GENERALIZATION OF THE KLEIN-GORDON EQUATION FOR THE FREE TACHYON INTERACTING WITH THE CLASSICAL VACUUM

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The hypothetical de Broglie waves of the scalar bradyons and tachyons are considered. To describe them a new variant of the Klein–Gordon equation is proposed. For tachyons, it leads to unexpected freedom in choosing a variable scalar quantity and a constant four-vector. The former takes the place of the "rest mass" in the tachyon theory, while the latter corresponds to the tachyon shock wave and expresses the tachyon-vacuum interaction. The resulting image of the tachyon differs from that usually accepted in special relativity and corresponds to that given by general relativity.

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1. *Introduction*

The generalization of the Klein–Gordon (K–G) equation presented in this paper concerns the objects of two types, slower and faster than light, commonly called bradyons and tachyons. Though the formalism is general, the main attention is focused on the tachyonic case, which exhibits some peculiar properties.

Even in the simplest special relativistic picture (in which the bare free particles of both types are represented by the straight world lines in the flat spacetime) there are essential differences between bradyons and tachyons concerning the concept of rest and the direction of motion in various reference frames[1]. The differences become greater when the free particles are considered to be the sources of fields. It is especially so in the general relativistic

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[1] The concept of rest for the tachyon cannot be defined in terms of the theory of relativity [1–4], though it can be defined in certain extensions of that theory, but in the latter cases we obtain very strange pictures of the universe, e.g., more than one state of rest in one reference frame must exist. These problems were analysed in Sections 4.2–4.4 of Ref. [3], and less precisely but in a simpler and shorter way in Ref. [4]. For the problem of direction of tachyonic motion, related to the so-called creation point of the tachyon, see pp. 36 and 37 in Ref. [3], and Ref. [5]. As regards the perturbing problem of the tachyonic causal paradoxes, it is widely discussed in Ref. [3, 6], where the most representative literature of the subject is also given, and it is shown [3] that the existence of tachyons would not be contradictory to the theory of relativity.

description, where the source of fields is (in space) a point in the bradyonic case and a conical surface [2, 5] expanding along its normals with the speed of light in the tachyonic case (the vertex of the cone, moving faster than light, is interpreted as a particle) [5]. These sources are irremovable singularities [7] of certain exact solutions of the Einstein and Einstein–Maxwell equations. These solutions are rotation-free, i.e, they represent "spinless objects" what justifies the search for a generalization just of the K–G equation. When properly combined with the flat spacetime, the tachyonic solutions reveal "comet--like objects" expanding into the flat space (see Fig. 1) with the mass that could be continuously transformed into the field (including the shock wave), or vice versa, due to the hypothetical interaction with vacuum, e.g., with the Dirac sea. Two of the tachyonic solutions [7, 8] seem to be particularly pertinent since they yield a realistic model of the tachyon with a well defined creation point and a shock surface closed in space (see Fig. 1c). As yet we have not seen such a particle but the tachyon might be it, if it existed.

In the known special relativistic descriptions of the tachyon the above properties do not appear, what makes the impression that the tachyon is an ordinary particle having the spacelike world line. This has been the inspiration to make a different description (in terms of quantum mechanics in the flat spacetime) in which the specific properties of the tachyon find some reflection.

The subsequent material is organized as follows. In Section 2, beside the introductory remarks on formalism, some interpretation problems of the tachyonic dynámical quantities

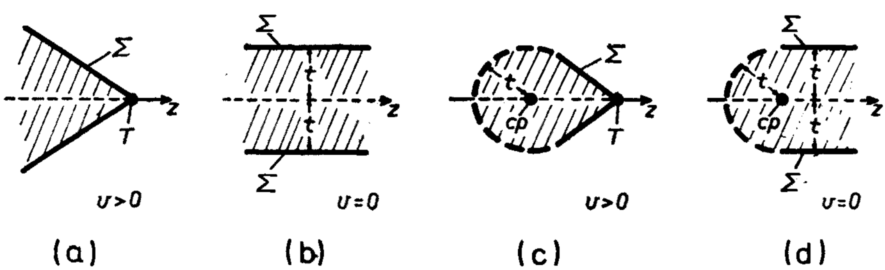

Fig. 1. The pointlike free tachyon T moving in space along axis z with velocity $w = v^{-1} > 1$ ($c = 1$) together with its "ballistic" shock wave Σ (singular in terms of general relativity) presented in the longitudinal section, in accordance with the general relativistic description. The cases (a) and (b) correspond to the Levi-Civita (or Levi-Civita–Reissner–Nordström) solution of the Einstein (or Einstein–Maxwell) equations, while (c) and (d) correspond to the solutions given in Ref. [7, 8]. The cases (a) and (b) represent an unphysical model since the tachyon comes from infinity. In the (c) and (d) cases the tachyon is going from its creation point cp. The internal shaded regions are curved spaces filled with fields and they are irrelevant to this paper. The present paper concerns the external blank regions representing the flat space into which the tachyon moves. Σ expands along its normals into the flat space with the speed of light. In the reference frames characterized by parameter $v > 0$ the shock wave Σ is an infinite (a) or finite (c) circular cone, and in the critical reference frame ($v = 0$, i.e. $w = \infty$) it is an infinite (b) or semi-infinite (d) circular cylinder having radius equal to t at every moment $t > 0$ (since $c = 1$). In the (c) and (d) cases the dashed part of Σ represents a tangent fragment of a sphere having the centre at cp and radius equal to t at every moment $t > 0$. This fragment is a luxonic signal of creation of the tachyon, and it is irrelevant to this paper. Only the conical or cylindrical part of Σ (continuous thick lines) is relevant to this paper, since only this part is generated by the tachyon. For more details see Ref. [5]

are discussed. In Section 3 the generalized K–G equation with three assumptions is presented. In Section 4 the general properties of the equation are briefly discussed. In Section 5 the main results are presented. Owing to an additional restriction the equation becomes invalid for bradyons and reveals the properties of tachyons (relations (25) and (27a)) that agree with the mentioned description given by general relativity.

2. *Formal and methodological introductory remarks*

Our considerations concern the flat spacetime. Let us assume its signature as $(+\ +\ +\ -)$ and choose an inertial reference frame endowed with the standard Lorentz coordinates $x^\mu = x, y, z, t$. Then, for every four-vector n^μ and four-vector operator $\hat{n}^\mu$, where $\mu = x, y, z, t$, there are $n_\mu = n^\mu$ and $\hat{n}_\mu = \hat{n}^\mu$ for $\mu = x, y, z$, and $n_t = -n^t$ and $\hat{n}_t = -\hat{n}^t$ in terms of the chosen coordinate system in which all our formulae will be presented.

We assume the speed of light $c = 1$.

From the standard definition of the four-velocity u^μ of a free particle, we have

$$u_\mu u^\mu + \varepsilon = 0, \tag{1}$$

where $\varepsilon = 1$ for the timelike world line (bradyon) and $\varepsilon = -1$ for the spacelike world line (tachyon).

Defining

$$p^\mu := m u^\mu, \tag{2}$$

where m is an arbitrary real constant, we get by multiplying Eq. (1) by m^2 that

$$p_\mu p^\mu + \varepsilon m^2 = 0, \tag{3}$$

which is the classical origin of the quantum K–G equation

$$\hat{p}_\mu \hat{p}^\mu \psi + \varepsilon m^2 \psi = 0, \tag{4}$$

where

$$\hat{p}^\mu := -i\hbar\partial^\mu, \tag{5}$$

for both the types of free particles; $\hat{p}^\mu$ being here the four-momentum operator.

Constant m, that plays the role of a parameter in Eqs. (2) and (3), has the dimension of mass (or energy or momentum since velocity is dimensionless in our units $(c = 1)$), and is commonly interpreted in the bradyonic case $(\varepsilon = 1)$ as the particle rest mass. In the tachyonic case $(\varepsilon = -1)$, when we cannot reasonably define the state of rest for the particle (see footnote 1), the meaning of m is not clear (cf. also p. 46 in Ref. [1]). Anyway, for both the types of particles the constant m is, generally speaking, a dynamical quantity transforming the kinematical quantities u^μ into dynamical quantities p^μ (Eqs. (2)), or kinematical Eq. (1) into dynamical Eq. (3). We shall call m (and its generalization M below) a *masslike quantity*.

For free bradyons (and luxons, i.e. objects moving with the speed of light) the components p^x, p^y, and p^z are interpreted as those of momentum and p^t as energy (in our units).

Such an interpretation is not obvious for tachyons, especially when we consider a tachyon in its critical reference frame, i.e. in the frame where the tachyon has infinite velocity. (Such a frame exists for every tachyon by virtue of the known property of the Lorentz transformation.) However, simple analysis shows that we have to interpret p^μ in the same way also for the tachyons[2].

Now let us discuss a few well-known concepts and relations. Consider a plane harmonic wave having an amplitude equal to unity and constant frequency ν and length λ, that moves with a constant phase velocity v_{ph}, i.e. $v_{ph} = \nu\lambda$. We shall refer to such a wave as basic wave, since it is a groundwork to construct a modulation moving with constant group velocity v_{gr}. The standard calculation gives $v_{gr} = d\nu/d(\lambda^{-1})$. If the basic wave is assumed to be the de Broglie wave (when it is an auxiliary unphysical object) and its modulation is constructed so as to form a wave packet, then such a modulation represents a particle moving with velocity v_{gr}. It should be emphasized that these kinematical relations are general properties of wave motion and are *independent* of the relativistic or nonrelativistic points of view, and thus they are *independent* of the division into bradyons, luxons, and tachyons. This and the congeneric description of all three types of particles in terms of the theory of relativity given by Eqs. (1)–(3) (remarks on luxons are summarized in footnote 7) suggest the need to assume the existence of the de Broglie wave also for the tachyon, as it is the case for the bradyon and luxon.

We make such an assumption.

Using the mentioned kinematical equations $v_{ph} = \nu\lambda$, $v_{gr} = d\nu/d(\lambda^{-1})$, and the known de Broglie dynamical relations, i.e. momentum $h\lambda^{-1}$ and energy $h\nu$, we find from Eq. (3) that

$$v_{ph}v_{gr} = 1. \tag{6}$$

Henceforth we assume, without loss of generality, that each particle considered in the further text moves along the axis z of our frame.

Let v and w be velocities constant in our frame and parallel to its axis z, and such that

$$vw = 1, \quad 0 \leqslant v^2 < 1, \quad w^2 > 1. \tag{7}$$

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[2] Assuming that a pointlike tachyon moves along z-axis with velocity $w > 1$ and calculating u^μ we get from Eqs. (2) that $p^x = p^y = 0$, $p^z = mw(w^2-1)^{-1/2}$, and $p^t = m(w^2-1)^{-1/2}$. This implies $p^z = m$ and $p^t = 0$ in the critical frame, in which the tachyon world line coincides with z-axis. The latter equations may be interpreted in two ways. In the first way, the tachyon moves with infinite velocity and has momentum $p^z = m$ and zero energy ($p^t = 0$). In the second way, there is no motion in the frame and thus no momentum ($p^t = 0$), but there is only an instantaneous flash stretched on axis z and having the total energy $p^z = m$. The second interpretation seems to be intuitively much more obvious than the first one, however, it leads to nonsense. In fact, rotating only the spatial axes and leaving the time axis intact we make $p^x \neq 0$ or/and $p^y \neq 0$ while there is still $p^t = 0$. Thus, in terms of the second interpretation, we obtain, by invariant Eq. (3), either more than one energy component, what contradicts our concept of energy, or the appearance of momentum and the alteration of energy by means of formal rotation of the spatial axes, what is nonsense. Note that this would take place in the usual subluminal reference frame. (We reject the unfortunate concept of the so-called superluminal reference frame; see Section 4.4 in Ref. [3] or Ref. [4], cf. also Ref. [9] and appropriate references in Refs. [3, 9].) Concluding, we have to assume the first interpretation, i.e. p^z as momentum and p^t as energy.

In accordance with the known rule that the velocity of a particle is the group velocity of the de Broglie wave of the particle and with relations (6) and (7), we assume that in our frame we have

$$v = v_{\mathrm{gr}}, \quad w = v_{\mathrm{ph}} \quad \text{for} \quad \varepsilon = 1, \tag{8a}$$

$$v = v_{\mathrm{ph}}, \quad w = v_{\mathrm{gr}} \quad \text{for} \quad \varepsilon = -1, \tag{8b}$$

for a chosen bradyon and tachyon, respectively.

The assumption on motion along axis z, relations (1), (7), and (8), and the condition $v_{\mathrm{gr}} = dz/dt = u^z/u^t$ determining the world line of our particle, give

$$u_x = u^x = u_y = u^y = 0, \tag{9a}$$

and

$$u_z = u^z = \gamma v, \quad u_t = -u^t = -\gamma \quad \text{for} \quad \varepsilon = 1, \tag{9b}$$

$$u_z = u^z = \gamma, \quad u_t = -u^t = -\gamma v \quad \text{for} \quad \varepsilon = -1, \tag{9c}$$

where

$$\gamma := (1-v^2)^{-1/2} \geqslant 1. \tag{9d}$$

Equation (1) gives u_μ's with accuracy up to the sign, which can be chosen arbitrarily without loss of generality. Above we chose γ positive and the signs of u_μ's in accordance with the rule for $d\zeta$ given just below Eqs. (10).

Let $x^\mu = x^\mu(\zeta)$ be equations of the world line of our particle, where the real variable ζ is an affine parameter of this line. Thus, by definition, ζ is independent of the choice of coordinate system as being dependent only on spacetime points, in this case on the points of the world line. We can determine ζ more precisely in the standard way when defining the four-velocity, i.e. assuming $u^\mu := dx^\mu/d\zeta$. This gives $-\varepsilon(d\zeta)^2 = dx_\mu dx^\mu = ds^2$ by Eq. (1), where ds^2 is the standard infinitesimal invariant of the world line. These relations enable us to express ζ in terms of our coordinate system, namely we have

$$-\varepsilon d\zeta = u_\mu dx^\mu = u^\mu dx_\mu. \tag{10}$$

Equation $ds^2 = -\varepsilon(d\zeta)^2$ gives us the freedom in choice, without loss of generality, of the sign of $d\zeta$ (do not confuse ε). Here we choose the $d\zeta$ sign so as to have ζ increasing with t for $\varepsilon = 1$ and with z for $\varepsilon = -1$.

Relations (7)–(10) give $d\zeta \propto dz - v_{\mathrm{ph}} dt$.

For every differentiable function $f = f(\zeta)$, Eqs. (5) and (10) give

$$\hat{p}^\mu f = i\hbar\varepsilon \dot{f} u^\mu, \tag{11}$$

where an overdot denotes throughout the differentiation with respect to ζ.

The basic de Broglie wave fulfilling Eq. (4) in terms of p_μ and ζ is $\psi = \phi$, where

$$\phi := \exp i\hbar^{-1} p_\mu x^\mu = \exp -i\hbar^{-1}\varepsilon m\zeta, \tag{12}$$

an insignificant constant phase shift having been neglected.

Note that we use two languages in this paper, namely the language of classical relativity when we speak of pointlike particle, its world line, or affine parameter, and the language of quantum mechanics when we speak of de Broglie wave or operator. This will be held in the further text to make our considerations clearer, since our subject belongs to quantum mechanics but it is based on the results obtained in general relativity (see Section 1).

The present section was dealing with a constant masslike quantity. The following will be dealing with one that can be a function of ζ.

3. *The generalized equation and three assumptions*

We consider the equation

$$\hat{P}_\mu \hat{P}^\mu \psi + \varepsilon M^2 \psi = 0, \tag{13a}$$

from which by substituting $\hat{P}_\mu = \hat{p}_\mu$ and $M = m$ we get the free particle K–G equation (4).

The general and obvious restriction of $\hat{P}^\mu$, M, and ψ (and thus of H, k^μ, and F in the following) consists in assumption that Eq. (13a) is Lorentz invariable (cf. footnote 4). Other assumptions are given below.

3.1. Assumption on constant velocity and function M

We assume that Eq. (13a) describes free particles in classical vacuum. The condition of being free is expressed by the assumption that the particles move with constant velocities, i.e. their world lines are straight. The de Broglie wave model for the particles and relations (6)–(10) are assumed here.

The hypothetical interaction between classical vacuum and the particle moving with a constant velocity may be manifested by a change of the masslike quantity (cf. Section 1). Thus we assume a disposable differentiable real function M instead of the constant m in the K–G equation (4). Variability of something in spacetime means dependence on coordinates. It seems to be obvious that the variable intrinsic properties of particle should depend on coordinates only through the affine parameter of the particle world line (cf. Eqs. (10)). The masslike quantity can be treated as an intrinsic property of particle. Thus we assume that

$$M = M(\zeta). \tag{13b}$$

3.2. Assumption on operator $\hat{P}^\mu$

The interaction between a free particle and classical vacuum considered here is defined on the analogy of the interaction between a charged particle and electromagnetic field, namely we introduce a four-vector operator

$$\hat{P}^\mu := \hat{p}^\mu - Hk^\mu \tag{13c}$$

instead of $\hat{p}^\mu$ in the K–G equation (4).

It is seen that H and k^μ take the place of the electric charge and electromagnetic four--vector potential, respectively. Thus, by virtue of this analogy, $-Hk^\mu$ is an interaction term of $\hat{P}^\mu$, and H should be a kind of dynamical charge. Therefore H may be treated as an intrinsic property of the particle and in accordance with what was said in Section 3.1 we assume that

$$H = H(\zeta), \tag{13d}$$

where H is a disposable (temporarily, lest we lose the generality) differentiable real function.

As regards the four-vector k^μ, let us note that the field potential is assumed to give interaction forces by differentiation of that potential with respect to coordinates, while in our case we should have no forces since our Eq. (13a) is expected to describe free particles having constant velocities in the classical vacuum. Thus k^μ should be coordinate independent and we assume that

$$k^\mu = \text{constant}. \tag{13e}$$

We assume here a situation opposite to the standard ones, namely we assume a variable (in general) dynamical "charge" H and a constant "potential" k^μ. We do not determine precisely the meanings of the quantities H and k^μ since Eqs. (13a)–(13e) represent only a scheme containing various theories, in which H and k^μ may have various senses (see the last paragraph of Section 4). For instance, in Section 5, where we assume $H = M$, k^μ is interpreted as a kind of local reaction of the vacuum to the presence of the particle and not as a four-vector field in spacetime.

Now, thanks to the direct affinity between Eqs. (4) and (13a), $\hat{P}^\mu$ has acquired the direct physical meaning of a four-momentum operator, while $\hat{p}^\mu$ given by the standard definition (5) has lost that meaning. Now, by definition (13c), $\hat{p}^\mu$ is the generalized four--momentum operator that need not have a direct physical sense. Such a standpoint is just the same as the one in the case of charged particle in the electromagnetic field[3], and is in direct agreement with the common interpretation used in quantum (and classical) mechanics.

In the further text Eq. (13a) together with Eqs. (13b)–(13e) will be referred to as Eq. (13).

3.3. Assumption on the existence of a generalized basic solution

The basic solution (12) of Eq. (4) need not be a solution of Eq. (13), although, as we shall see in Section 4, it is the one in particular cases. On the other hand, the existence of a basic de Broglie wave solving Eq. (13) is necessary to construct a wave packet as a realistic model of the particle. Thus we generalize the concept of basic wave replacing only the term $-\varepsilon m\zeta$ with a function $F(\zeta)$ in ϕ given by Eqs. (12).

Thus we assume that there exists a solution ψ of Eq. (13) such that $\psi = \theta$, where

$$\theta := \exp i\hbar^{-1}F, \qquad F = F(\zeta), \quad \dot{F} \neq 0, \tag{14}$$

and where F is a disposable (temporarily, lest we lose the generality) differentiable real function. An overdot denotes throughout the differentiation with respect to ζ. Condition $\dot{F} \neq 0$ is obvious, since for $F =$ constant there would be no wave motion.

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[3] For instance, considering a charged particle moving along a constant magnetic field we have the real momentum parallel to the direction of motion, while formally there are transverse (and even nonunique) components of the generalized momentum given by the field vector potential.

The use of a disposable (imaginary) exponent in the basic wave expression, instead of the standard linear one (cf. relations (12) and (14))[4], agrees with assumption 3.1 by which m is replaced by $M(\zeta)$, v_{ph} being constant. In terms of the de Broglie wave, taking into account that the exponent in the expression of ϕ is proportional[5] to $\lambda^{-1}\zeta$, θ could be understood as a basic wave (being an abstract groundwork for constructing a wave packet) with $\lambda = \lambda(\zeta)$ and v_{ph} = constant, and $\nu\lambda = v_{ph}$.

4. *General results*

Substituting $\psi = \theta$ in Eq. (13) and dividing the result into real and imaginary parts we obtain two equations

$$\varepsilon(M^2 - \dot{F}^2) - 2a_\varepsilon \dot{F}H + H^2 k_\mu k^\mu = 0, \tag{15a}$$

$$\varepsilon\ddot{F} + a_\varepsilon \dot{H} = 0, \tag{15b}$$

which determine more accurately Eq. (13), and where $k_\mu k^\mu \equiv k_x^2 + k_y^2 + k_z^2 - k_t^2$ and the constants a_ε are for $\varepsilon = \pm 1$ the following:

$$a_1 = -\gamma(vk_z + k_t), \tag{16a}$$

$$a_{-1} = \gamma(k_z + vk_t). \tag{16b}$$

If $H = 0$ or all the k_μ's are equal to zero, then M = constant and $\theta = \phi$ (with accuracy up to the insignificant constant phase shift), and thus Eq. (13) becomes the K–G equation. If $H \neq 0$ and $\dot{H} = 0$, when we can put $H = 1$, then also M = constant and $\theta = \phi$ (up to a constant phase shift). In this case, therefore, Eq. (13) is also the simple K–G equation presented in the shape of a constant shift of the four-momentum ($p^\mu \to p^\mu + k^\mu$)[6].

Thus, to obtain a true generalization of the K–G equation in the framework of Eq. (13), we should assume k^μ as a nonzero four-vector[7] and

$$\dot{H} \neq 0, \tag{17}$$

what we do.

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[4] Having $\dot{F} \neq 0$, one can always make the transformation $F(\zeta) = -\varepsilon m\zeta'$ that gives $\theta(\zeta) = \phi(\zeta')$, i.e. one can always have a linear exponent in the basic wave expression, if one were, e.g., to construct a wave packet in the standard way. However, if $F(\zeta)$ is not linear and one uses the new variable ζ', then to preserve the invariant form of Eq. (13) one has to replace $\hat{p}^\mu$ by $-i\hbar\delta^\mu$ in definition (13c), where δ_μ is the covariant derivative, and then some quantities may lose their direct physical sense in the flat spacetime.

[5] Note that for ϕ we have λ = constant and $\lambda^{-1}\zeta = \lambda^{-1}\int d\zeta = \int \lambda^{-1} d\zeta$, cf. Eq. (22).

[6] The use of such a shape of the K-G equation would be justified if the existence of a hypothetical free object having four-momentum calculated from Eqs. (2) different from its four-momentum directly measured were assumed.

[7] Let us make some remarks on luxons. Assuming $v_{ph} = v_{gr} = \pm 1$ and $\varepsilon = 0$ we pass to the luxonic cases described by relations (1)–(6), (9a), (13), (14), and (15). Then Eq. (4) becomes the wave equation. To obtain the dynamical Eq. (3) from the kinematical Eq. (1) we have to assume $m \propto h\nu$. (Note that the phrase commonly used saying that "luxons have the zero rest mass" does not seem to make sense in terms of relativity since we cannot define the rest for the luxon in that theory and the term εm^2 is absent in Eq. (3) because

Relations (15) and (17) give

$$M^2 - m^2 + H^2(\varepsilon k_\mu k^\mu + a_\varepsilon^2) = 0, \quad (18a)$$

$$F = -\varepsilon a_\varepsilon \int H d\zeta - \varepsilon m \zeta, \quad (18b)$$

an insignificant additive constant having been neglected in Eq. (18b).

From relations (17) and (18) we conclude the following:

$\theta = \phi$ if and only if $a_\varepsilon = 0$.

If $a_1 = 0$, then $k_\mu k^\mu > 0$ since k^μ has been assumed to be a nonzero four-vector, and then $m \neq 0$ and $\dot{M} \neq 0$.

If $a_{-1} = 0$ or $a_\varepsilon \neq 0$, then there are various possibilities that are seen from Eqs. (18). Among others, there is a tachyonic case created by the assumptions $a_{-1} = 0$ and $k_\mu k^\mu = 0$, where we have $M^2 = m^2 \neq 0$, $\theta = \phi$ (correspondence to the K–G equation), arbitrary $H \neq$ constant, $k_t \neq 0$, and $k_x^2 + k_y^2 > 0$ (cf. inequality (25) and the comments directly below it).

The theory presented above, and its special case in Section 5, does not precisely determine all the functions of ζ and the constants under consideration, giving us the freedom in choice of additional assumptions for these quantities, within the framework of relations (16)–(18a) of course. Note that our quantities may also depend on parameters that are coordinate independent but can depend on certain physical conditions (e.g., our quantities may be different for different particles). Generally speaking, our theory is a scheme containing various more particular theories.

5. *A restriction of assumption 3.2*

It seems reasonable to assume that the interaction between the particle and vacuum is proportional to the particle masslike quantity, i.e. to put

$$H = M, \quad (19a)$$

what by inequality (17) gives

$$\dot{M} \neq 0. \quad (19b)$$

of the property $\varepsilon = 0$, already on the kinematical level of Eq. (1), and not because of $m = 0$.) To obtain Eq. (13) different from Eq. (4) we have to assume k^μ as a nonzero four-vector and $H \neq 0$. Choosing a null coordinate as the variable ζ such that $\partial_x \zeta = \partial_y \zeta = 0$ and $(\partial_z \zeta)^2 = (\partial_t \zeta)^2 = 1$, for the luxonic motion along axis z, we get $a_0 = k^\mu \partial_\mu \zeta$, where $a_0 = a_\varepsilon$ for $\varepsilon = 0$ in Eqs. (15). Assuming $a_0 \neq 0$ we obtain $H = 1$, $k_\mu k^\mu \neq 0$, and $F \propto \zeta$, and thus Eq. (13) becomes the wave equation in the shape of a constant shift of four-momentum (see footnote 6). Assuming $a_0 = 0$ we get F and H as arbitrary functions, $k_x = k_y = 0$, $k_z^2 = k_t^2 \neq 0$, i.e. $k_\mu k^\mu = 0$, and we find that Eq. (13) is fulfilled by every complex function $\psi(\zeta)$. Would therefore Eq. (13) be, for $\varepsilon = 0$ and $a_0 = 0$, a generalization of the wave equation in the spirit of the interaction between the spinless luxon (if it existed) and classical vacuum? Perhaps it would, if something similar to that mentioned in footnote 6 were admitted. However, it seems to be reasonable to acknowledge Eq. (13), for $\varepsilon = 0$ and $a_0 = 0$, as a simple wave equation in a transformed form since $k_x = k_y = 0$. In connection with these considerations, note the existence of the rotation-free plane luxons in the theory of relativity (Section IV in Ref. [7]).

It is easy to see that relations (18a) and (19) give together a self-contradictory system for $\varepsilon = 1$. In other words, Eq. (13) with restrictions (19) *is invalid for the bradyons*. This result seems to be important since a free bradyon interacting with classical vacuum by changing its rest mass would be a strange thing in our contemporary image of matter.

On the other hand, the variability of the masslike quantity M (i.e., $\dot{M} \neq 0$) during the interaction seems to be a reasonable assumption for the tachyons, if we assume the tachyon description given by general relativity (see Section 1).

Henceforth we shall consider only the tachyonic case $\varepsilon = -1$.

5.1. Properties of the tachyonic case

For $\varepsilon = -1$ we get from relations (13c), (14), (16b), (18), and (19) the following:

$$m = 0, \tag{20}$$

$$1 - k_\mu k^\mu + a_{-1}^2 = 0, \tag{21}$$

$$F = a \int_{-1} M d\zeta, \tag{22}$$

$$a_{-1} \neq 0, \tag{23}$$

$$\hat{P}^x\theta = -Mk_x\theta, \quad \hat{P}^y\theta = -Mk_y\theta, \tag{24a}$$

$$\hat{P}^z\theta = v\hat{P}^t\theta, \tag{24b}$$

$$\hat{P}^t\theta = M(a_{-1}\gamma v + k_t)\theta = M\gamma^2(vk_z + k_t)\theta. \tag{24c}$$

Function M is *arbitrary* here (with the condition $\dot{M} \neq 0$), i.e. the theory presented here gives us the freedom in choice of $M(\zeta)$. Also the values of k_μ, which are determined only by relations (16b), (21), and (23) (and as a result by (25) below), are not unique. In other words, we have here a set of arbitrary descriptions of the interaction between the tachyon and vacuum, partially determined by the just mentioned relations involving k_μ's. To obtain an explicit description we have to make additional assumptions or use data from another theory to determine uniquely $M(\zeta)$ and all the k_μ's.

Equation (21) means that k^μ is a spacelike four-vector and constant a_{-1}^2 is a scalar. Thus a_{-1} can be regarded as a constant characterizing the interaction between the free tachyon and vacuum, and then Eq. (16b) can be applied to determine, e.g., k_z in terms of k_t, v, and a_{-1}. Equation (21) determines the relationship between scalars a_{-1}^2 and $k_\mu k^\mu$, one of which should be given. For instance, if we assume the possibility of small variability of M (i.e. $M \cong$ constant; note that $M =$ constant is forbidden by inequality (19b)) and of condition $\theta \cong \phi$, what seems to be reasonable by correspondence to the K–G equation, then equality $a_{-1} = 1$, and thereby $k_\mu k^\mu = 2$, can be regarded as a conclusion from Eq. (22). Note that if $a_{-1} = 1$, then $\hat{P}^\mu\theta = M(u^\mu - k^\mu)\theta$.

By virtue of assumption 3.3, the basic solution of Eq. (13) concerns only a motion in the z direction since θ is assumed to be independent of x and y. When the pointlike tachyon moves in the z direction, its shock wave expands in other directions by virtue of the general relativistic description (Fig. 1). In the tachyonic case we are obliged therefore to interpret Eq. (13) as one concerning only the pointlike free tachyon (i.e. describing the interaction only on the tachyon world line, in terms of relativity) and not the tachyon shock wave, which is a luxon[8]. The existence of the shock wave can, however, be interpreted in terms of the quantum mechanical description presented here as an *effect* of the interaction on the tachyon world line. This can be seen as follows.

Equations (16b) and (21) give

$$k_x^2+k_y^2 \geqslant 1, \tag{25}$$

what means that there must exist certain interaction between the tachyon and vacuum *transverse* to the direction of the tachyon motion. Components k_x and k_y appear in the relations determining k_μ's only as a term $k_x^2+k_y^2$. Thus we can assume the circular radial symmetry (with axis z) of the transverse interaction. This can be figuratively represented, at every point of the tachyon world line, as a continuous set of interaction "arrows". The "arrows" have equal and constant length $(k_x^2+k_y^2)^{1/2}$, they are perpendicular in space to the z axis and fixed to it, and they are directed in all the transverse directions. This and relations (24a) and (25) mean that a radial (in space) momentum impulse exists at every point of the tachyon world line. Such impulses can be directly interpreted as creation of the shock wave. Thus, the theory presented here describes the creation of the tachyon shock wave, being thereby in direct agreement with the general relativistic description. In fact, in terms of the latter the shock wave is an irremovable singularity [5, 7], thus it inseparably accompanies the pointlike tachyon.

Choosing the above picture, we have at every point of the tachyon world line not one but an infinite number of four-vectors k^μ having the same $k_x^2+k_y^2$, k_z, and k_t (thus the same $k_\mu k^\mu$), and different k_x and k_y. The set of constant four-vectors k^μ does not represent a four-vector field in the spacetime but it is such a potential property of the vacuum that is revealed only on the world line of the tachyon. It can be regarded as a kind of local reaction of the vacuum to the presence of the tachyon. Let us remind that the tachyon is never at rest (see footnote 1).

5.2. Situation in the critical reference frame

Putting $v = 0$ we assume that our reference frame is critical for our tachyon. This is of course admissible since we have chosen our frame arbitrarily. Then there are $k_z(v = 0) = a_{-1}$, i.e. $k_z^2(v = 0)$ is a scalar, and

$$k_x^2+k_y^2-k_t^2(v = 0) = 1, \tag{26}$$

$$\hat{P}^z\theta = 0, \tag{27a}$$

$$\hat{P}^t\theta = Mk_t(v = 0)\theta. \tag{27b}$$

[8] Note that we can reverse that argumentation as follows. Since Eq. (13) concerns the pointlike tachyon (for $\varepsilon = -1$) moving in the z direction, while the shock wave is a luxon, θ should be independent of x and y. Both the approaches are connected with the general relativistic description of the tachyon as a foundation of our considerations.

In terms of relativity the tachyon has in this frame the infinite velocity, it exists only in the instant $t = 0$, and its world line coincides with axis z (or its half). In time $t > 0$ the tachyon does not exist and there is only its shock wave that is a radially expanding cylinder (infinite or semi-infinite, see Fig. 1b, d).

In accordance with what was said in Section 5.1, Eq. (13) concerns in the critical frame only the situation on axis z in the instant $t = 0$, i.e., the one-dimensional flash without the longitudinal momentum component (by Eq. (27a)), with transverse momentum components (by Eqs. (24a) and (26)), and with energy (by Eq. (27b)) (distributions of the dynamical quantities along axis z are determined by function $M(\zeta)$; note that for $\varepsilon = -1$ and $v = 0$ we have $d\zeta = dz$). However, this causes of course a subsequent situation in time $t > 0$ in the neighbourhood of axis z, what is not described by Eq. (13). We should therefore have there a luxonic signal radially expanding from axis z. These agree very well with the general relativistic description of the situation in the critical frame (Fig. 1b, d).

Such agreement is due to Eq. (27a), which therefore should be acknowledged as an important property of our theory.

Assumptions that $k_t(v = 0)$ is equal to or different from zero are equivalent to those that relation (25) is an equality or a strong inequality, respectively (see relations (25) and (26)), and each of the assumptions is formally admitted by our theory. Assumption $k_t(v = 0) = 0$ would lead, however, to difficulties in interpretation of Eqs. (24a) and (26), and of the general relativistic description. Assumption $k_t(v = 0) \neq 0$ seems to be reasonable since it means that the tachyonic flash and luxonic signal have energy in the critical frame.

Finally, let us note that the description of the free tachyon presented here (Eq. (13)) does correspond to the general relativistic picture, but does not to the special relativistic image of the bare free tachyon with constant masslike quantity (cf. Eqs. (27) and footnote 2) corresponding to Eq. (4). This reflects in terms of quantum mechanics, the differences between the general and special relativistic descriptions of the tachyon.

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