n\cdot p)}(\partial^\mu H_\nu)L^c_{-1}\right\}|0\rangle \doteq 0$$

which is equivalent to the following differential equation

$$\partial^\mu H^\nu \doteq \frac{g^{\mu\nu}}{(n\cdot p)}(p\cdot H). \tag{5.29}$$

By contracting (5.29) with p_ν one sees that the only solution of (5.29) is

$$H^\mu \doteq H^\mu(p) \quad \text{where} \quad p^\mu H_\mu = 0. \tag{5.30}$$

Thus the wave-function depends only on the physical momentum p^μ as before, and is orthogonal to it.

Finally parametrizing the grade-two sates by

$$|\Phi_2\rangle = H_\mu(n, p)\beta^\mu_{-2}|0\rangle + K_{\mu\nu}(n, p)\beta^\mu_{-1}\beta^\nu_{-1}|0\rangle \tag{5.31}$$

one finds in a similar manner that the solution of (5.25) is given by

$$K_{\mu\nu} \doteq K_{\mu\nu}(p), \qquad H_\mu \doteq -\left(p^\nu + \frac{n^\nu}{n\cdot p}\right)K_{\mu\nu}, \tag{5.32a}$$

where

$$K_{\mu\nu}(g^{\mu\nu} - 2p^\mu p^\nu) = 0. \tag{5.32b}$$

Note that the physical states (5.30) and (5.32) coincide with the ones we have found in Section 4 by demanding k-independence.

6. *Ghost fields and connection with BRST formalism*

In conventional string theory the oscillators are accompanied by ghost fields which are not merely auxiliary fields that are introduced to make the BRST formalism of the theory work, but are an intrinsic part of the structure introduced by diagonalization of the two-dimensional metric — indeed their functional integral is just the Jacobian for this diagonalization [1]. Hence it is natural to ask what role the ghosts play in the k-formalism. In this section we show that they play a very natural role in that they can be used to cancel the Brower states, provided that the ghosts are regarded as conformal scalars rather than vectors.

To show this we first recall that the ghost algebra takes the form

$$\{c_n, b_m\} = \delta_N \qquad c_n^+ = c_{-n} \qquad b_n^+ = b_{-n} \tag{6.1}$$

and differs from the oscillator algebra in that the brackets are anti-commutators rather than commutators, the b's and c's are self-adjoint rather than adjoints of each other, and that there is no factor n on the right-hand side of the anti-commutator relation. Due to the absence of the factor n, the ghost algebra admits a larger automorphism group than the usual Virasoro one, namely the semi-direct product group with algebra $L \wedge R$ generated by

$$L_n^{(0)}(g) = \sum_k k :c_{n-k} b_k: \tag{6.2a}$$

$$R_n(g) = \sum_k :c_{n-k} b_k: \tag{6.2b}$$

where $L_n^{(0)}(g)$ is a Virasoro algebra with centre -2 i.e.

$$[L_n^{(0)}(g), L_m^{(0)}(g)] = (n-m)L_N^{(0)}(g) - \tfrac{2}{12} n(n^2-1)\delta_N, \tag{6.3}$$

$R_n(g)$ is a one-dimensional oscillator algebra with unit centre i.e.

$$[R_n(g), R_m(g)] = n\delta_N \tag{6.4}$$

and the commutators of the $L_n^{(0)}$ and R_n are of the semi-direct sum form

$$[L_n^{(0)}(g), R_m(g)] = -mR_N(g) + \tfrac{1}{2} n^2 \delta_N. \tag{6.5}$$

One sees that $L_n^{(0)}$ is actually the usual (matter-like) Virasoro algebra for the ghost fields c_n, b_n with $n \neq 0$ (since for these a factor n can be introduced in the anticommutator (6.1) by a rescaling of the ghost fields) whereas R_n is the number-operator density for all the ghost fields. Thus the ghost field representation of $L^{(0)}(g)$ is partially-reducible into c_0 and the full set of c_n's respectively,

$$[L_n^{(0)}(g), c_0] = 0,$$

$$[L_n^{(0)}(g), c_m] = -mc_{n+m} \quad \text{for} \quad m \neq 0 \tag{6.6}$$

whereas the ghost field representation of $R(g)$ is irreducible:

$$[R_n(g), c_m] = c_{n+m}. \tag{6.7}$$

One also sees that the algebra $L \wedge R$ contains a whole family of Virasoro algebras $L_n^{(j)}$, namely,

$$L_n^{(j)} = L_n^{(0)} - jnR_n, \quad j = \dots -1, 0, 1 \dots \tag{6.8}$$

whose centres are

$$c(j) = -2 - 12j(j-1) = -2, -26, -74\dots \tag{6.9}$$

In configuration space $L_n^{(0)}$ generates the purely orbital part of the conformal transformation while $(-jnR_n)$ generates the pure spin part.

The member of the family (6.8) which is the Virasoro subalgebra of the reparametrization algebra is not $L^{(0)}$ but $L^{(2)}$, which corresponds to ghost fields of conformal spin 2. Actually $L^{(2)}$ is the gradient of the BRST operator Q and the centre of the $L^{(2)}$ algebra is the Hessian of Q^2 [7] i.e.

$$L_n^{(2)} = \{b_n, Q\},$$

where

$$Q = \sum_k c_{-k} L_k^D(\alpha) + \tfrac{1}{2} \sum_{k,l} (l-k) : c_{-k} c_{-l} b_{k+l} :.$$

and

$$(n-m) L_{n+m}^{(2)} - [L_n^{(2)}, L_m^{(2)}] = \{[Q^2, b_n], b_m\}.$$

Note that in the critical dimension $D = 26$ for which Q^2 is zero, $L^{(2)}$ commutes with Q and thus Q may be thought of as the Higgs-like operator for the reduction $L \wedge R \to L^{(2)}$.

After this brief recapitulation let us return to the relationship between ghosts and Brower states. If in the origional α-algebra one uses the natural Virasoro algebra, that is to say, the Sommerfield-Sugawara one $L^D(\alpha)$, then the decoupled Brower Virasoro algebra $L_n(\beta)$ has centre $c = 2$, and the question is whether there exists in the family $L^{(j)}$ a member with centre -2 to cancel $L_n(\beta)$, and whether there is a Higgs-like mechanism to select it. One sees at once from (6.3) that the Virasoro algebra $L^{(0)}$ itself has the required property, and from (6.6) that $L^{(0)}$ is just the little algebra of c_0 in $L \wedge R$. Thus the answer is in the affirmative and c_0 is the required Higgs operator. It is remarkable that the sequence of centres (6.9), which is so very special, should have the required centre -2 and it means that the algebra defined as the little algebra of $k \cdot X(z)$ and c_0 in the combined oscillator and ghost algebra can be reduced to exactly the algebra of the light-cone gauge by setting $L_n(\beta) + L_n(g)$ equal to zero.

The choice of c_0 as a Higgs operator may seem a little ad hoc, but it becomes more natural if it is introduced by considering first the operator

$$Y = \sum_k c_{-k} \chi_k.$$

This operator is BRST invariant and like the BRST charge itself it is nilpotent. Thus we actually have a Q, Y algebra of the form

$$Q^2 = 0, \qquad Y^2 = 0, \qquad \{Q, Y\} = 0.$$

The condition $Y = 0$ reduces the Brower plus ghost-automorphism algebra $L \wedge R \oplus L(\beta)$ to $L^{(0)} \oplus L(\beta)$. Furthermore, since c_n and χ_n transform linearly with respect to

$$L^c = L^D(\alpha) + L^{(0)}(g),$$

and c_0 and χ_0 are L^c-invariant, one sees that $Y = 0$ defines two L^c orbits, namely

$$\{c_k = 0 \ (k \neq 0), \quad \chi_0 = 0\} \quad \text{and} \quad \{\chi_k = 0 \ (k \neq 0), \quad c_0 = 0\}.$$

Neither of these orbits is separately BRST invariant, and the second one corresponds to the above choice $\{c_0, k \cdot X(z)\}$ of Higgs-like operators.

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