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A STUDY OF INFRARED ASYMPTOTICS OF GREEN'S FUNCTION IN A FOUR-FERMION MODEL

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It is shown by the method of functional integration that in the infrared energy region the Green function of the four-fermion model has a simple pole.

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One of the currently topical problems is the investigation of quantum chromodynamics (QCD) at low energies when nonperturbative effects manifest themselves and the quark confinement problem exists. In this energy region there is no rigorous calculation scheme in QCD due to the large constant of the quark-gluon interaction α_S. At the same time there is a belief [1, 2] that at low energies the QCD passes to a theory with an effective four-quark interaction. Such an approach describes quite well an experimental situation in the low-energy physics of mesons [3, 4]. Therefore, it is interesting to study four-fermion schemes in the infrared region of energies. It should be noted that in this case the requirement for renormalization is not placed upon this model, since the latter describes only the low-energy limit of the whole theory. To eliminate the ultraviolet divergences, cutoff at large momenta will be used. With this approach the physical quantities — masses of particles are expressed in terms of the cutoff momentum which therefore has a physical meaning. Note that in QCD there appears also a characteristic energy scale which is due to the phenomenon of "dimensional transmutation" [5].

We shall study the infrared behaviour of the fermion Green function in an elemental four-fermion massless model with the Lagrangian

$$L = -\bar{\psi}\gamma_\mu\partial_\mu\psi + \frac{\kappa}{2}(\bar{\psi}\psi)^2, \qquad (1)$$

where κ is the fermion field self-action constant with dimensionality m^{-2}.

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On introduction of the collective field φ and after functional integration with respect to the Fermi-fields $\bar{\psi}$, ψ the generating functional for Green's functions can be expressed as [6–8]

$$Z[\bar{\eta}, \eta] = N \int \mathscr{D}\varphi \det(-\gamma_\mu \partial_\mu + g_0 \varphi)$$
$$\times \exp\{i \int dxdy[-\tfrac{1}{2}\mu_0^2 \varphi^2 \delta(x-y) + \bar{\eta}(x) K(x, y|\varphi)\eta(y)]\}. \qquad (2)$$

Here $\kappa = g_0^2/\mu_0^2$; g_0, μ_0 are the dimensionless and the dimensional constants, respectively. The Green function of the fermion in the external collective field $K(x, y|\varphi)$ satisfies the equation

$$(\gamma_\mu \partial_\mu - g_0 \varphi(x)) K(x, y|\varphi) = \delta(x-y). \qquad (3)$$

Making C-numerical shift of the Bose-field $\varphi(x) = \varphi_0 + \varphi'(x)$ caused by the phase transition where $\varphi_0 = \text{const}$ and expressing the fermion determinant as a series in loops, we arrive at the expression [6–8]

$$Z[\bar{\eta}, \eta] = N_1 \int \mathscr{D}\varphi' \exp\{i \int dxdy[\tfrac{1}{2}\varphi'(\Box - M^2)\varphi' Z_3^{-1}\delta(x-y)$$
$$+ \bar{\eta}(x) K(x, y|\varphi)\eta(y) + \sum_{n=3}^{\infty} \frac{i}{n} \operatorname{tr}[K_0 g_0 \varphi']^n, \qquad (4)$$

where the integral $Z_3^{-1} = -\dfrac{ig_0^2}{8\pi^4}\displaystyle\int \frac{dp}{p^2+m^2}$ is cut off at a large momentum $p^2 = \Lambda^2$, $m = -g_0\varphi_0$ is the dynamic mass fermion (condensate), $M = 2m$ is the mass of the collective field $\varphi' Z_3^{-1/2}$ which is a bound fermion-antifermion state, and the free Green function of the fermion is $K_0(p) = (-i\hat{p}+m)/(p^2+m^2)$. As a result of taking into account the vacuum polarization (fermion determinant) in the field $\varphi' Z_3^{-1/2}$ there appeared a kinetic term $(\varphi' \Box \varphi' Z_3^{-1})$ and the fermion has acquired a dynamic mass m. The sum in (4) represents an expansion in loops and contributes to the effective action of the form [7] $2mg_0\varphi'^3 Z_3^{-1} - \frac{1}{2} g_0^2 \varphi'^4 Z_3^{-1} + O(g_0^2)$. In the infrared region, the contribution of the loops is usually ignored and this sum therefore will not be taken into account.

Changing the variable $\Phi = \varphi' Z_3^{-1/2}$, neglecting the last term in (4), we shall write the expression for the quantum Green function $G(x, y) = \delta^2 Z[\bar{\eta}, \eta]/\delta\bar{\eta}(x)\delta\eta(y)|_{\eta=\bar{\eta}=0}$

$$G(x, y) = N_2 \int \mathscr{D}\Phi K(x, y|\Phi) \exp\{i \int dx \tfrac{1}{2} \Phi(\Box - M^2)\Phi\}. \qquad (5)$$

The Green function (5) effectively corresponds to the Yukawa interaction $g\bar{\psi}\psi\Phi$ if the vacuum polarization is neglected. The infrared asymptotics of the Green function from the point of view of the spinor field model with the Yukawa interaction has been studied in [9]. To investigate (5), we shall employ another method involving the solution of (3) in the form of the functional integral of [10]. To this end we quadrate equation (3)

$$(\Box - (m - g\Phi)^2 + g\gamma_\mu \partial_\mu \Phi)\mathscr{D}(x, y|\Phi) = \delta(x-y), \qquad (6)$$

where

$$K(x, y|\Phi) = (\gamma_\mu\partial_\mu - m + g\Phi(x))\mathcal{D}(x, y|\Phi),$$

$$g_0\varphi(x) = -m + g\Phi(x), \quad \Phi = \varphi' Z_3^{-1/2}, \quad g = g_0 Z_3^{1/2}.$$

Let us write the solution of equation (6) as [10]

$$\mathcal{D}(x, y|\Phi) = -i \int_0^\infty dSC \int \mathcal{D}v \exp \{i \int_0^S d\xi[v_\mu^2(\xi) - (m - g\Phi(x - 2\int_\xi^S v(\eta)d\eta))^2$$

$$+ g\gamma_\mu(\xi)\partial_\mu\Phi(x - 2\int_\xi^S v(\eta)d\eta)]\}\delta(x - y - 2\int_0^S v(\xi)d\xi). \tag{7}$$

Here functional integration with respect to the additional field is used and the constant C satisfies the condition

$$C \int \mathcal{D}v \exp \{i \int_0^S v_\mu^2(\xi)d\xi\} = 1. \tag{8}$$

Let us introduce the following designations:

$$j(z) = \int_0^S d\xi\delta(z - x + 2\int_\xi^S v(\eta)d\eta),$$

$$[\mathcal{D}v]_0^S = C\mathcal{D}v \exp \{i \int_0^S v_\mu^2(\eta)d\eta\} \tag{9}$$

with the account of which equation (7) will take on the following compact form:

$$\mathcal{D}(x, y|\Phi) = -i \int_0^\infty ds \int [\mathcal{D}v]_0^S \exp \{-i \int dzj(z)$$

$$\times [(m - g\Phi(z))^2 - g\gamma_\mu\partial_\mu\Phi(z)]\}\delta(x - y - 2\int_0^S v(\xi)d\xi). \tag{10}$$

As a result, we arrive at the solution of equation (3)

$$K(x, y|\Phi) = \left(\gamma_\mu\partial_\mu - m + ig\frac{\delta}{\delta J(x)}\right)(-i)\int_0^\infty dS \int [\mathcal{D}v]_0^S \exp$$

$$\{-i \int dz[j(z)((m - g\Phi(z))^2 - g\gamma_\mu\partial_\mu\Phi(z)) + J(z)\Phi(z)]\}\delta(x - y - 2\int_0^S v(\xi)d\xi), \tag{11}$$

where for convenience the Schwinger source $J(z)$ is introduced. We shall hereinafter neglect the term $g\gamma_\mu\partial_\mu\Phi(z)$ in the exponent of (11) responsible for the spin effects, since it contains matrices γ_μ. In the infrared region of energies this is justified (see [10]). On substituting (11) into (5) there appears a functional integral of the Gaussian type

$$\int \mathscr{D}\Phi \exp\{-i\int dz[j(z)(m-g\Phi(z))^2+J(z)\Phi(z)-\tfrac{1}{2}\Phi(\Box-M^2)\Phi\}$$
$$= e^{-iSm^2}\det{}^{-1/2}(-\Box_E+M^2+2g^2j_E(z))\exp[\tfrac{1}{2}\int dz_E dy_E$$
$$\times(J_E(z)-2mgj_E(z))\Delta(z-y)(J_E(y)-2mgj_E(y))]. \tag{12}$$

Here $\Delta(z-y)$ satisfies the equation

$$(\Box_E-M^2-2g^2j_E(z))\Delta(z-y) = -\delta_E(z-y) \tag{13}$$

and the passage to Euclidean space-time has been realized. Hereafter we shall assume that $g^2 < 1$. This condition is fulfilled in four-quark schemes describing the low-energy meson physics [3, 4]. In this case, the solution of equation (13) can be sought by the iteration method. The zeroth and the first term of Δ expansion by the coupling constant g have the following form in the momentum space:

$$\Delta^0(p) = \frac{1}{p^2+M^2}, \quad \Delta^1(p) = -\frac{2g^2j_E(p)}{p^2+M^2}\int\frac{dk_E}{(2\pi)^4}\frac{e^{ik(x-2\int_\xi^S v(\eta)d\eta)}}{k^2+M^2}. \tag{14}$$

We shall use hereafter only $\Delta^0(p)$ and, therefore, assume that the constant N_2 has been chosen from the condition $N_2\det^{-1/2}(-\Box_E+M^2+2g^2j_E(z)) = 1$. This requirement complies with the fact that the free (at $g = 0$) Green function had the standard form. Using (11), (12) and the approximations made, we shall arrive at the complete Green function in the infrared region

$$G(x,y) = -i\int_0^\infty dSe^{-iSm^2}\int[\mathscr{D}v]_0^S\left(\gamma_\mu\partial_\mu-m-g\frac{\delta}{\delta J_E(x)}\right)\delta(x-y-2\int_0^S v(\xi)d\xi)$$
$$\times\exp[-2mg\int dz_E J_E(z)\int_0^S d\xi\Delta^0(z-x+2\int_\xi^S v(\eta)d\eta)$$
$$+2m^2g^2\int_0^S d\xi\int_0^S d\xi_1\Delta^0(2\int_{\xi_1}^{\xi} v(\eta)d\eta)+\tfrac{1}{2}\int dz_E dy_E J_E(z)\Delta^0(z-y)J_E(y)]. \tag{15}$$

Taking the variation derivative in (15), then assuming $J = 0$ and passing to the momentum space, we find

$$G(p) = -i\int_0^\infty dSe^{-iSm^2}\int[\mathscr{D}v]_0^S(i\hat{p}-m+2mg^2\int_0^S d\xi\Delta^0(2\int_\xi^S v(\eta)d\eta))e^{-2ip\int_0^S v(\xi)d\xi+I(S)} \tag{16}$$

Here

$$I(S) = 2m^2g^2\int_0^S d\xi\int_0^S d\xi_1\Delta^0(2\int_{\xi_1}^{\xi} v(\eta)d\eta). \tag{17}$$

If we assume $g = 0$ in (16) we shall arrive at the free Green function $G^0(p) = (-i\hat{p}+m)/(p^2+m^2)$, where $\hat{p} = p_\mu \gamma_\mu$.

In order to calculate the functional integral of (16), we use the approximate formula [10–12]

$$\int [\mathcal{D}v] F_1(v) \exp F_2(v) \approx \langle F_1 \rangle \exp \langle F_2 \rangle,$$

where

$$\langle F_i \rangle = \int [\mathcal{D}v] F_i(v) \quad (i = 1, 2). \tag{18}$$

Note that approximation (18) fairly well describes not only the infrared region, but the high energy region, too [12, 13]. Applying (18) to expression (16), dropping the last term enclosed in parentheses in (16) which gives a correction to the fermion mass and making a shift of the integration variable $v_\mu(\xi) - p_\mu = \omega_\mu(\xi)$, we obtain

$$G(p) = -i \int_0^\infty dS e^{-iS(p^2+m^2)} (i\hat{p} - m) e^{F(S)}, \tag{19}$$

$$F(S) = 2m^2 g^2 \int_0^S d\xi \int_0^S d\xi_1 \int [\mathcal{D}\omega]_0^S \Delta^0 (2 \int_{\xi_1}^{\xi} \omega(\eta) d\eta + 2p|\xi - \xi_1|)$$

$$= 2m^2 g^2 \int_0^S d\xi \int_0^S d\xi_1 \int \frac{dk_E}{(2\pi)^4} \frac{\exp\{-i|\xi - \xi_1|(k^2 - 2pk)\}}{k^2 + M^2}. \tag{20}$$

In (19), the presence of a negative imaginary addition ($m^2 \to m^2 - i\varepsilon$) in the exponent is implicit. Expressions (19), (20) generalize the formula obtained in [9], since they take into account the values quadratic in boson momenta k, too. This is important in taking into account g^2 corrections [10]. To estimate the integral of (20), we act as does the author of [9]. On substituting the variables $|\vec{k}| = M \operatorname{sh} u$, $k_0 = M \operatorname{ch} u$, $d|\vec{k}|/k_0 = du$, passing to the system of fermion rest and after adequate integration, we find

$$F(S) = \frac{Sg^2 m\Lambda}{4\pi^2} + \frac{iSg^2 M^2}{8\pi^2} \ln \frac{2\Lambda}{M} + \frac{ig^2}{8\pi^2} \ln \frac{2\Lambda}{M}$$

$$- \frac{g^2}{16\pi} e^{iSM^2} H_0^{(2)}(2mMS) - \frac{ig^2 M^2}{8\pi} \int_0^S dS_1 e^{iS_1 M^2} H_0^{(2)}(2mMS_1), \tag{21}$$

where the Hankel function is $H_0^{(2)}(x) = \frac{2i}{\pi} \int_0^\infty e^{-ix \operatorname{ch} u} du$. To obtain (21), we have used the relations $M = 2m$, $p^2 \simeq -m^2$. The first three terms of (21) are eliminated by renormalizing the fermion mass m and the corresponding fields.

Being interested only in low energies, we correctly take into account the infrared region when $p^2 \simeq -m^2$. In this case, the main contribution to the integral of (19) is from the region of large S. We can therefore replace the function $F(S)$ by its asymptotic value $\lim_{S\to\infty} F(S)$.

Taking further into account that $\lim_{x\to\infty} H_0^{(2)}(x) = 0$ and reasoning as in [9], we come to the conclusion that the integral of (19) yields

$$G(p) = \frac{-i\hat{p}+m}{p^2+m^2} + \text{const}\,(-i\hat{p}+m). \tag{22}$$

Thus, the Green function of the fermion in the model involved has a simple pole. In this approximation, there is neither strengthening nor weakening of the pole. The conclusion obtained is in excellent agreement with the results obtained in [9] which considered the infrared asymptotics of the fermion Green function in scalar mesodynamics.

In [14], there is a suggestion that consistent treatment in quantum electrodynamics leads also to a characteristic feature of the spinor propagator in the form of a simple pole in the infrared region.

In conclusion, note that in [15] (see also [16]) a principal possibility of observing colour from the point of view of QCD is shown. This suggests the existence of a pole in the asymptotic state of the quark propagator. So, the investigation discussed in the present paper points to the same asymptotic behaviour of the fermion Green function in the initial model.

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