RANDOM DYNAMICS AND RELATIONS BETWEEN THE NUMBER OF FERMION GENERATIONS AND THE FINE STRUCTURE CONSTANTS*

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The main topic of my talks is a set of relations (D. Bennet, H. B. Nielsen, I. Picek, *Phys. Lett.* **208B**, 275 (1988)) between the fine structure constants of the standard model which fit the experimental couplings surprisingly well. These relations were inspired from 'random dynamics', a project (H. B. Nielsen, D. Bennett, N. Brene, in *Recent Developments in Quantum Field Theory*, Proc. of the Niels Bohr Centennial Conference, Copenhagen 1985, eds. J. Ambjørn, B. J. Durhuus, J. L. Petersen, North Holland, Amsterdam 1985) on which I and several collaborators and also others have worked since long, and in particular from ideas about what we call confusion (H. B. Nielsen, N. Brene, in Proc. of the XVIII International Symposium, Ahrenshoop, Institut für Hochenenergiephysik, Akademie der Wissenschaften der DDR, Berlin-Zeuthen 1985). The basic assumption of the random dynamics project is that the fundamental physical 'laws' are so complicated that it is best to treat them as random, and furthermore that for phenomena that are practically accessible it does not matter precisely which random model Nature happens to make use of. As a subject that is in a sense not even physics but yet inspired from considerations of random dynamics I shall also talk about a prediction saying that the human race should be extinct or decrease appreciably in population within a few hundred years!

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1st Lecture

1. What does the standard model teach us?

1.1. Introduction

In the first lecture I shall not make any assumptions of random fundamental laws but rather proceed more phenomenologically in order to be more convincing. More precisely I want to look at the structure of the standard model, i.e. the Glashow-Weinberg-Salam

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[4, 5, 6] electroweak theory combined with QCD [7], and attempt from that to suggest a picture of the deeper physics. The standard model is in a certain sense our only reliable window to the deeper physics. As a tool for obtaining the relations involving the coupling constants we need the technique of mean field approximation on a lattice. I therefore give some introduction to this technique in Lecture 2. In Lecture 3 I start using the random dynamics project openly as a theory of everything, but I digress to a prediction of Doomsday. The fourth Lecture, finally, is about the 'derivation' of the standard model from a random dynamics point of view.

As an apéritif I would like to present to you our surprisingly well-satisfied relations between coupling constants and the number of generations N_{gen}. By generations or families we refer to the apparently dull repetition of fermions in the standard model. The first generation consists of the electron e, the electron neutrino ν_e, the up-quark u and the down--quark d. The second generation consists similarly of μ, ν_μ, s and c while the third consists of τ, ν_τ, b and the yet to be found top-quark t. As we have already found most of three generations it is very natural to wonder whether there should be more. According to our prediction, however, there are only these three.

It is an important feature of our scheme that the number N_{gen} is equal to the number of direct product factors in the essential part of *our* gauge group

$$G_N = \underbrace{\mathrm{SMG} \times \mathrm{SMG} \times \ \ldots \ \times \mathrm{SMG}}_{N_{gen}} \tag{1}$$

at a more fundamental (but not necessarily the most fundamental) level. Here

$$\mathrm{SMG} = \mathrm{S(U(2) \times U(3))} \sim \mathrm{U(1) \times SU(2) \times SU(3)} \tag{2}$$

stands for the group of the standard model, i.e. the gauge group having the Lie algebra U(1) × SU(2) (electroweak) together with SU(3) (QCD). Our model may be called anti--Grand Unified in the sense that the gauge group (1) is not a *simple* group as it is in the most popular Grand Unification models such as the SU(5) model of Georgi and Glashow [8]. Actually we would claim that the relations between coupling constants as given by this model are satisfied by accident only. Our competing model instead gives a relation between the number of generations and the absolute values for the 3 gauge coupling constants.

Our model is essentially based on the assumption that the coupling constants for all the SMG factors in (1) have a critical size at a fundamental energy scale at which the symmetry is broken. This is probably the Planck scale μ_{Planck}. By a critical size of a coupling constant we *here* mean the value separating two different phases for waves of wavelength corresponding to the scale μ^{-1}. This use of the word critical deviates from usual convention. The critical coupling here separates a 'Coulomb' phase and a 'confining' phase *at a scale* μ_{Planck}. The idea of such critical coupling at a certain scale is in practice implemented by using the Mean Field Approximation (M.F.A.) scheme. Our basic relation is

$$N_{gen}\alpha_i(\mu_{Planck}) = \alpha_{iPeter}(\mu_{Planck}) = \alpha_{icrit.M.F.A.}, \tag{3}$$

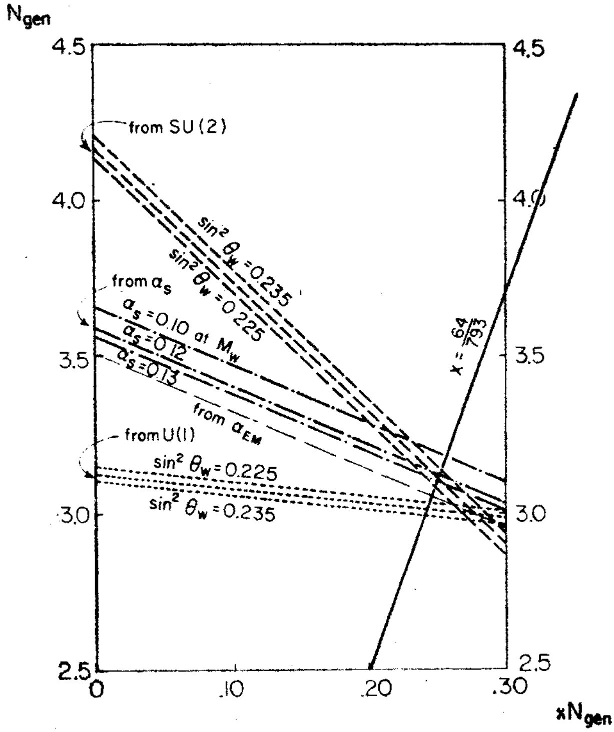

Fig. 1. Relation between the parameters xN_{gen} and N_{gen} corresponding to the experimental values for the 3 gauge coupling constants in the standard model

where the subscript i denotes one of the groups U(1), SU(2) or SU(3) from which the standard model Group SMG is built, and the subscript Peter denotes a factor $\mathrm{SMG}_{\mathrm{Peter}}$ in the direct product group

$$\underbrace{\mathrm{SMG} \times \dots \times \mathrm{SMG}}_{N_{gen}} = \mathrm{SMG}_{\mathrm{Peter}} \times \mathrm{SMG}_{\mathrm{Paul}} \times \dots \times \mathrm{SMG}_{N_{gen}}. \tag{4}$$

The factor N_{gen} in (3) is just a factor that turns out to relate the coupling constants squared in this situation, where a direct product group of isomorphic factors break down to the diagonal subgroup. The details will be explained later. Fig. 1 illustrates the confrontation of our model with the gauge couplings constants obtained from experiments. In this figure the three triplets of lines represent the relation between the parameters $x \times N_{gen}$ and N_{gen} for experimental values of the three 'fine structure constants'. Here x is a parameter which represents an ambiguity in putting the theory on a lattice such that the mean field approximation can be applied. We expect x to be rather small and it is indeed small in the fit, although in a sense just as large as it can be; I call it 'marginal convergence'.

The single line marked α_{EM} ($= 137.03604^{-1}$) and derived from the genuine (i.e. electromagnetic) fine structure constant is not independent of the couplings for two electroweak coupling constants α_1 for U(1) and α_2 for SU(2). Hence the crossing of the lines corresponding to these three couplings represents a mathematical triviality. However, we consider it a victory for our model that, to a good approximation, also the line marked α_S ($= \alpha_3$) given by the strong (QCD) coupling constant goes through the same point (0.27,3). This shows that the model is consistent with experiments.

The next victory is that the crossing of the three independent lines occurs at an integer value for the N_{gen}-parameter, namely $N_{gen} = 3$. Thus we predict three generations. It is surprising that the crossing points coincide so well and the value for N_{gen} is so near an integer. It fits within the experimental uncertainties for the Weinberg angle θ_W and the QCD coupling α_3. A posteriori I invented an argument saying that the ad hoc parameter x should not be bigger than the value given by

$$\frac{x}{1-x} = \left(\sqrt{\tfrac{2}{3}}\right)^{12}. \tag{5}$$

This happens to be very close to the fitted value.

1.2. Significant features of the standard model

The standard model is a rather loose unification of the electroweak $U(1) \times SU(2)$ gauge theory and the SU(3) gauge theory for QCD. Together with Einstein's theory of gravity the standard model form a theory of everything in the sense that essentially all experiments that can be performed with presently accessible instruments should be explainable in terms of this combined model. There are, however, huge calculational troubles in making a solid check of the QCD-part of the standard model. Various claims of new physics have been forwarded but at present no reliable results disagree with the standard model. Only very exceptional experiments look beyond the standard model such as e.g. the proton decay experiment. Cosmological investigations may also give information beyond the standard model.

As long as all experiments agree with the standard model and the classical theory of gravity these models constitute our only window to physics at a higher energy level, to a possible Theory of Everything, which might for instance be a superstring theory.

Thus it becomes very important to squeeze as many suggestions as possible out of the experimentally fitted parameters (masses and coupling constants) and the structure (groups and their representations) of the standard model and of gravity.

Let us now look at various salient features of the standard model and attempt to see what they may teach us:

1) The standard model is a Yang Mills theory. This means that it is a (quantum) field theory characterized by a set of symmetry operations acting on the fields, one for each choice of a gauge function $\Lambda(x)$ mapping the Minkowski space-time into the 'gauge group' G. The characterstic fields of Yang Mills theories are the gauge fields $A^a_\mu(x)$ which are 4-vector fields marked by the name of a basis vector in the Lie algebra of the gauge group G. Representing the basis vectors by matrices $\lambda^a/2$ the Yang Mills fields can be collected into

$$A_\mu(x) = A^a_\mu(x)\,\frac{\lambda^a}{2}\,. \tag{6}$$

The gauge transformation may be written

$$gA_\mu(x) \to \Lambda(x)gA_\mu(x)\Lambda^{-1}(x)+\Lambda(x)\partial_\mu\Lambda^{-1}(x), \tag{7}$$

where $\Lambda(x)$ is the gauge function in the representation $\lambda^a/2$. The matter fields: fermion fields and the possible but unobserved, doubtful Higgs field transform as

$$\Psi(x) \to \Lambda_r(x)\Psi(x), \tag{8}$$

where $\Lambda_r(x)$ represent the gauge function in the (possibly reducible) representation r. The mutual interaction of the gauge fields as well as their interactions with the matter fields are almost uniquely given by the requirement of symmetry under these local gauge transformations. The only unspecified parameters for these interactions are the values of the gauge coupling constants, one for each factor in the Lie algebra when resolved into simple and abelian Lie algebras.

The importance of gauge symmetry in the standard model and of reparametrization invariance (diffeomorphism symmetry) in General Relativity suggests that whatever type of theory that might be proposed on a fundamental level it should either be a gauge field theory or lead to gauge field theory in a natural way. The latter is true for string theories. They were originally investigated as candidates for models of hadron physics. A major problem was that they kept on giving massless gauge bosons instead of the massive vector mesons ϱ, ω, etc. Although there now are string models without massless gauge particles on the market [9] it is still true that string theories naturally leads to gauge particles. We [10, 11] have argued, however, that it *is* possible to obtain massless gauge particles without assuming exact gauge symmetry a priori. This is a warning against taking the appearance of gauge particles as a very strong evidence for a fundamental string theory.

Whatever mechanism is responsable for the observed gauge symmetries it may presumably give rise to further gauge symmetries.

2) The standard model comprises only particles which are massless (apart from Higgs particles) in the gauge symmetric limit. This means that, were it not for the spontaneous breakdown of the gauge symmetry caused by the Higgs field, all observed particles would be massless. The Higgs mechanism is just invented to cause this breakdown and no Higgs particles have been observed up to now.

Thus the 'observed' particles of the standard model are only:

1. the gauge particles themselves:
   the photon γ, the intermediate bosons of weak interaction W^+, W^-, and Z^0 and the gluons of QCD,
2. the spin $\frac{1}{2}$ fermions:
   the leptons: e^-, μ^-, τ^-, and the corresponding neutrinos,
   and the quarks: u, d, c, s, b (t is not yet observed) each occuring in three colors.

We count all these particles as observed although some of them have only been seen in a rather indirect way as jets or as components in composite hadrons.

1. The gauge particles would be massless provided the symmetry breaking mechanism is switched off and effects of infrared slavery are ignored i.e. one considers a classical approximation.

2. The fermions of the standard model have such properties under gauge transformations that a gauge invariant Lagrangian has no place for mass terms. In the standard model the left- and right-handed fermions belong to different representations of the gauge group (2). This explains parity violation in weak interactions. Left and right handed fermions are usually described by projection operators

$$(1 \pm \gamma_5)/2 \tag{9}$$

acting on 4-component Dirac spinor fields $\Psi(x)$. In the Weyl representation we have

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tag{10}$$

where I is a (2×2) unit matrix. Hence the left- and right-handed fermion fields can be written in the form

$$\Psi_L = \tfrac{1}{2}(1+\gamma_5)\Psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix},$$

$$\Psi_R = \tfrac{1}{2}(1-\gamma_5)\Psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}. \tag{11}$$

Thus the standard model makes use of the two component Weyl fields ψ_L and ψ_R.

While the kinetic term in the Lagrangian

$$i\overline{\Psi}(x)\gamma^\mu D_\mu \Psi(x) = i\overline{\Psi_L}(x)\gamma^\mu D_\mu \Psi_L(x) + i\overline{\Psi_R}(x)\gamma^\mu D_\mu \Psi_R(x) \tag{12}$$

keeps the left and right components separated, the usual mass term

$$m\overline{\Psi}(x)\Psi(x) = m(\overline{\Psi_L}(x)\Psi_R(x) + \overline{\Psi_R}(x)\Psi_L(x)) \tag{13}$$

mixes these components. Hence a Weyl particle, i.e. a particle described by either a left or a right field (ψ_L or ψ_R) but not both, cannot have a mass. The symbol

$$D_\mu = \partial_\mu - iA_\mu(x) \tag{14}$$

in (12) denotes the covariant derivative. One can write a Majorana mass term

$$mC\overline{\Psi_L}^{\mathrm{T}}(x)\Psi_L(x) \tag{15}$$

using a two component field ψ_L. Here C is the matrix for charge conjugation. A Majorana mass term, however, requires that the (Majorana) fermion is totally neutral because it is its own antiparticle.

None of the fermions in the standard model are totally neutral, hence they cannot have Majorana masses. As the left- and right-handed fermions in this model never have coincident gauge quantum numbers a mass term of the type (13) would destroy the gauge symmetry and therefore cannot occur either. We say that all the fermions of the standard model are mass protected. On the other hand, any pair of particles with gauge quantum numbers allowing a mass term is expected to have a mass which probably is of the order one on a fundamental mass scale. If this scale is high the absence of such particles at our low scale is understandable. All the observed particles of the standard model owe their low masses to some symmetry violating mechanism, e.g. the Higgs mechanism. This is even true for the gauge bosons which are mass protected by the requirement of gauge invariance. The 'observed' particles of the standard model do not include a Higgs particle.

The fact that all the observed particles of the standard model are mass protected suggests that the standard model is not a complete theory but rather a low energy limit of a more fundamental theory in which particles without mass protection do exist. The natural scale for such theory must be at least an order of magnitude higher than our presently accessible energies.

3) The standard model is relevant at energies less than about

$$\mu_{SM} \approx 100 \text{ GeV} \ll \mu_{Planck} \approx 10^{19} \text{ GeV}.$$

This fact supports the suggestion that the standard model is a low energy limit of a more fundamental theory.

4) The standard model is characterized by a Lie algebra (2) composed of abelian and simple algebras of low dimensionality, in fact the lowest possible algebras corresponding to compact groups. Let us first note the sense in which the three algebras of the standard model group are small. The dimensions of these algebras are

$$\dim(U(1)) = 1; \quad \dim(SU(2)) = 3; \quad \dim(SU(3)) = 8. \tag{16}$$

For comparison the dimensions of a number of other small simple algebras are

$$\begin{aligned} &\dim(SO(5)) \sim Sp(4)) = 10; && \dim(G_2) = 14; \\ &\dim(SU(4)) \sim SO(6)) = 15; && \dim(Sp(6)) = 21; \\ &\dim(SO(7)) = 21; && \dim(SU(5)) = 24. \end{aligned} \tag{17}$$

Nonabelian solvable Lie algebras [12] with small dimension do exist but they can at best lead to ordinary gauge theories of still smaller dimensions, as we shall show in a forthcoming paper.

The fact that the group (2) has only small factors

$$U(1), \quad SU(2), \quad SU(3)$$

does not allow us to draw any conclusion concerning a fundamental theory because it could have many reasons. One is just simplicity. Another might be phrased like this: If Nature were going to invent groups it would be easiest to invent small groups. A third involves the variation of the running coupling constants as functions of the energy scale. From rather loose arguments we expect that bigger groups usually confine at higher energy than smaller groups do. Here we implicitly assume the existence of a theory containing larger groups at a higher energy scale. A fourth reason involves Grand Unification. The known examples of Grand Unification groups all lead to the right SMG after break down. This, however, is just because their Higgs systems are tailored to do so. Only if the Grand Unification group itself is small can we be sure that it leads to small factors in the surviving group.

5) It is a fact that in the standard model each of the small factor groups occurs only

once. As mere chance alone would almost never lead to a group without some factor occurring several times we are tempted to seek a deep reason for this fact. We want to promote it to a principle that needs an explanation: At the energy scale $\mu_{\mathrm{LEP}} \approx 100$ GeV (LEP = Low Energy Phenomena) no factor group occurs more than once. To see that it is unlikely that this principle be satisfied by accident let us list all the Lie algebras of dimension 12 (the algebra of the standard model has dimension 12). There are 8 such algebras:

$$\mathrm{U}(1)^{12},\ \mathrm{U}(1)^{9}\times \mathrm{SU}(2),\ \mathrm{U}(1)^{6}\times \mathrm{SU}(2)^{2},\ \mathrm{U}(1)^{3}\times \mathrm{SU}(2)^{3},\ \mathrm{SU}(2)^{4},\ \mathrm{U}(1)^{4}\times \mathrm{SU}(3),$$
$$\mathrm{U}(1)\times \mathrm{SU}(2)\times \mathrm{SU}(3),\ \mathrm{U}(1)^{2}\times \mathrm{SO}(5). \tag{18}$$

Multiple occurrence of some factor algebra is clearly the rule, statistically. This leads to the conclusion that a fundamental theory either

A) must be very special in that it cannot lead to multiple occurrences of any factor group at any level, or

B) must have room for a mechanism which can break such multiples down to single factor algebras.

The first possibility A) appears difficult to realise in theories with sequential breakdown (several Grand Unifications). It is rather likely that a breakdown will lead to a U(1) subgroup among the surviving factors. A maximal subgroup contain very often a U(1) [15]. Thus several U(1)'s can easily occur in such theories. One might of cause imagine some Higgs-like mechanism which systematically eats the U(1) factors (or other groups) but why should it not eat the last factor and destroy even the standard model group!

Next we turn to possibility B). The kind of break down of a product of n_G groups

$$G_n = \underbrace{G\times G\times \ldots \times G}_{nG} \tag{19}$$

is in principle possible because of the existence of the diagonal subgroup (in which all the factors transform with the same group element)

$$G_{\mathrm{diag}} = \{\underbrace{(U, U, \ldots, U)}_{n_G} \mid U \in G\} \subseteq G_n. \tag{20}$$

In the last lecture we shall demonstrate one mechanism (confusion) for such breakdown. The Higgs mechanism is another possibility.

The difficulties of models of type A) suggest that Nature prefers models of type B) and that at some energy scale(s) we find a group structure of the kind

$$\begin{aligned}&\underbrace{\mathrm{U}(1)\times \ldots \times \mathrm{U}(1)}_{n_{\mathrm{U}(1)}}\\ &\times \underbrace{\mathrm{SU}(2)\times \ldots \times \mathrm{SU}(2)}_{n_{\mathrm{SU}(2)}}\times \underbrace{\mathrm{SU}(3)\times \ldots \times \mathrm{SU}(3)}_{n_{\mathrm{SU}(3)}}\times \ldots \times \underbrace{\mathrm{SU}(N)\times \ldots \times \mathrm{SU}(N)}_{n_{\mathrm{SU}(N)}}\\ &\times \ldots \times \underbrace{\mathrm{SO}(5)\times \ldots \times \mathrm{SO}(5)}_{n_{\mathrm{SO}(5)}}\times \ldots \times \underbrace{\mathrm{SO}(N)\times \ldots \times \mathrm{SO}(N)}_{n_{\mathrm{SO}(N)}}\times \ldots \times \ldots\end{aligned} \tag{21}$$

which tend to break (possibly stepwise) into

$$U(1)\times SU(2)\times SU(3)\times \ldots \times SU(N)\times SO(5)\times SO(7)\times \ldots \times \ldots . \tag{22}$$

6) The standard model shows the only known example of breakdown of a gauge symmetry in particle physics

$$U(1)\times SU(2) \Rightarrow U(1)_{EM} \subseteq U(1)\times SU(2) \tag{23}$$

which breaks the gauge symmetry of the Salam-Weinberg theory into the gauge symmetry of the electromagnetic theory. This symmetry breakdown resembles more to the example of (19) to (20) than to the most popular examples of breakdown in Grand Unification theories in the following sense. In the cases of Grand Unification breakdown a simple group is broken into a product of simple groups and U(1)'s which are trivially embedded in the original group. In the breakdown (23) the end product is a single U(1) which is not trivially embedded in any of the factors of the original group. In fact the surviving $U(1)_{EM}$ is the *diagonal* subgroup of the product of the original $U(1)_Y$ (weak hypercharge) and a subgroup $U(1)_{W^0}$ of the original $SU(2)_{WI}$ (weak isospin)

$$U(1)_{EM} = U(1)_{diag} \subseteq U(1)\times U(1)_{W^0} \subseteq U(1)\times SU(2). \tag{24}$$

We may formally contemplate that the breakdown of the gauge symmetry in the Salam-Weinberg theory occurs in two steps, the first leading to a product of two U(1) groups the second leading to a diagonal subgroup of this product. Similar sequences may be relevant at higher energy.

7) In the standard model we find big ratios between the masses of the fundamental fermions, both between the lepton masses and between the quark masses. This suggests the existence of (several) approximately conserved quantum numbers which suppress with various strengths the masses of the various fermions.

In the Salam-Weinberg theory the Higgs fields serve the double purpose of breaking the symmetry and providing masses for the fermions via Yukawa type couplings

$$h_{ij}\bar{\Psi}_{R,i}(x)\Psi_{L,j}(x)\phi(x)+\text{h.c.} \tag{25}$$

To provide large ratios between masses we need large ratios between coupling constants. A large number of coupling constants with wildly different magnitudes requires some explanation. Here the approximately conserved quantum numbers come in, leading to suppressed coupling constants and thereby reduced masses. It would be preferable to identify these quantum numbers with gauge quantum numbers at some higher level.

In usual unification models with a simple group (Grand Unification) all the generations belong to equivalent representations; at least in the SU(5) Grand Unification model it is so since all generations belong to $\underline{5} \oplus \underline{10}$. We need some extra quantum numbers which change from one generation to the next to explain the large mass ratios but we do not find them among the broken quantum numbers of SU(5). In models using cross-product groups which break down to diagonal subgroups it is, however, easy to construct models in which different generations have different gauge quantum numbers. An example is provided by the gauge group of (4). Simply let one generation transform nontrivially according to the Peter group only, the next according to the Paul group only, etc. One might extend the model imagining that the Peter and the Paul groups are subgroups of different groups at some higher level. Thus the need for extra (i.e. generation) quantum numbers to explain the mass ratios suggests that the standard model group results from a cross product of several groups breaking to their diagonal subgroup rather than from a simple Grand Unification group.

8) The fermions as well as the Higgs particles of the standard model belong to such representations of the gauge algebra that the electric charge

$$Q = t_3 + y \tag{26}$$

is quantized according to the rule

$$Q = -\tfrac{1}{3} \cdot \text{`triality'} \pmod 1). \tag{27}$$

Here t_3 is the third component of the weak isospin (SU(2)), y is the weak hypercharge (U(1)), and 'triality' is the number of triplets $\underline{3}$ needed to build up the SU(3) representation in question. According to this rule the leptons must have integer electric charge because the leptons have 'triality' zero. The quarks, on the other hand, have 'triality' equal $+1$; hence they can have electric charges 2/3 or $-1/3$. The rule of charge quantization (27) may be deduced from the assumption that all the representations of the gauge algebra realised on matter fields must be genuine representations of the *group*

$$S(U(2)\times U(3)) = (\mathbf{R}\times SU(2)\times SU(3))\Big/\left\{\left(2\pi,\ -I_{2\times 2}, \exp\left(i\frac{2\pi}{3}\right)I_{3\times 3}\right)^n\right\}. \tag{28}$$

This group can be defined as

$$S(U(2)\times U(3)) =$$

$$= \left\{ \begin{bmatrix} \begin{matrix} & \\ U_2 & \\ & \end{matrix} & \begin{matrix} 0 & 0 & 0 \\ & & \\ 0 & 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} & U_3 \end{bmatrix} \ U_2 \in U(2) \wedge U_3 \in U(3) \wedge \det U_2 \cdot \det U_3 = 1 \right\}. \tag{29}$$

It is the factor group obtained from the covering group $\mathbf{R}\times SU(2)\times SU(3)$ by dividing out the discrete invariant subgroup $\left\{\left(2\pi, -I_{2\times 2}, \exp\left(i\frac{2\pi}{3}\right)I_{3\times 3}\right)^n\right\}$ [12, 13]. Thus the rule of charge quantization (27) suggests that a) *groups* are important and b) the *group* behind the standard model is S(U(2)×U(3)) rather than its covering group $\mathbf{R}\times SU(2) \times SU(3)$. It is well-known that the charge quantization (27) is a consequence of the SU(5) model. That is also in accordance with the rules because the covering group of the SU(5) algebra has SMG as a subgroup but not $\mathbf{R}\times SU(2)\times SU(3)$. Other types of models exist for which groups rather than algebras are essential, e.g. lattice gauge theories or theories with monopoles.

9) The representations in which we find the matter fields of the standard model are

small. In fact a generation of fermions constitute the smallest anomaly-free and massprotected (reducible) representation of the standard model group [14]. Analogously, we may expect to find small group representations at a more fundamental level.

10) Consider the gauge coupling constants of the standard model. Two of them are rather small; the values for $4\pi\alpha_i(M_W) = g_i^2(M_W)$ ($i = 1, 2$ for U(1) and SU(2)) are 0.12 and 0.40, respectively. The third is larger: 1.51. After extrapolation to near the Planck energy level, however, they will all be rather small ≈ 0.3, provided there are not many particles heavier than the W particles. As a value closer to 1 would appear more likely this suggests some mild reduction factor. Such reduction is natural in a model where a cross product group breaks down to its diagonal subgroup

$$G_{\text{Peter}} \times G_{\text{Paul}} \times \ldots \times G_n \Rightarrow G_{\text{diag}}, \tag{30}$$

with coupling g_{diag}. In fact one can show the relation

$$\frac{1}{g_{\text{diag}}^2} = \frac{1}{g_{\text{Peter}}^2} + \frac{1}{g_{\text{Paul}}^2} + \ldots + \frac{1}{g_n^2}. \tag{31}$$

A good value for n is about 3–4.

At first sight the SU(5) Grand Unification does not seem to produce a corresponding reduction of the coupling constant because of the relation

$$\tfrac{5}{3}\, g_1^2 = g_2^2 = g_3^2 = g_5^2, \tag{32}$$

where g_5 is the SU(5) coupling constant. This relation, however, depends on the convention for normalizing the structure constants. To avoid problems about normalization we suggest to compare the couplings to the corresponding critical coupling (in the same group and the same normalization) in mean field approximation. With the normalization used in (32) we find $g_5^2 \approx 0.3$ which is to be compared with the critical coupling $g_{5\text{crit}}^2 \approx 0.5$. If we assume a bigger GUT group (e.g. SO(10)) the critical coupling is even smaller, i.e. closer to the corresponding ('experimental') coupling.

We conclude that in both a diagonal breaking scheme (30) and a simple group Grand Unification scheme the smallness of the coupling constants of the standard model is in accordance with values near the critical coupling (≈ 1) at a fundamental level. What really matters is that the (breaking down) group at the fundamental level is sufficiently large. The simple SU(5) is barely large enough for this purpose. The cross product group (30) requires around 3 factors.

2nd Lecture

2. *Mean field approximation and the inequality*

2.1. Mean field approximation

What is the fundamental quantity to describe the strength of a gauge coupling, g or g^2 or $g^2/(4\pi)$? What is the convention for normalizing the involved structure constants and thereby the coupling constants? These questions become essential when we want to

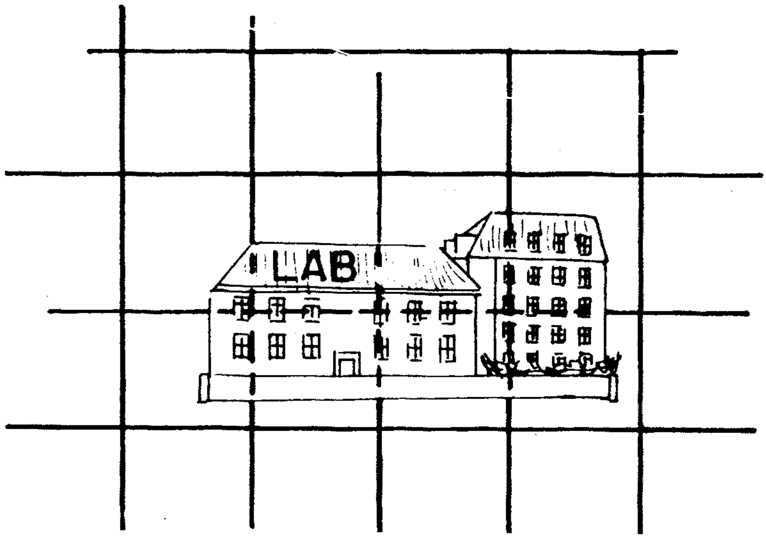

Fig. 2. A laboratory with dimensions comparable to the scale of fundamental lattice

compare values of coupling constants for different groups and give their absolute magnitude. One way to make a notation independent statement about a coupling constant is to compare it to a value characteristic of a physical feature such as a phase transition. In fact one might use some critical coupling as a unit for couplings. This procedure is useful even if the critical coupling is calculated only in a crude but well-defined approximation, e.g. the lowest order mean field approximation. We are going to use this approximation to decide whether physics *at a certain energy scale* μ behaves like being in a confining phase or in a Coulomb phase. By physics at a given scale we mean the behaviour of waves with wavelength near that scale.

One can imagine a laboratory (see Fig. 2) with dimensions comparable to μ^{-1} and thus unable to observe waves of wavelength larger than μ^{-1}. Would this laboratory be able to decide whether we are in a Coulomb or a confining phase? The answer is yes — for wavelengths shorter than μ^{-1}. It would not see confinement at a much longer scale. If the scale μ^{-1} is the fundamental scale $\approx \mu_{\text{Planck}}^{-1}$ it would be able to make statements about the phase referring to that scale only. The figure suggests that we assume physics at the fundamental scale can be treated as if it occurred on a kind of lattice; this assumption, however, is not necessarily of critical importance. Let me therefore give a short review of the mean field approximation for a gauge theory on a lattice.

In the following we shall for definiteness consider a hypercubic lattice for which the 'sites' are points in a 4-dimensional space time

$$x^\mu = an^\mu, \qquad n^\mu \in \mathbf{Z}^4. \tag{33}$$

The length a is called the lattice constant. Each pair of nearest neighbour sites is connected by a 'link'. In a lattice gauge theory the rôle of the field variables is played by the link variables, for which the values are elements in the gauge group G

$$U(-) \in G. \tag{34}$$

A Feynman path is identified with a group-valued function defined on the set of all links in the lattice

$$U: \{\text{links}\} \to G. \tag{35}$$

We often represent link elements $U(-)$ by matrices obeying the composition rules of the group G. Then a crude connection to the usual continuum Yang-Mills field theory is provided by the naive continuum limit

$$U\left(\underset{x \quad x+a\delta_\rho}{\bullet\!\!-\!\!\!-\!\!\bullet}\right) = \exp\{iag\mathbf{A}_\rho(x + \frac{1}{2}a\delta_\rho)\} \tag{36}$$

where $A_\varrho(x)$ is a matrix of gauge fields (6). A link and its inverse (going from x to $x+\delta_\varrho a$ and back) are represented by group elements which are inverse of each other.

In order to write the *action* S of a lattice gauge theory in a compact way one introduces the plaquette variables. To each plaquette, i.e. closed loop of four links, corresponds a variable which is the product of the four link variables, ordered according to the direction of traversing

$$U(\square) = U(\underline{\square})\, U(\square|)\, U(\overline{\square})\, U(|\square). \tag{37}$$

The value of a plaquette variable is a group element like the value of a link variable. The action (a real quantity; $\Re$ and $\Im$ denote real and imaginary parts) is then written

$$S = \beta \sum_{\square} \Re\, \mathrm{Tr}\,(U(\square)). \tag{38}$$

Expectation values and correlation functions can be derived from the functional integral, the *partition function*

$$Z = \int \mathscr{D}U(-) \exp(S), \tag{39}$$

where

$$\mathscr{D}U(-) = \prod_{\text{links}} d^{\text{Haar}}U(-) \tag{40}$$

and $d^{\text{Haar}}U(-)$ is the Haar measure on the gauge group G. Up to a normalization it is the only measure invariant under right (or left) multiplication

$$d^{\text{Haar}}U(-) = d^{\text{Haar}}(U(-) \cdot B)\ \forall\, B \in G. \tag{41}$$

The expectation value of some functional of the fields $A[U]$ is

$$\langle A \rangle = \frac{\int \mathscr{D}U(-) A[U] \exp(S)}{\int \mathscr{D}U(-) \exp(S)}. \tag{42}$$

For the sake of simplicity we shall in the following restrict ourselves to the gauge group U(1). One way to introduce the mean field approximation is to start from the identity

$$1 = \int \mathscr{D}V^{R}\mathscr{D}V^{I} \prod_{\text{links}} \delta(V^{R}-\Re U(-)) \prod_{\text{links}} \delta(V^{I}-\Im U(-)). \tag{43}$$

Here

$$\mathscr{D}V^{R} = \prod_{\text{links}} dV^{R}(-); \quad \mathscr{D}V^{I} = \prod_{\text{links}} dV^{I}(-),$$

and

$$V^{R}(-), \quad V^{I}(-)$$

are real variables defined on all links. Representing the δ-functions by their Fourier expansion one obtains

$$1 = \int \mathscr{D}V^{R}\mathscr{D}V^{I}\mathscr{D}\alpha^{R}\mathscr{D}\alpha^{I}$$

$$\exp\{i \sum_{\text{links}} (\alpha^{I}(-)(V^{I}(-)-\Im U(-))$$

$$+\alpha^{R}(-)(V^{R}(-)-\Re U(-)))\}, \tag{44}$$

where

$$\mathscr{D}\alpha^{I,R} = \prod_{\text{links}} \frac{d\alpha^{I,R}}{2\pi}.$$

This expression for the unit operator is introduced in the partition function

$$Z = \int \mathscr{D}U \exp(S[U])$$

$$= \int \mathscr{D}U\mathscr{D}V^{R}\mathscr{D}V^{I}\mathscr{D}\alpha^{R}\mathscr{D}\alpha^{I} \exp\{S[U]+i\sum_{\text{links}}(\alpha^{R}(-)(V^{R}(-)-\Re U(-))$$

$$+\alpha^{I}(-)(V^{I}(-)-\Im U(-)))\}. \tag{45}$$

Using the δ-functions (44) we are allowed to replace U by V in the expression for $S[U]$. We may even use our freedom in this insertion to replace $U^{-1} = U^{*}$ by $V^{R}-iV^{I}$ instead of $(V^{R}+iV^{I})^{-1}$ and avoid having $V^{R,I}$ in the denominator. If we define

$$\exp\{W(\alpha^{R}(-), \alpha^{I}(-))\}$$

$$= \int d^{\text{Haar}}U(-) \exp\{-i\alpha^{R}(-)\Re U(-) - i\alpha^{I}(-)\Im U(-)\}$$

$$= (2\pi)^{-1} \int d\theta(-) \exp\{-i\alpha^{R}(-)\cos\theta(-) - i\alpha^{I}(-)\sin\theta(-)\}$$

$$= (2\pi)^{-1} \int d\hat{\theta} \exp\{-i\sqrt{(\alpha^{R}(-))^{2}+(\alpha^{I}(-))^{2}}\cos\hat{\theta}\}$$

$$= I_{0}(-i\sqrt{(\alpha^{R}(-))^{2}+(\alpha^{I}(-))^{2}}) \tag{46}$$

or

$$W(\alpha^{\mathrm{R}}, \alpha^{\mathrm{I}}) = \ln I_0(-i\sqrt{(\alpha^{\mathrm{R}})^2 + (\alpha^{\mathrm{I}})^2}), \tag{47}$$

where I_0 is the modified Bessel function

$$I_0(z) = \int \frac{d\theta}{2\pi} \exp(z\cos\theta), \tag{48}$$

we obtain

$$Z = \int \mathscr{D}V^{\mathrm{R}}\mathscr{D}V^{\mathrm{I}}\mathscr{D}\alpha^{\mathrm{R}}\mathscr{D}\alpha^{\mathrm{I}} \exp(S_{\mathrm{eff}}). \tag{49}$$

Here

$$S_{\mathrm{eff}}[V, \alpha] = S[V^{\mathrm{I}}, V^{\mathrm{R}}] + \sum_{\mathrm{links}} [i\alpha^{\mathrm{R}}(-)V^{\mathrm{R}}(-) + i\alpha^{\mathrm{I}}(-)V^{\mathrm{I}}(-) + W(\alpha^{\mathrm{R}}(-), \alpha^{\mathrm{I}}(-))]. \tag{50}$$

We can define the lowest order mean field approximation as the classical approximation using this effective action S_{eff}.

The choice of having no $V^{\mathrm{R,I}}$ in the denominator corresponds to the form

$$S[V^{\mathrm{I}}, V^{\mathrm{R}}] = \beta \sum_{\square} \Re V(\square), \tag{51}$$

where

$$V(\square) = \left(V^R(\rightarrow) + \mathrm{i}V^I(\rightarrow)\right)\left(V^R(\uparrow) + \mathrm{i}V^I(\uparrow)\right)\left(V^R(\leftarrow) - \mathrm{i}V^I(\leftarrow)\right)\left(V^R(\downarrow) - \mathrm{i}V^I(\downarrow)\right). \tag{52}$$

In seeking a translational invariant vacuum we may choose the 'imaginary parts' of the V's and the α's to be zero

$$\begin{aligned} V^{\mathrm{R}}(-) &= V^{\mathrm{R}} \quad \text{(same value for all links)} \\ V^{\mathrm{I}}(-) &= 0 \\ \alpha^{\mathrm{R}}(-) &= \alpha^{\mathrm{R}} \quad \text{(same value for all links)} \\ \alpha^{\mathrm{I}}(-) &= 0. \end{aligned} \tag{53}$$

The (classical) equations of motion obtained from

$$\delta S_{\mathrm{eff}} = 0$$

become

$$V^{\mathrm{R}} + \frac{\partial W(\alpha^{\mathrm{R}}, 0)}{i\partial\alpha^{\mathrm{R}}} = 0, \tag{54}$$

$$i\alpha^{\mathrm{R}} + 4\beta \sum_{\square}{}' (V^{\mathrm{R}})^3 = 0, \tag{55}$$

where the sum Σ' is taken over all plaquettes having a particular link in common. From the definition (46)–(47) of W we get

$$\frac{\partial W(\alpha^{\mathrm{R}},0)}{\partial(i\alpha^{\mathrm{R}})}=\frac{\partial \ln I_0(i\alpha^{\mathrm{R}})}{\partial(i\alpha^{\mathrm{R}})}=\frac{I_0'(i\alpha^{\mathrm{R}})}{I_0(i\alpha^{\mathrm{R}})}=\frac{I_1(i\alpha^{\mathrm{R}})}{I_0(i\alpha^{\mathrm{R}})}, \tag{56}$$

where we have used the modified Bessel function

$$I_n(z)=\int\frac{d\theta}{2\pi}\exp(z\cos\theta+in\theta). \tag{57}$$

The freedom in choosing the gauge can be used to improve the accuracy of the approximation. The most preferable choice for this purpose is the axial gauge. Here one requires that all links in one direction have unit value

$$U(-)=1 \quad \text{(for all links in the chosen direction).} \tag{58}$$

With this gauge the Euler-Lagrange equations (54)-(55) are modified into

$$V^{\mathrm{R}}+\frac{I_1(i\alpha^{\mathrm{R}})}{I_0(i\alpha^{\mathrm{R}})}=0, \tag{59}$$

$$i\alpha^{\mathrm{R}}+4\beta(2(d-2)(V^{\mathrm{R}})^3+2V^{\mathrm{R}})=0 \tag{60}$$

from which follows (d is the dimension of space time; we use the value $d=4$)

$$i\alpha^{\mathrm{R}}=-4\beta(2(d-2)(V^{\mathrm{R}})^3+2V^{\mathrm{R}})=4\beta\left(4\left(\frac{I_1(i\alpha^{\mathrm{R}})}{I_0(i\alpha^{\mathrm{R}})}\right)^3+2\frac{I_1(i\alpha^{\mathrm{R}})}{I_0(i\alpha^{\mathrm{R}})}\right). \tag{61}$$

This transcendental equation can be solved grafically. Consider the curves corresponding to two functions of $|V^{\mathrm{R}}|$:

1. $f_1(|V^{\mathrm{R}}|)=\ln(i\alpha^{\mathrm{R}})$ with α_i determined by (59);
2. $f_2(|V^{\mathrm{R}}|;\beta)=\ln(4|V^{\mathrm{R}}|^3+2|V^{\mathrm{R}}|)+\ln(4\beta)$. Curves with different values of the constant β are generated by displacing the curve along the ordinate.

A crossing point for the two curves provides a solution of the equation (61). Fig. 3 shows curves for $f_1(|V^{\mathrm{R}}|)$ and for $f(|V^{\mathrm{R}}|;\beta)$ with various values of β. For $\beta\to 0$, i.e. for very strong coupling, there is only one, viz. the trivial, solution

$$\alpha^{\mathrm{R}}=0, \qquad V^{\mathrm{R}}=0. \tag{62}$$

This solution is indeed possible for all values of β. For increasing values the curves develop in addition first one point of contact which successively splits into two cross-over points. For such β-values the partition function (39) is dominated by that solution of (59) and (60) or (61) for which the functional integral Z assumes its largest value. Hence it is important to calculate Z or $\ln Z$ (which is essentially the free energy with opposite sign) in the three approximations corresponding to taking either one of the three solutions alone. For the trivial solution (62) one finds $\ln Z=0$ in the classical approximation. A better

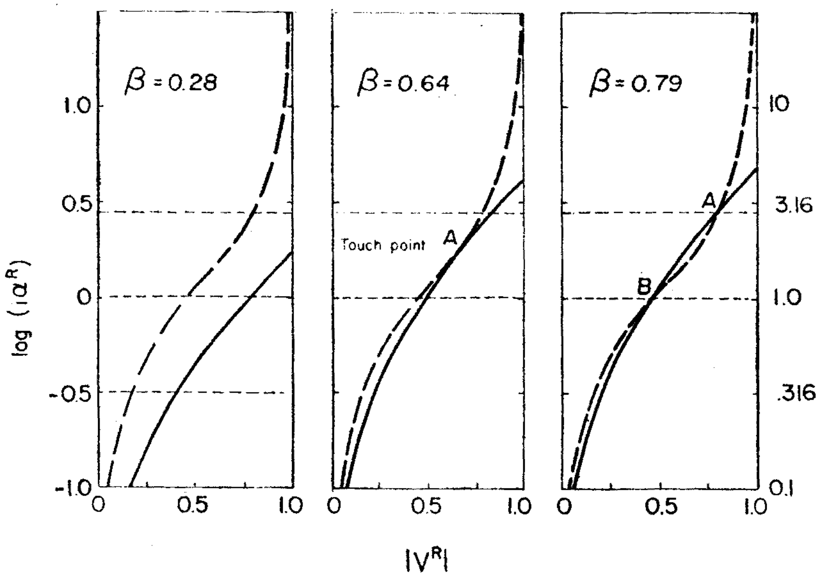

Fig. 3. The value of ln ($i\alpha^{R}$) as a function of $|V^{R}|$. The dashed curve f_1 represents equation (59) and the full curve f_2 equation (60)

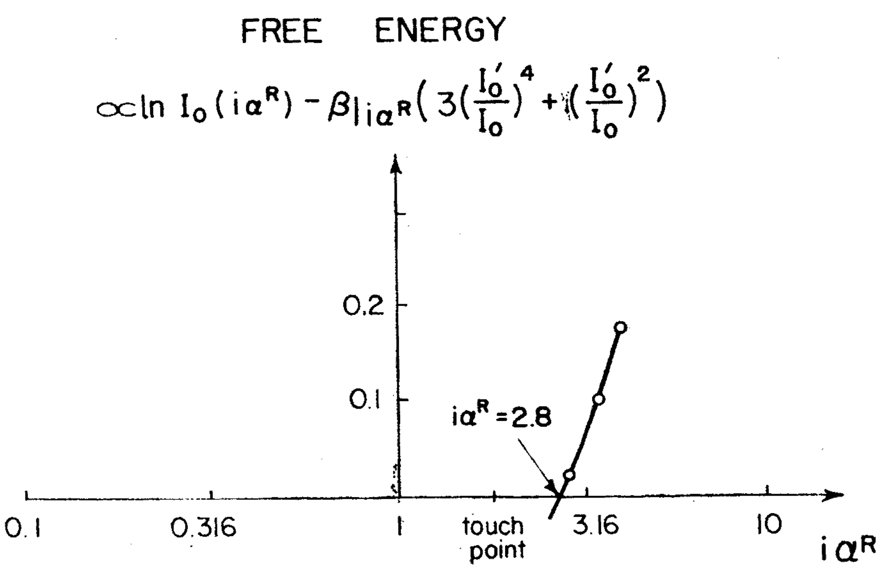

Fig. 4. ln Z (up to a constant factor) as a function of $i\alpha^{R}$ for the solution corresponding to the biggest value of $|V^{R}|$

approximation might be achieved by making a strong coupling expansion, a power series in β. Of the two remaining solutions only the one with the biggest value of $|V^{R}|$ has a chance in the competition for giving a maximal value of Z. In Figure 4 we have shown ln Z (up to a constant factor) as a function of $i\alpha^{R}$ for the solution corresponding to the biggest value of $|V^R|$ (In this we have considered β as a function of $i\alpha^R$). At the value of $i\alpha^R$ for which $\ln Z$ passes from negative to positive the dominance changes from the trivial to this other solution. For this critical point I found (for the group U(1))

$$i\alpha^R = 2.8 \quad (U(1)) \tag{63}$$

and the corresponding value

$$\beta_{\text{crit}} = 0.79 \quad (U(1)). \tag{64}$$

Drouffe and Zuber [16] give the values of the critical couplings in the mean field approximation with strong coupling corrections. When their values are translated to our notation in which in the lattice action is

$$S = \beta \sum_{\square} \Re \operatorname{Tr}(U_N(\square)), \quad \beta = \frac{2}{g_N^2} \tag{65}$$

we obtain for U(N) groups, $N = 1, 2, \ldots, 5$

$$\begin{aligned}
\beta_{\text{crit.1}} &= 2/g^2_{\text{crit.1}} = 1.03, \\
\beta_{\text{crit.2}} &= 2/g^2_{\text{crit.2}} = 3.45/2 = 1.72, \quad \beta_{\text{crit.2}}/2 = 0.86; \quad g^2_{\text{crit.2}} = 1.16, \\
\beta_{\text{crit.3}} &= 2/g^2_{\text{crit.3}} = 7.30/3 = 2.43, \quad \beta_{\text{crit.3}}/3 = 0.81; \quad g^2_{\text{crit.3}} = 0.82, \\
\beta_{\text{crit.4}} &= 2/g^2_{\text{crit.4}} = 12.5/4 = 3.13, \quad \beta_{\text{crit.4}}/4 = 0.78; \quad g^2_{\text{crit.2}} = 0.64, \\
\beta_{\text{crit.5}} &= 2/g^2_{\text{crit.5}} = 19.5/5 = 3.90, \quad \beta_{\text{crit.5}}/5 = 0.78; \quad g^2_{\text{crit.5}} = 0.51, \\
& \qquad \beta_{\text{crit.}N}/N|_{N\to\infty} = 2 \cdot 0.38 = 0.76.
\end{aligned} \tag{66}$$

We see that to a good approximation

$$g^2_{\text{crit.}N} \cdot N = 0.38^{-1} = 2.63. \tag{67}$$

2.2. Change of theory with energy scale

The lowest order mean field approximation is very local in the sense that it does not include effects of long range correlations. In fact it is the approximation in which link variables $U(-)$ are replaced by their average

$$\langle U(-) \rangle = V^R + iV^I$$

which is constant in space and time. Hence the calculation of critical values of coupling constants involves only a very limited region of the lattice, of size one or two lattice length. This property of locality suggests that we may use the mean field approximation to study the properties of a theory *at a definite scale*. The property might e.g. be whether the theory describes one or another phase. The answer to this question may depend on the scale. This language is in contradiction to the conventional use of the concept of phase transition which refers to long distance behaviour, only. We, however emphasize the concept of scales and try, a la renormalization group arguments, to find the relation between effective theories at different scales. Knowing the effective Lagrangian at an energy scale μ we may obtain an effective Lagrangian at the scale $\mu/2$ by functional integration over modes of wave number higher than $\mu/2$. According to the philosophy of Cadanov, Wilson etc. [17] successive small steps of this type may connect effective theories at widely different scales. It often happens that the Lagrangian at the lower level to a good approximation has the same functional form as the Lagrangian at the higher level, just with other values of the parameters (coupling constants, masses). The relation between the parameters at the low level and at the high level is given by the β function of Callan and Symanzik. In some cases, however, the functional form may change drastically, some fields may disappear and new fields appear. An example is provided by QCD. At scales higher than Λ_{QCD} the effective Lagrangian contains gluon fields. Below this scale the effective Lagrangian contains only hadron fields and at still lower energy no fields and no strong contribution to the Lagrangian at all. The theory of QCD is 'dead' at low energy scales — from confinement. Another example is the Salam-Weinberg theory which suffers a partial death at 100 GeV — from Higgs'ing. We may compare to a kind of time development. The QCD dies from old age: its coupling has grown large. The Salam-Weinberg theory dies young: it is killed by Higgs before the couplings have grown large. However, before death it has delivered a child QED. We see a group die: $U(1)\times SU(2)$ and another being born: $U(1)_{QED}$ (cf. 24)). Another example, although not experimentally observed, is provided by confusion: a group $G\times G\times$ ... dies leaving the group G_{diag} as its inheritor. It seems plausible to assume that the ability of delivering subgroups is lost at a transition age, i.e. when the coupling constant exceeds a critical value, e.g. the critical value given by the mean field approximation (66) which may predict death to be near. Thus all groups from which gauge symmetry can be unherited at a lower level must satisfy the condition

$$\alpha_G(\mu) \leqslant \alpha_{G\mathrm{critMFA}}. \tag{68}$$

This relation is true in particular for all the members of a set of groups undergoing confusion collapse into the diagonal subgroup.

2.3. The inequality

In the following we assume that the physics at the Planck scale is given ab initio and that we need not care about physics at a higher scale, i.e. we do not require conditions like (68) to be satisfied for $\mu > \mu_{\mathrm{Planck}}$. We also assume that the breakdown of the gauge group (1) to the group SMG occurs at the scale μ_{grav}, where gravity is becoming critical in a sense similar to the mean field criticallity; in my calculations I used the value $\mu_{\mathrm{grav}} = \mu_{\mathrm{Planck}}/2.2$.

To compare our speculative (critical) values of the (running) gauge coupling constants at the scale μ_{grav} to experimental values at the scale of experiments $\mu_{\mathrm{LEP}} \approx 100$ GeV we use the conventional renormalization group procedure, applying the Callan-Symanzik β function [19, 20] defined by

$$\frac{dg(\mu)}{d\ln\mu} = \beta(g(\mu), \ldots). \tag{69}$$

The β function has positive contributions from all the matter fields which have the coupling in question and unavoidable negative contributions from the selfcoupling of the gauge particles. We can define a set of semiexperimental fine structure constants

$$\alpha_i^{\text{desert}}(\mu_{\text{grav}}) \tag{70}$$

extrapolated to the critical scale under the assumption that the β function will be influenced by no more matter fields than we know about today; that is indicated by the superscript *desert*. The subscript $i = 1, 2\ 3$ refers to the 3 factors in SMG. This represents a conservative lower limit as more matter fields might exist

$$\alpha_i(\mu_{\text{grav}}) \geqslant \alpha_i^{\text{desert}}(\mu_{\text{grav}}). \tag{71}$$

We are now armed to set up a sequence of inequalities:

$$\begin{aligned} \alpha_i^{\text{desert}}(\mu_{\text{grav}})^{-1} &\overset{(1)}{\geqslant} \alpha_i(\mu_{\text{grav}})^{-1} \overset{(2)}{=} n_i \cdot \alpha_{i,NN}(\mu_{\text{grav}})^{-1} \\ &\overset{(3)}{\geqslant} N_{\text{gen}} \cdot \alpha_{i,NN}(\mu_{\text{grav}})^{-1} \overset{(4)}{\geqslant} N_{\text{gen}} \cdot \alpha_{i,\text{crit MFA}}^{-1}. \end{aligned} \tag{72}$$

The index i, NN refers to an average member of the set of isomorphic groups of the i^{th} kind: $G_{i,\text{Peter}}$, $G_{i,\text{Paul}}$, etc. while n_i is the number of such members. In the sequence (72) is built in some tentative results from the first lecture. In the second relation $(\overset{(2)}{=})$ we have utilized (31) and the third relation $(\overset{(3)}{\geqslant})$ is based on point 7) of the first lecture where the large mass ratio between the generations led to the suggestion that different generations are characterized by different gauge quantum numbers at some high level. The last relation $(\overset{(4)}{\geqslant})$ is just an application of (68) which expresses that no group can participate in a confusion breakdown if it has grown so old that its coupling exceeds the critical value.

From the above sequence we derive the combined inequality

$$\alpha_i^{\text{desert}}(\mu_{\text{grav}})^{-1} \geqslant N_{\text{gen}} \cdot \alpha_{i,\text{crit MFA}}^{-1}(x). \tag{73}$$

In the critical fine structure constant we have introduced a new parameter x which will be explained later. We want to confront the combined inequality with experiment. To lowest order the left hand side is linear in the number of generations

$$\alpha_i^{\text{desert}}(\mu_{\text{grav}})^{-1} = \alpha_i^{N_{\text{gen}}=0}(\mu_{\text{grav}})^{-1} - N_{\text{gen}} \cdot c_i(\mu_{\text{grav}}). \tag{74}$$

Inserting this we finally obtain

$$\frac{\alpha_i^{N_{\text{gen}}=0}(\mu_{\text{grav}})^{-1}}{c_i(\mu_{\text{grav}}) + \alpha_{i,\text{crit MFA}}^{-1}(x)} \geqslant N_{\text{gen}}. \tag{75}$$

In Fig. 1 the left hand side is represented by the straight lines. For the favoured value of x the number of generations must lie below these curves. The figure tells us that there is no place for more than 3 generations; actually the relation is very nearly satisfied as an equality when x takes the value 0.09.

We now return to the parameter x. The plaquette action is in principle a sum over representations of the given group

$$S_{\square} = \sum_{r} \beta_r \Re \operatorname{Tr}(U_r(\square)). \tag{76}$$

From simplicity and the requirement that the plaquette action be a smooth function of the plaquette variable we expect that only the lowest nontrivial representations have significant coefficient β_r. If the standard model group were embedded in SU(5) then the lowest nontrivial representations would be $\underline{5}$ and $\underline{10}$. To each of these corresponds a reducible representation of SMG and associated values of the β_r's. The ratio $x/(1-x)$ gives the relative size of the coefficients corresponding to $\underline{10}$ with respect to those corresponding to $\underline{5}$. From considerations of convergence properties of the series indicated by (76) one might get a vague feeling that the ratio $x/(1-x)$ should not exceed $\sqrt[12]{2/3} = 0.09$.

If our assumptions are true the near saturation of (73) shows that each of the relations in the sequence (72) is satisfied as an equality. This allows the following conclusions:

- From the saturation of the first relation $(\overset{(1)}{\geqslant})$ follows that there is indeed a desert; no more generations should be found up to near the Planck scale. In particular we can exclude supersymmetry unless broken near this scale. Similarly, technicolor would be hard to accomodate. Higgs bosons give very small contributions to the β-function; In fact our best fit corresponds to 1.9 Higgs boson sets (doublets).
- From the saturation of the third relation $(\overset{(3)}{\geqslant})$ follows that no more than 3 standard model groups participate in the process of confusion leading to the LEP-physics.
- From the saturation of the fourth relation $(\overset{(4)}{\geqslant})$ follows that the gauge couplings of all the confusing groups have maximum size, i.e. the critical size.

3rd Lecture

3. *Random dynamics*

3.1. Linear laws

In the first two lectures I argued for the anti-Grand Unification scheme and outlined the derivation of inequalities involving the fine structure constants. Our starting point was the standard model. In reality, however, we were led to our scheme and the relations from considerations based on the philosophy that the true model of fundamental physics is so complicated that we are forced to treat it as random.

In the last two lectures I would like to show that the anti-Grand Unification picture and the standard model are almost unavoidable consequences if we choose the fundamental model at random from a set of very complicated models.

A sidetrack starts from consideration of hermiticity. In the random dynamics picture the Hamiltonian which controls the development of the wave function for the whole world is only approximately hermitean. This leads to questions like: does it make any sense to ask 'is it more probable to live at any particular moment than at any other moment'. Thus one may be led to investigate the consequences of the hypothesis that we live at a *random* moment in time. However, to avoid any confusion, let me stress that the prediction of Doomsday (*Ostatni Dzień*), based on the hypothesis that we live at a random moment of time, has no logical connection to our random dynamics project. It could be right while random dynamics were wrong or oppositely without any logical difficulty. I included the Doomsday story together with the problems of hermiticity of the Hamiltonian in random dynamics because the ideas involved are similar and because the discussion of hermiticity in random dynamics led me to think about the moment *now* being random.

The philosophy of random dynamics is that all laws of presently known physics follow from almost any model which is complicated enough provided we go to some limit, as a rule the long wave length limit. Hence we should be able to derive both

- general laws, e.g. quantum mechanics (in an abstract sense), theory of relativity, locality, and
- more specific properties as the existence of the standard model with its gauge group $S(U(2) \times U(3))$, fermion representations, and various coupling constants.

Now if one really throws away all known physics we have a difficulty in finding a good starting point because we even lack a language in which to formulate the random model; even the language would be random. That means that only the mathematical structure would have content and I presume that the final procedure in the project of random dynamics will be to start from a 'random mathematical structure', whatever meaning can be asigned to this concept. One would presumably take some sets of sets and relations and impose some random rules (axioms) for handling these sets and relations. It requires a little thought to construct a random mathematical structure. But next there is still a question of interpretation: Which features of the (random) mathematical structure should be identified with which physical quantities? I guess that we shall eventually interprete deviations from some 'standard' state of the structure as the wave functional of the whole world — a quantum field theory. I shall not attempt to present such a derivation yet.

A 'derivation' of the time development of the Schrödinger wave functional [18] may suggest how to set up a description of Nature by a random mathematical structure.

There is an almost standard way of deriving all linearity laws: take for instance the law of Hook, that the extension Δl of a piece of material (a bar say) is proportional to the force pulling it out

$$\Delta l = \frac{1}{k} F. \tag{77}$$

This can be explained from very general assumptions, such as analyticity. The most important assumption needed is that the force is very small and the extension Δl therefore also small. First one must assume that the length l of the bar is a function of the force $l = l(F)$. In the case of Hook's law it really means that we perform the measurements so slowly that the bar is always in equilibrium with the force F. Next it is a rather generally accepted principle in physics that, unless there is some reason for the opposite, any physical function such as here $l(F)$ is smooth enough to be expanded in a Taylor series, i.e. it is for instance analytical. The great hopes of applying such analyticity principles to obtain all of physics was put forcefully forward by G. Chew [21] and you can therefore consider the project of random dynamics as my personal version of Chew's great bootstrap project. Anyway we assume that the length $l(F)$ of the bar is an analytical function of the force so that we can write a Taylor expansion around $F = 0$

$$l(F) = l(0) + F \left.\frac{dl}{dF}\right|_{F=0} + \tfrac{1}{2} F^2 \left.\frac{d^2 l(F)}{dF^2}\right|_{F=0} + \dots . \tag{78}$$

Now the trick of talking about the extension

$$\Delta l(F) \stackrel{\text{def}}{=} l(F) - l(0) \tag{79}$$

instead of just about $l(F)$ allows us to get rid of the force-independent term $l(0)$ in the Taylor expansion

$$\Delta l(F) = F \left.\frac{dl}{dF}\right|_{F=0} + \tfrac{1}{2} F^2 \left.\frac{d^2 l}{dF^2}\right|_{F=0} + \dots \tag{80}$$

and the restriction to small forces allows us to get rid of all the higher order terms in the Taylor expansion. Thus the limit of low force gives us Hook's law

$$\Delta l(F) = F \left.\frac{dl}{dF}\right|_{F=0} \tag{81}$$

if we identify the constant $\left.\frac{dl}{dF}\right|_{F=0}$ with the constant k^{-1} in Hook's law.

We used the steps:

1) write a Taylor expansion based on a general analyticity,
2) define away the first term by considering only a deviation (e.g. $\Delta l(F) = l(F) - l(0)$),
3) throw away as small all terms of order higher than first order.

This method is also applicable to the derivation of many other physical laws, e.g. Ohm's law and the law of linear expansion under heating.

One would strongly suspect that this method may also lead to an understanding of the principles behind quantum theory, at least the principle of superposition or linearity of the Schrödinger equation. Here we consider quantum theory in its most general form describing single particle motion as well as many particle problems, i.e. quantum field theory. We [18] actually attempted to explain the linearity of the Schrödinger equation (in an abstract sense) in this way.

We take as the starting model a world described (really a whole system of tremendously many worlds) by a point $\vec{x}(t)$ on some manifold $\mathcal{M}$. Here t is the time — a concept which is put in by hand in this formulation. For this derivation we use the language of the many world interpretation of Everett, Graham and Wheeler [22] according to which all components of a wave function are realized when a measurement is performed. We see a certain definite result of the measurement because we happen to be the persons living in the 'world' corresponding to the component of the wave function obtained by projection onto the eigenspace of that particular eigenvalue.

In the many world interpretation the state of all the many worlds is represented by a point $\vec{x}(t)$ on the manifold $\mathcal{M}$. The dimension of this manifold equals the number of quantum states of the whole world (it may be infinite). Assuming time translational invariance a very general set of differential equations for a point on a manifold consists of the autonomous equations

$$\dot{\vec{x}}(t) = \vec{F}(\vec{x}(t)) \tag{82}$$

meaning that the time-derivative $\dot{\vec{x}}(t)$ is given by a vector field (velocity field) $\vec{F}$ on the manifold. This $\vec{F}$ represents, one can say, the fundamental law of nature; in the spirit of the random dynamics we assume that it is a random field although with a distribution such that it is analytical or at least can be approximated by a Taylor expansion.

In computer simulations we used a random number generator to create a random vector field $\vec{F}$ on a torus and let $\vec{x}(t)$ develop in time according to the equation (82). In a typical run the point $\vec{x}(t)$ approached a fixed point $\vec{x}_{fix}$ on the torus. Here a fixed point means a point in which the vector field vanishes

$$\vec{F}(\vec{x}_{fix}) = \vec{0}. \tag{83}$$

After long time, i.e. for $t \to \infty$ the running point $\vec{x}(t)$ is close to the fix point $\vec{x}_{fix}$; hence a Taylor expansion gives

$$\dot{\vec{x}}(t) = \vec{F}(\vec{x}_{fix}) + \frac{\partial \vec{F}}{d\vec{x}}(\vec{x}_{fix}) \cdot (\vec{x}(t) - (\vec{x}_{fix})) + \frac{1}{2!}\frac{\partial^2 \vec{F}}{\partial \vec{x}^2}(\vec{x}_{fix}) \cdot (\vec{x}(t) - \vec{x}_{fix})^2 + \dots . \tag{84}$$

We now make the assumption that the wave function $\vec{\Psi}(t)$ can be identified with the deviation

$$\vec{\Psi}(t) = \vec{x}(t) - \vec{x}_{fix}.$$

Ignoring higher order terms in the Taylor expansion we obtain

$$\dot{\vec{\Psi}}(t) = \dot{\vec{x}}(t) = \vec{F}(\vec{x}(t)) = \frac{\partial \vec{F}}{\partial \vec{x}}(\vec{x}_{fix}) \cdot \vec{\Psi}(t). \tag{85}$$

We here also used $\vec{F}(\vec{x}_{fix}) = 0$. If we now make the identification

$$\frac{\partial \vec{F}}{\partial \vec{x}}(\vec{x}_{fix}) = -iH, \tag{86}$$

where the matrix H is the Hamiltonian we obtain:

$$i\dot{\vec{\Psi}}(t) = H \cdot \vec{\Psi}(t) \tag{87}$$

which looks quite like the Schrödinger equation. There is however, no a priori reason why the Hamiltonian defined via equation (86) should he hermitean. The $-i$ which we introduced was quite arbitrarily put in just to make the equation get the appearance of the usual Schrödinger equation, but the problem of lack of hermiticity is not only a matter of an overall factor since the operator (matrix) H will have eigenvalues of many different phases.

To have a stable fix point all the eigenvalues λ must have nonpositive imaginary part

$$\Im\lambda \leqslant 0.$$

There is, however, a mechanism by which the eigenvalues can approach real values after a long time so that hermiticity is at least not excluded. Really nothing happens to the eigenvalues but as time passes those components in the wave function die out which correspond to eigenvalues with negative imaginary parts that are not extremely small. The restriction of the Hamiltonian H to the subspace of wave functions that survive after long time will have approximately real eigenvalues. So the effective Hamiltonian will after some time have improved with respect to hermiticity.

However, even real eigenvalues does not mean hermiticity, as can be seen by an example. The two by two real matrix

$$\begin{pmatrix} a & b \\ -b & -a \end{pmatrix} \tag{88}$$

has the eigenvalues

$$\left.\begin{matrix}\lambda_1 \\ \lambda_2\end{matrix}\right\} = \pm\sqrt{a^2-b^2} \tag{89}$$

which for $a > b > 0$ are both real and the eigenvectors

$$\left.\begin{matrix}|\lambda_1\rangle \\ |\lambda_2\rangle\end{matrix}\right\} = \begin{pmatrix} b \\ -a\pm\sqrt{a^2-b^2}\end{pmatrix}. \tag{90}$$

However,

$$\begin{pmatrix} a & b \\ -b & -a \end{pmatrix}^{\dagger} = \begin{pmatrix} a & -b \\ b & -a \end{pmatrix} \neq \begin{pmatrix} a & b \\ -b & -a \end{pmatrix} \tag{91}$$

so that it is not hermitean as is also implied by the fact that the two eigenvectors $|\lambda_1\rangle$ and and $|\lambda_2\rangle$ are not orthogonal

$$\langle\lambda_1|\lambda_2\rangle = 2b^2 \neq 0. \tag{92}$$

Since eigenvalues of the Hamiltonian are only approximately real, it is actually a prediction of random dynamics that the hermiticity of the world Hamiltonian H can at most be approximate. Hence it becomes important to make experiments looking for a possible antihermitean part

$$\Im H \overset{\text{def}}{=} \frac{H-H^{\dagger}}{2i} \tag{93}$$

of the Hamiltonian for the world(s). The meaning is clearly a lack of conservation of probability in time since the Schrödinger equation

$$i\dot{\Psi}(t) = H\Psi(t) \tag{94}$$

(more strictly the equation for the time development of the Schrödinger functional) implies:

$$\begin{aligned}\frac{d}{dt}\langle\Psi(t)\,|\Psi(t)\rangle \\ &= \langle\dot{\Psi}(t)\,|\Psi(t)\rangle + \langle\Psi(t)\,|\dot{\Psi}(t)\rangle \\ &= i\langle\Psi(t)\,|H^{\dagger}|\Psi(t)\rangle - i\langle\Psi(t)\,|H|\Psi(t)\rangle \\ &= 2\langle\Psi(t)|\,(H-H^{\dagger})/(2i)\,|\Psi(t)\rangle \\ &= 2\langle\Psi(t)|\,\Im H\,|\Psi(t)\rangle. \end{aligned}\tag{95}$$

Thus, if the expectation value of the antihermitean part $\Im H$ is positive the probability for the existence of the 'world' increases with time, while the world has a chance to disappear at every moment if the expectation value is negative. Now we will usually renormalize the total probability to one at every moment of time. It is therefore not so easy to measure such variations in the norm of the wave function.

We do not even know if there should exist some extremely dangerous decay of say the proton which caused erradication of the earth, because if it happens we would no longer be there to observe it and if it does not happen there is nothing to observe.

How can one possibly look for such effects as an antihermitean part or some viscious decay of the proton with the same destructive effect? Well if, in the language of Everett, Graham, and Wheeler, we can split the world into several worlds by the measurement of some (quantum mechanical) dynamical variable one may hope that the antihermitean part is different in the different resulting worlds. This might lead to observable effects. Let us now assume that it really is possible to manipulate the antihermitean part. For instance it could be that some particle carried an amount of say negative $\Im H$ meaning that it had a decay mode making the world disappear (or proliferate if $\Im H$ is positive). Now from Lorentz invariance we would expect that the rate for such a world killing decay is modified by relativistic motion just as the rate for any other decay is modified by a factor $\gamma(v)^{-1} = \sqrt{1-v^2}$, where v is the velocity of the decaying particle.

Here I shall propose an experiment which might detect such effects.

The first part of the experiment is a quantum random number experiment which splits the world in the many world sense. We may, e.g. use light polarized in one direction and measure its polarization in another direction, rotated say 45°. By measurement on many photons, counting every other photon opposite it is possible to construct a quantum random number ξ with a very reliable (insensitive to systematic error) distribution. The result ξ of this experiment may even be published.

When the number ξ has been published we go to the second part of the experiment. Here we try to change the average of the antihermitean part of the Hamiltonian $\langle \mathfrak{J} H\rangle$ by an amount proportional to ξ.

To do that we take some particles suspected to carry $\mathfrak{J}H$ and change their relativistic factor γ^{-1}, integrated over some time interval, by an amount proportional to ξ

$$\sum \int dt \left(1 - \frac{1}{\gamma(v)}\right) = c \cdot \xi. \tag{96}$$

This may be done simply by loading the suspect particles on a car and driving a distance ξ with a prescribed speed. This drive changes time dilatation factor of the driver, the passengers, and the car itself by an extra factor relative to the value it would have had if they stayed at rest, thereby changing $\mathfrak{J}H$, i.e. the decay rate of the world.

Thus it might happen that worlds in which the car drive is short (because the value of ξ is small) decay faster (or proliferate slower) than those in which the car drive is long. This should be so if the material of the car and its cargo has a world decaying effect which is diminished by the time dilatation when the car moves. If it so then we should expect, after the car trip, that the distribution of ξ is shifted towards bigger ξ values than according to the usual quantum mechanics theory. If we then study the literature we should find a publication containing ξ-values which are surprisingly large compared to the usual predictions.

One may doubt whether it is even logically possible that there could be such an effect of an antihermitean part $\mathfrak{J}H$ since it may seem surprising that one could influence the past by making car trips at a later time.

However, the logic is similar to that of an experiment involving the intake of some poison in which one should certainly experience a biased distribution. Suppose that, instead of the car trip, you courageously take an amount of poison proportional to the random ξ. Then the distribution of ξ which you should experience after the poison has had its time to act should have a lower average than the one determined from usual quantum mechanics. The reason is of course that we strictly speaking talk about another distribution namely the distribution of ξ under the restriction of your survival and that is of course quite different from the unrestricted ξ-distribution, because of the effect of the poison. In an analogous way the ξ-distribution experienced after the car trip is the ξ-distribution under the restriction that the world survives the trip. You may here even imagine some sort of proliferation of the world and the ξ-distribution being weighted by factor representing the number of worlds produced in the proliferation. Such considerations may lead to the idea that the probability for the world to exist is not always the same.

Think for instance of a Hamiltonian similar to the 2×2 matrix (88) mentioned above as an example of a nonhermitean matrix with real eigenvalues for a, b real and $|a| > |b| > 0$. It would lead to a $|\Psi(t)|^2 = \langle \Psi(t) \,|\Psi(t)\rangle$ which oscillates as a function of the time. Would that mean that we would be more likely to exist at times when $|\Psi(t)|^2$ is large than when it is small? Yes, that must be the logical interpretation. This brings us to discuss whether it is a good and meaningfull hypothesis that we live at a random moment of time.

3.2. Hypothesis: we live at a random time

Science has several times taught us that we live at a rather random place. It is e.g. no longer true that the Earth is the center of the Universe. Rather it has turned out that we live on a planet (among several) circulating around a star of rather average type at a rather random place in a galaxy which is itself just one among many in the Universe. Also the belief that our own country has a special status as the center seems hard to be upheld with todays knowledge. Thus the consideration that China — as indicated by the name given by its inhabitants — should really be the center of the world or even the earth cannot be taken seriously. Our near relationships to apes and monkeys may also be taken to mean that the human race is not so extremely special.

In analogy to these considerations it is a natural hypothesis that our time is not special either, that we in some sense live at a random moment of time.

This hypothesis, that we should just now live at a random number of hours after Big Bang, immediately runs into the trouble that at most moments of time until now there were no human beings but only dinosaurs or trilobites or not even that. This does not really mean that the hypothesis is wrong since the species homo sapiens may have appeared at a time that could be considered random. Let us, however, rather study the hypothesis that the combination ('now', 'I') or the combination ('now', 'you') is random. Thus I suggest the following hypothesis: Make the little experiment of finding what the ordered pair (now, you) is, i.e. you read off your watch and put the reading in as the left element of an ordered pair and you put your name as the right element. Now the hypothesis is that this ordered pair can be considered random. Let me define from which set L I suggest the combination (pair) to be picked and with what measure:

First define the set of all human beings $\{humans\}$ and the set of all possible moments of time $\{time\}$. Essentially $\{time\}$ is an interval on the real axis asigned the second as the unit. Taking say the birth of Christ as $t = 0$ it is presumably:

$$\{time\} = \{t| -3 \cdot 10^{17}\,\text{sec} < t < \infty\}. \tag{97}$$

The set $\{humans\}$ should include people who have not yet been born, people presently alive, and people who already died. The usual definition of a cross product of two sets allows us to write

$$\{time\} \times \{humans\} = \{(t, p) | t \in \{time\},\ p \in \{humans\}\} \tag{98}$$

where t is a moment in time and p is a person.

Next define the subset L of this cross product consisting of the living combinations

$$L \stackrel{\text{def}}{=} \{(t, p) \in \{time\} \times \{humans\} | p \text{ alive at time } t\}. \tag{99}$$

As you clearly are alive since you hear me talk to you we have

$$(\text{now, you}) \in L. \tag{100}$$

In order to make sense of a random element of the set L we must choose a probability measure on this set. I suppose it is obvious that the natural measure to choose on L is so that the measure $\mu(A)$ of a set $A \subseteq L$ is given by

$$\mu(A) = \sum_{p \in \{humans\}} \text{Lebesque measure } (\{t | (t, p) \in A\}). \tag{101}$$

The number of persons ever to exist or having existed is presumably finite so that the set $\{humans\}$ is finite and the sum defining $\mu(A)$ therefore will be defined whenever the Lebesque measures of the sets $\{t|(t, p) \in A\}$ make sense. That I defined the measure μ on L using the Lebesque measure on the time axis is not essential; any measure on the time axis invariant under translations in time would for our purposes be identical, and identical to the Lebesque measure.

As an example of a subset $A \subseteq L$ for which one might like to ask for the measure $\mu(A)$, we could take the set of such ordered pairs (t, p) spent in the present seminar

$$A_{\text{seminar}} = \{(t, p) \mid \text{At time } t \text{ the person } p \text{ attends the present seminar}\}. \tag{102}$$

If we ignore corrections due to people coming late or leaving before the end of the seminar we obtain

$$\mu(A_{\text{seminar}}) = \text{'length of seminar'} \cdot \text{'number of participants'}. \tag{103}$$

This measure μ is measured in say person-hours or person-seconds. If the human race like most species of the past die out some day the set L will have a finite measure

$$\mu(L) < \infty. \tag{104}$$

In this case we can define a probability measure, i.e. normalized measure on L by

$$\mu_{\text{prob}}(A) \stackrel{\text{def}}{=} \frac{\mu(A)}{\mu(L)}. \tag{105}$$

The precise meaning of the statement that *the ordered pair* (*now*, *you*) *is random* is that the set (now, you) is picked randomly from L with the probability distribution μ_{prob}.

You may make various statistical tests of this hypothesis by investigating if there should be any very remarkable feature of the combination (now, you). For instance if some of you just have birthday today I should advise him (or her) to be very suspicious of the hypothesis that (now, you) is randomly picked with probability measure μ_{prob}. The point is that with this distribution the probability for having birthday is very close to $\frac{1}{365}$ since it only happens one day of the year. It would be too remarkable that (now, you) should accidentially just fall inside the birthday subset $B \subseteq L$ of L defined as

$$B = \{(t, p) \mid t \in \text{a birthday for } p\} \tag{106}$$

since as just said

$$\mu_{\text{prob}}(B) = \frac{\mu(B)}{\mu(L)} \approx \frac{1}{365} \ll 1. \tag{107}$$

In the same way you should be very suspicious of my hypothesis if (now, you) has in some other way a very special and remarkable property. If for instance you are older than 95 years you should also be suspicious of my hypothesis. If you just now are living in extremely remarkable moment you should discard my hypothesis of a random (now, you) totally. This might be what you should do if you have been just married within the last hour say, or if you have won a premium in some lotterie say of more than 100,000 dollars today.

Now there is something which, depending on the picture of the development of the population of human beings, may or may not be remarkable for all of us under the hypothesis of a random (now, I):

It could be rather remarkable that we just now live in a time when there are more people on the earth than ever before, and it even increase rather rapidly, doubling in less than 100 years. We know from historical and archeological data, that the crude main feature of the past is a significant increase of population with time, modified by huge fluctuations. Let us then investigate our hypothesis from the point of view of different reasonable sceneraios for the development of the population in the past and the future. For each proposed scenario we should then estimate the probability that a supposed random (now, you) or (now, I) would have the property that the now means a time at which there has never before existed more people on earth. That means we should estimate $\mu_{\text{prob}}(C)$ for the subset $C \subseteq L$ defined as

$$C = \{(t, p) \in L \mid \text{Population at } t \text{ higher than population at } t', \ \forall\, t' < t\}. \tag{108}$$

If in a certain scenario $\mu_{\text{prob}}(C) \ll 1$, there must be something suspicious either with the scenario in which this happens or with the hypothesis of random (now, you); in the latter case this hypothesis should be discarded.

Now my point is that this procedure leads us to discard all sensible scenarios with the exception of those which either have a violent end, Doomsday (*Ostatni Dzień*) or at least such a strong reduction of the population that it would never again rise to the present height, i.e. almost a Doomsday. Estimates show that this 'Doomsday' must (even) come not later than a few hundred years from now. So the main point is to see that in all reasonable scenarios which do not have such a soon (violent or soft) Doomsday we live now at a very special time and not at a random time as the hypothesis says. A typical scenario without the menace of a near Doomsday is illustrated in Fig. 5: After some time, a century say, the population of the earth will roughly stabilize at a level somewhat higher than today although there will be quite large fluctuations. The average stability may then last for perhaps several thousands of years. This rather optimistic scenario is not in accordance with our hypothesis. The point is that in this scenario most of the set L of alive combinations $(t, p) \in L \subseteq \{time\} \times \{humans\}$ counted by the measure μ lies below the plateau of the long future.

So with our hypothesis it would be most likely that (now, I) or (now, you) should be at this plateau. If it was already the time of the plateau that should however, have been easily observed in the study of historical demography.

Most likely there would have been fluctuations with a population higher than today and for several thousand years the average population would have been approximately equal to that of today. So from demographic historical studies it is known that we do not live at some moment deep inside a plateau in the population development. So if there is a future plateau lasting thousands of years we are just at the beginning of it now. That however, would be very unlikely with a long plateau. So the optimistic scenario with the long plateau in the future is not likely with our hypothesis of random (now, you) or (now, I) and hence we must discard it.

Scenarios that seem to fit best to the hypothesis of random (now, you) or (now, I) are of the Doomsday type, i.e. scenarios for which the near future (a few hundred years from now) will see either a total extinction of all of humanity or a strong decrease of the population, for instance as a mirror image in time of the way the population grew up to the present level. It may be difficult to imagine a mechanism that would be able to keep the human population down to a mirror image of the rather low population in the paleolithicum over even millions of years. Then a total extinction may be the more probable result. The Doomsday scenarios are illustrated by Fig. 6 and Fig. 7. In such scenarios all the people living before the peak will live at a time when there are more people than ever before, ignoring fluctuations on top of the main trend. In the true Doomsday scenario with a big final catastrophy this will be practically all points in L. In a more soft Doomsday scenario, in which the population falls in a similar way as it grew up, the probability that the present population is higher than ever before is roughly 50%. So the soft Doomsday scenario is all right from the random (now, you) or (now, I) hypothesis point of view.

Let us now ask whether we could have a scenario of the Doomsday type but one for

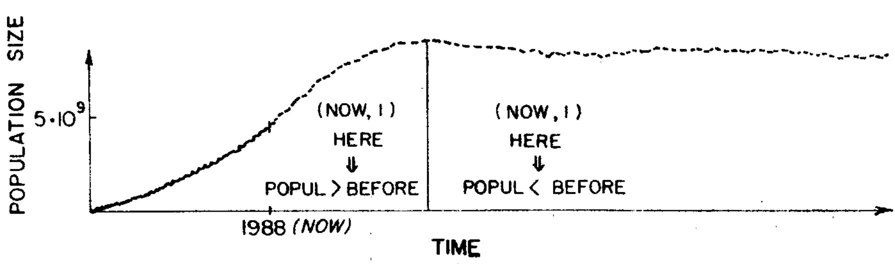

Fig. 5. Evolution of the population in a typical non-Doomsday scenario

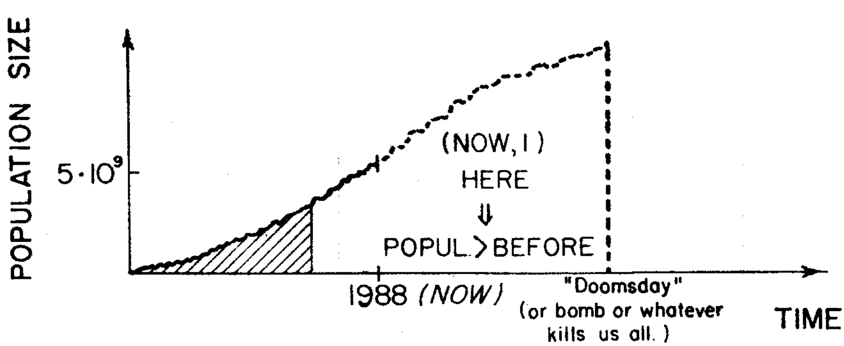

Fig. 6. Evolution of the population in a typical true Doomsday scenario. The probability for living in the extremely early shaded region is very small for the random (now, you) hypothesis

which the population will reach its maximum in a distant future, perhaps several thousands of years from now (see Fig. 8). This scenario fits the fact that the present population is higher than ever before. Although that is not against our hypothesis it does not fit very well. The points is that seen from this scenario we live now extremely early compared to the very huge maximum population. If you think of the population distribution on the time axis in this scenario as an approximate Gaussian distribution we would live many standard deviations too early for it to be likely. Now from the way the population raised in the past a Gaussian could not be true but it is nevertheless so that a very huge peak in the population in thousands of years would mean that so many people would live in the future that it would be unlikely that (now, you) should fall so early in time (as already).

So such a peak of very huge dimensions is also not consistent with our hypothesis of random (now, you) or (now, I). Consequently the only scenario consistent with our hypothesis is a Doomsday like scenario with a Doomsday or a peak comming after a time of the order of magnitude of the time scale given by the present rate of growth. Hence

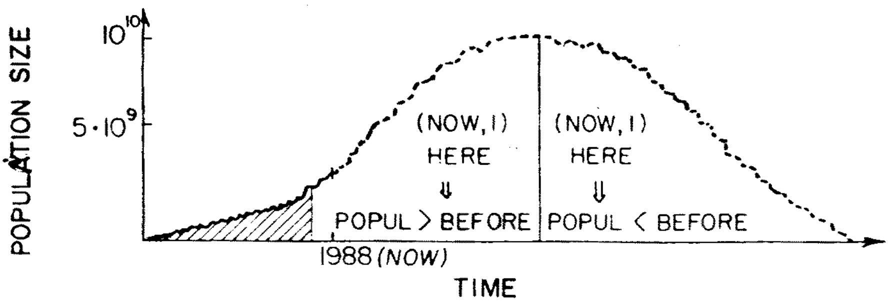

Fig. 7. Evolution of the population in a typical soft Doomsday scenario

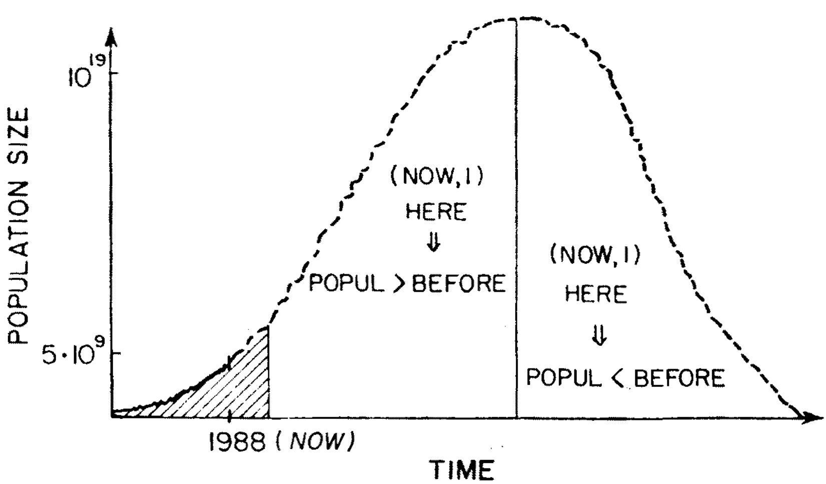

Fig. 8. Evolution of the population in a delayed Doomsday scenario. The probability for living in the extremely early shaded region (as we happen to do) is very small for the random (now, you) hypothesis

the peak (or Doomsday) will come within a time roughly of the order of the present time scale for doubling of the population, i.e. hundred years.

It is interesting that we by means of our random (now, you) or (now, I) hypothesis can make conclusions about the future quite independent of the usual way of extrapolating into the future. If one for some reason do not want to believe our somewhat pessimistic prediction one has of course the option of totally discarding the hypothesis of random (now, you) or (now, I) but that is to admit that we live at a very special time, just before the plateau starts (if one prefers the plateau scenario). Or one must admit it is a statistical fluctuation that we live just now. One may also challenge, as Jens Bang suggested, whether the concepts *now*, *I*, and *you* are defined well enough for our application; they are not objective notions. Still another route to escape the conclusion of a near Doomsday goes via the many world interpretation: one might assume that the worlds die with a lifetime of the order of hundreds of years, but that would really just mean that we have replaced the extinction of humanity by the disappearance of the whole world. That might, however, be the logical conclusion if other knowledge would assure us that humanity is stable against all kinds of realistic attacks on the magnitude of the population, such as wars, epidemies, catastrophs, family planning, or other cultural developments influencing the proliferation.

Actually I suppose we cannot at all be sure of the survival of humanity since polution, wars and family planning etc. could easily reduce the population. But if we could be sure we might have to conclude that the whole world should disappear. That would bring a weak link to the original subject which inspired me: that the random dynamics project suggested an imaginary (i.e. antihermitean) part of the Hamiltonian of the world.

4th Lecture

4. The standard model from random dynamics and confusion

4.1. Field glass and gauge glass

In this lecture I want to sketch how random dynamics may lead to an explanation of the standard model and many of the features suggested in the first lecture. Already in the third lecture I have loosely sketched a route for deriving very general principles, the superposition principle of quantum mechanics and the approximative hermiticity of the Hamiltonian. In an early work on random dynamics [23] we have shown that the Weyl equation with $3+1$ dimensions is the natural equation for a fermion field — assuming translational invariance, but not Lorentz invariance. Thus one might hope that it would be possible to derive the general principles of physics at some fundamental scale (μ_{Planck}) from random dynamics. We may imagine that such principles would justify the very general field theory glass model I shall now sketch.

The field theory glass is a close analogon to the spin glass. The generalized field ϕ is defined on a random basic set of sites $\{j\}$ in space-time as a mapping from this set into a corresponding set of random manifolds $\mathcal{M}(j)$

$$\phi\colon \{j\} \to U_j\{\mathcal{M}(j)\} \tag{109}$$

with the restriction

$$\phi(j) \in \mathcal{M}(j). \tag{110}$$

Perhaps the following statement may clarify the meaning of this field: Each mapping ϕ represents a set of field amplitudes on all sites. The target space of ϕ varies from point to point; $\mathcal{M}(j)$ depends on j. The action is not assumed to be strictly local, but only *semilocal*, i.e. of the form $S[\phi] = \sum_B S_B[\phi|_B)$. Here the summation is taken over regions B of the order of the Planck size and $\phi|_B$ denotes the restriction of the function ϕ to the set of points in the region B. In reality $\phi|_B$ is a point in the product $X_{j\in B}\mathcal{M}(j)$. The regional action

$$S_B: X_{j\in B}\mathcal{M}(j) \to \boldsymbol{R} \tag{111}$$

is assumed to have random functional form; a random function S_B on $X_{j\in B}\mathcal{M}(j)$ may be constructed as an expansion with fixed (quenched), random coefficients on a basis of functions on this set. In fact all points, manifolds, regions as well as these coefficients are quenched random in the sense that they are kept fixed under the functional integration giving the partition function

$$Z = \int \mathcal{D}\phi \exp(S[\phi]). \tag{112}$$

Having set up this model we ought to use it as the basis for a computer Monte Carlo calculation, hoping that the output would reproduce the standard model in some long wavelength limit. Since we want to identify the typical distance between sites with the Planck length we would apparently need of the order of $(10^{18})^4$ points to reach the LEP length scale. The considerations of the two first lectures led us to the tentative conclusion that the region between the LEP scale and the Planck scale is rather arid and that the standard model should reign even near the Planck scale making the thought of a computer calculation considerably less terrible. Still, we have not yet dared to try. Instead we turned to speculations.

First we note some rather discouraging points.

1) The quenched randomness means that translation invariance and Lorentz invariance are broken, at least at small distances. One may hope that these invariances become valid either as some average effect at large distances or as a consequence of quantum fluctuations [24].

2) Even if we succeded to obtain these (Poincare) invariances, we would still have the problem that the typical scale for particle masses is the Planck mass scale. Particles with mass so high would hardly ever be observable unless for some reason they are stable enough to have survived since the creation of the universe. There are, however, exceptions. In gauge theories particles exist for which it is impossible to construct a mass term without breaking the symmetry. These particles have zero mass on the Planck scale; we call them mass protected. The gauge bosons are obviously mass protected; also fermions and Nambu--Goldstone bosons may be mass protected and one massless particle may give masslessness to a supersymmetric partner. In all these cases some symmetry is needed. In a field theory glass characterized by randomness it seems a priori extremely unlikely to find any symmetry

at all. We can, however, give arguments telling that even a field theory glass may develop gauge symmetry [10, 25]. Hence mass protection is nevertheless very likely but only of the kind that follows from gauge symmetry.

Starting from a field theory glass and supposing that no other type of symmetry than gauge symmetry can arise directly in a spontaneous way we conclude that all known particles and particles yet to be observed at the LEP level are mass protected by gauge symmetry down to this level. This result fits very well with point 2) in the list of observations regarding the standard model (lecture 1). To be honest, however, we must admit that potential Higgs bosons would have mass comparable to the Planck mass — if they exist at all.

We here show in an example how a field theory glass may develop gauge symmetry. In this example we assume for simplicity that the sites of the field theory glass are ordered in a regular lattice (this is irrelevant for our argument). Also for simplicity we shall concentrate on the group U(1). The action for this field theory glass has two types of terms

$$S = \beta \sum_{\square} \Re U(\square) + \alpha \sum_{-} \Re U(-). \tag{113}$$

The first

$$\beta \sum_{\square} \Re U(\square) = \beta \sum_{\square} \Re \{U(\square)\, U(\square)\, U(\square)\, U(\square)\}. \tag{114}$$

corresponds to an exactly invariant U(1) gauge theory. The other,

$$\alpha \sum_{-} \Re U(-) \tag{115}$$

corresponds to a symmetry breaking 'photon mass' part. Let us replace the original, physical, set of variables

$$U(-), \quad (U(-) \in \mathrm{U}(1))$$

by a new and larger set of formal variables (subscript h for 'human'):

$$U_{\mathrm{h}}(-), \quad H(\bullet), \quad (U_{\mathrm{h}}(-) \in \mathrm{U}(1); \quad H(\bullet) \in \mathrm{U}(1))$$

where $U_{\mathrm{h}}(-)$ refers to links and $H(\bullet)$ to sites.

The introduction of the apparently superfluous field $H(\bullet)$ opens the possibility of making the complete action invariant under a U(1) gauge symmetry. Then one can show that $U_{\mathrm{h}}(-)$ describes a massless 'photon', provided α is not too large and β ($= 2/g^2$, twice the inverse squared coupling constant) not too small. The new variable $H(\bullet)$ describes a priori a Higgs field, but quantum effects may lead to a positive renormalized mass square, i.e. to an ordinary physical particle. This is a reversed Higgs mechanism: a massive 'photon' has become a massless 'photon' plus a massive scalar particle. To see how this works we define the relation between old and new variables

$$U\left(\underset{x \;\; x+a\delta_\mu}{\bullet\!\!-\!\!\!-\!\!\bullet}\right) = H(x)\, U\left(\underset{x \;\; x+a\delta_\mu}{\bullet\!\!-\!\!\!-\!\!\bullet}\right) H^{-1}(x+a\delta_\mu) \tag{116}$$

and insert it in (113) to give

$$S = \beta \sum_{\square} \Re U_h(\square) + \alpha \sum_{-} \Re\{H(x)\, U\left(\underset{x\;\; x+a\delta_\mu}{\bullet\!\!-\!\!\!-\!\!\bullet}\right) H^{-1}(x+a\delta_\mu)\}. \qquad (117)$$

This action is by construction exactly invariant under the gauge transformation

$$U_h\left(\underset{x\;\; x+a\delta_\mu}{\bullet\!\!-\!\!\!-\!\!\bullet}\right) \to \Lambda(x)\, U_h\left(\underset{x\;\; x+a\delta_\mu}{\bullet\!\!-\!\!\!-\!\!\bullet}\right) \Lambda^{-1}(x+a\delta_\mu),$$

$$H(x) \to H(x)\Lambda^{-1}(x), \qquad \Lambda(x) \in U(1). \qquad (118)$$

The first term of (117) corresponds to the kinetic contribution of the 'electromagnetic' field. For the second term, the naive continuum limit gives

$$\Re\{H(x)\exp(iaA_\mu)H^\dagger(x+a\delta_\mu)\}$$

$$\approx \Re\{H(x)\,(1+iaA_\mu-\tfrac{1}{2}\,a^2A_\mu^2)\,(H^\dagger(x)+a\partial_\mu H^\dagger(x)+\tfrac{1}{2}\,a^2\partial_\mu^2 H^\dagger(x))\}$$

$$= H^\dagger H - \tfrac{1}{2}\,a^2 D_\mu H^\dagger D_\mu H \text{ (modulo boundary terms).} \qquad (119)$$

Hence it contains the kinetic term for a Higgs field. The restriction $H^\dagger(x)H(x) = 1$ suggests that at this stage the model is called a nonlinear Higgs model.

It would be natural to study this nonlinear Higgs model on a lattice by means of a block spin method. We here approximate the model defined on the original lattice by a model defined on a more coarse grained lattice. To each site on the coarse grained lattice corresponds a block of sites in the original lattice and a field H_{block} defined in terms of the original fields:

$$H_{\text{block}} = \frac{1}{N} \sum_{\bullet \in \text{block}} H(\bullet) \qquad (120)$$

(summed over all original sites $\bullet$ of the block; N is a normalization factor). In order to give a meaning to (120) one must choose some gauge, e.g. the Landau gauge. While the original $H(\bullet)$ can vary with respect to phase only, the new H_{block} can vary with respect to both phase and modulus; $H_{\text{block}} \in C$ as in the usual Higgs model. In this formulation a normal mass term (for H) may arise for small values of α:

The condition $H^\dagger H = 1$ may be imposed by adding a (smeared) δ-function $\approx c\delta(H^\dagger H - 1)$ to the action Eq. (117). The sign of c is such that the Hamiltonian will be minimal for $H^\dagger H = 1$. This corresponds to a negative mass square. However, iterating to larger and larger blocks the δ-function (-potential) is smeared out and may finally give rise even to a positive mass square.

We can argue in the following way for this transition to a phase containing a massless photon and an ordinary charged particle:

If the (nonzero) coefficient α is sufficiently small, the quantum mechanical fluctuations of the $H(\bullet)$ fields within a block will be large. (In the extreme case of $\alpha = 0$ $H(\bullet)$ does not even appear in the action.) Then $H(\bullet)$ will be uniformly distributed over U(1). As a result the average value of $H_{\text{block}} \in C$ will tend to zero. For $\alpha \neq 0$ this does not, however, prevent H_{block} field oscillations. In this case the field does not act as a Higgs field but rather as the field of an ordinary charged particle. For small enough α-values and large enough β-values the theory has changed from a Higgs phase to a Coulomb phase. By means of a mean field approximation one can show that a Coulomb phase really does exist in a model with the action (117).

The remarkable point is that the mechanism works for a whole range of values for (α, β); no fine tuning is needed to make the 'photon' massless.

It is possible to extend this reversed Higgs mechanism to work (in a limited sense), also for nonabelian groups. However, in the nonabelian case, we do not expect a Coulomb phase to exist unless the number of fermion fields (or other matter fields) is large enough to imply infrared freedom. In general, we should be satisfied with a theory which extrapolates to continuum Yang-Mills theory that confines only at a very low (exponentially low) energy scale.

This mechanism which may be termed *exactification* allows a field theory glass to develop into a gauge glass. A gauge glass is like the field theory glass based on a random set of sites and fields but now the fields obey gauge symmetries which may vary in a random manner from region to region.

4.2. Confusion

Let us assume some space in which regions have random groups. The space may be a continuous space or a discrete gauge glass. If several isomorphic groups are present all over the space one would like to distinguish them by giving them names, e.g. Peter, Paul, etc. A procedure would be to give the names arbitrarily in one region and then expand through chains of neighbouring regions, such that a travelling gauge quantum does not shift name on its way. The problem arises when a path closes: is a quantum bearing the name Peter during all of the travel received as a Peter quantum or would it be mistaken as a Paul quantum? does it interact with the Peter or with the Paul degrees of freedom on return to the original site? In a gauge glass such mistake may often happen. If it does happen the Peter and Paul degrees of freedom are mixed; a consistent system of baptisation does not exist; we say that the groups are *confused*. The original group was symmetric under permutation of the isomorphic subgroup named Peter, Paul, etc.; this permutation symmetry is an outer automorphism, a (special) mapping of the group onto itself.

In a continuous space one would expect that confusion does not exist. One may, however, imagine surfaces along which the boundary conditions are such that Peter degrees of freedom on one side are continued as Paul degress of freedom on the other. If a path passes an odd number of times through such *confusion surfaces* these degrees of freedom are confused. If the confusion surface has a boundary it is possible for a closed curve to pass it an odd number of times even in a topologically trivial (e.g. flat) space. If the confusion surface has no boundary (i.e. is closed) like a sphere it is impossible to pass it an odd number of times and still return to the starting point, as long as the space is topologically trivial.

But if the topology is not trivial it *is* possible for a closed curve to pass a closed confusion surface an odd number of times. This kind of topology is often illustrated by the (2-dimensional) surface of a cup with a handle; a line around the handle may symbolise a closed (1-dimensional) confusion surface; confusion is possible in a space with handles.

The ordinary Minkowski space has a trivial topology but quantum gravity suggests a nontrivial space-time foam at the scale of the Planck length. String theories often involve high dimensional space. To relate such theories with physics in (3 + 1 dimensional) Minkowski space one applies the trick of compactification which in many models leads to a nontrivial (high dimensional) topology. We conclude that confusion is a logical possibility in many models.

The mere existence of confusion surfaces challenges translational invariance. This challenge is in part met by the property that the position of a confusion surface has no physical significance: The surface can be deformed continuously by change of names in the region swept by the deformation. The boundary of the confusion surface, if any, cannot be moved; it has a physical significance. The existence of boundaries breaks translational invariance. Confusion surfaces bound to handles also lead to breakdown of translational invariance unless quantum fluctuation insures a translational invariant state of (the set of) handles.

We now imagine that the space is filled up (in a translational invariant way) with small handles of which many (half) have confusion surfaces. At a scale small compared to the dimensions of the handles one may give names without troubles; there is local gauge invariance under the product group. The problem of giving names arises in regions large enough to allow closed curves to be confused. At long distances one would effectively see only one group; all the isomorphic groups are mixed up by confusion, it has no meaning to say whether a transformation is of type Peter or Paul, etc. What *has* meaning is a common transformation of the Peter group, and the Paul group, etc., i.e. a transformation of the diagonal subgroup. This means that the original direct product group has been broken into its diagonal subgroup. We have shown in a simple example (Ahrenshoop 84) that (superpositions of) gauge particles not belonging to the diagonal subgroup acquire mass, presumably at the scale of symmetry breaking.

The property of having outer automorphisms is not restricted to product groups. Also the group U(1) and some of the simple groups have this property. For such groups we have a problem analogous to that of giving names. The problem is here to distinguish the group elements from their images under the outer automorphism. Thus, if space (space-time) has a nonsimple topology, there may exist confusion surfaces along which all elements are continued in their automorphic images. All SU(N) groups for $N > 2$ and all SO($2N$) groups have outer automorphism. Also the group of the standard model has an outer automorphism, viz. complex conjugation of the matrices in the 5×5 representation. These groups are therefore susceptible to breakdown due to confusion with the consequence that some or perhaps all the gauge bosons acquire mass. There is, however, a possibility for avoiding this charge conjugation collapse: These groups allow conjugated pairs of complex representations which are transformed into each other by the outer automorphism. If the theory contains Weyl particles belonging to a complex representation then the conjugated representation consists of antiparticles of the opposite helicity, and the symmetry under the automorphism is removed.

The presence of gauge anomalies is disastrous for a gauge theory. The standard model avoids the disaster by combining Weyl fermion representations, each with unavoidable anomalies, in generations such that all the anomalies of a generation cancel each other. The composition of a generation solves not only the anomaly problem but also the problem of collapse due to symmetry under complex conjugation. Thus we may expect each SMG at the fundamental level to be accompagnied by (at least) one generation. This does fit into the list of properties of the standard model given in the first lecture.

Within the frames of random dynamics it is natural to find rather small groups with rather small representations. The argument is based on the properties of field theory glass and gauge glass. In the first glass the local action or the action of the entire glass may by accident be approximately invariant under some group. The chances for a group being an approximate invariance group is better the smaller is its effect in transforming the fields around. Hence it is an advantage for a group to have few generators, i.e. to have low dimension and to have few and small representations to act on. Once the field theory glass has an approximate symmetry it may become an exact gauge symmetry by the exactification mechanism argued for above. The inferred SMG's at the Planck level are indeed rather small groups with the fewest and smallest possible representations.

We now want to prove the relation (3) between the couplings of the original groups in a product of isomorphic groups and the coupling of the surviving diagonal group. We start from the Lagrangian for a continuum gauge theory with all the original symmetries; it is a sum of terms ($p \in names = \{Peter, Paul, \ldots\}$)

$$\mathscr{L} = -\tfrac{1}{4} \sum_{names} (F^a_{\mu\nu,p})^2 = -\tfrac{1}{4} \sum_{names} \frac{1}{g_p^2} (g_p F^a_{\mu\nu,p})^2. \tag{121}$$

Using

$$U\left(\underset{x\ \ x+a\delta_\mu}{\bullet\!\!-\!\!-\!\!\bullet}\right) = \exp\{iag\mathbf{A}_\mu(x + \frac{1}{2}a\delta_\mu)\} \tag{122}$$

and the fact that for the diagonal subgroup

$$U_p = U_{\text{diag}}\ \forall\ p$$

we find

$$\begin{aligned} g_p A_{\mu,p}(x) &= g_{\text{diag}} A_{\mu,\text{diag}}\ \forall\ p \text{ or} \\ g_p F_{\mu\nu,p}(x) &= g_{\text{diag}} F_{\mu\nu,\text{diag}}\ \forall\ p. \end{aligned} \tag{123}$$

Hence we can rewrite the Lagrangian

$$-\tfrac{1}{4} \sum_{names} \frac{1}{g_p^2} (g_{\text{diag}} F^a_{\mu\nu,\text{diag}})^2 = -\tfrac{1}{4} \frac{1}{g^2_{\text{diag}}} (g_{\text{diag}} F^a_{\mu\nu,\text{diag}})^2 = -\tfrac{1}{4} (F^a_{\mu\nu,\text{diag}})^2, \tag{124}$$

with

$$\frac{1}{g_{\mathrm{diag}}^2} = \sum_{names} \frac{1}{g_p^2}. \qquad (125)$$

The next step is to show that the p^{th} generation which originally coupled to the gauge bosons of the p^{th} group with coupling constants g_p couples to the gauge bosons of the diagonal group with the common constant g_{diag}. This follows immediately from (123) and the original fermion Lagrangian

$$\sum_{names} \bar{\Psi}_p \gamma^\mu (\partial_\mu - i g_p A^a_{\mu,p}(x) T^a) \Psi_p = \sum_{names} \bar{\Psi}_p \gamma^\mu (\partial_\mu - i g_{\mathrm{diag}} A^a_{\mu,\mathrm{diag}}(x) T^a) \Psi_p. \qquad (126)$$

4.3. Concluding remarks

What have we learned from these considerations?

In the first lecture we have considered the standard model and tried to squeeze from it some information about the physics at higher energy levels. The information suggests that the group of the standard model descends from a group which, at a level near the Planck energy, is a direct product of groups of the same type as those found in the standard model group. This disagrees with Grand Unification models based on simple groups; on the contrary it may be termed anti-Grand Unification.

This picture compares well with the picture derived in the fourth lecture. The random dynamics point of view does allow a level characterized by direct products of gauge symmetries arising in a transition from a field theory glass to a gauge glass. Furthermore the confusion mechanism may well describe the reduction from many isomorphic groups to a single diagonal group and, at the same time, give a relation between the many original coupling constants, the number of generations, and the resulting diagonal coupling constants which finally lead to the couplings at the LEP level.

In the second lecture we used arguments about criticality to estimate plausible values of the original coupling constants and showed that the relation is to good accuracy satisfied if the number of generations is taken to be 3.

The third lecture contained a speculative derivation of some of the main principles behind quantum theory. As a side track without logical connection to the main content of these lecture I predicted doomsday to come within a few hundreds of years! This follows from the assumption that (you, now) is a random combination.

The picture that emerges from these lectures is in a sense rather discouraging. There will be a vast desert from the LEP level up to the Planck level, at least as seen by us. One may speculate that since the SMG contains the 3 lowest possible compact groups there may well be some oases of higher compact groups in the midle of the desert, but the near saturation of the relation (73) means that there is no place for matter fields with nontrivial quantum numbers both with respect to the SMG and with respect to such higher group. Hence these possible oases would be hidden for us.

A problem for the random dynamics point of view is that we have no natural way of explaining the breakdown of the Salam Weinberg symmetry at the LEP scale although the relation (73) leaves space for a few Higgs particles. The difficulty is that Higgs particles are not mass protected (see however [26]); why should they have mass as low as $\approx$ 100 GeV? Attempts to replace Higgs particles by technicolour or supersymmetry seem also to give troubles with the relation (73).

Apart from the difficulties just mentioned the random dynamics point of view seems to be quite fruitful. This, however, does not imply that physics has to be random on the true fundamental level. One may easily imagine that Nature is governed by a simple fundamental theory TOE, which at some level (Planck?) becomes so complicated that it effectively behaves as if it were random.

It is a pleasure to thank N. Brene for producing these notes after my draft. He does not, however, feel responsible for the content of the third lecture. In fact he finds the danger of an extinction of humanity due to random combinations of time and person far smaller than the danger of a Doomsday for single or a few human beings due to *certain* combinations of time derivative (e.g. car speed) and person.

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