ON AN ANALOG OF THE HIGGS-LEPTON SECTOR IN A $SU(2)_L \otimes U(1)$-FOUR-FERMION MODEL

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The present paper considers a $SU(2)_L \otimes U(1)$-invariant model with the most general four-fermion interaction of fermions of different generations. It is shown that as a result of dynamic break of symmetry the "upper" fermions (neutrinos) of different generations remain massless whereas the "lower" fermions (leptons e, μ, τ ...) acquire different masses. The spectrum of collective excitations has been found which includes Goldstone fields and fields of massive particles, analogs of Higgs particles representing fermion-antifermion bound states. We have found an effective action of the form similar to the Higgs form, with the difference that for each generation of fermions the mass of the fields, analogs of the Higgs fields, is twice the mass of the fermion.

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1. Introduction

The main problem of the standard theory of electro-weak interactions is at the present time the existence and mass of Higgs particles. Attempts to replace the Higgs sector by introducing technicolor [1, 2] and by using the Schwinger mechanism [3–6] have been reported. The former approach required amplification [7, 8], but it still involved difficulties [9–15]. The status of the latter approach is not clear due to the difficulty of obtaining nonperturbative solutions to the Schwinger–Dyson equations.

There exists another possibility of generating particle masses by introducing the four-fermion interaction [16–18]. Based on this idea, the authors of [19–20] considered a globally $SU(3)_c \otimes SU(2)_L \otimes U(1)$-invariant four-fermion model, where the role of gauge fields was played by compound fermion-antifermion states. Later the problem associated with the existence of massless composite vector fields was indicated [21].

In [22, 23] we proposed a model based on the replacement, in the standard theory of electrically weak interactions, of the Higgs doublet by the quantity

$$\varphi = \bar{R}L = \begin{pmatrix} \bar{\psi}_R \nu_L \\ \bar{\psi}_R \psi_L \end{pmatrix}, \tag{1.1}$$

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where

$$L = \tfrac{1}{2}(1+\gamma_5)\begin{pmatrix}\nu\\ \psi\end{pmatrix} \equiv \begin{pmatrix}\nu_L\\ \psi_L\end{pmatrix}, \quad R = \tfrac{1}{2}(1-\gamma_5)\psi \equiv \psi_R, \quad \bar{R} = R^+\gamma_4, \quad \bar{L} = L^+\gamma_4,$$

ψ is the four-component lepton field and ν is its corresponding neutrino. It should be noted that we did not abandon the requirement for local gauge invariance and used the quantity $\lambda\varphi^+\varphi$ with one dimensional coupling constant as the Higgs' Lagrangian analog. Thus, the scalar Higgs fields were not introduced initially but there appeared compound collective excitations, analogs of the Higgs fields. In this case, the massless phase was unstable and the dynamic $SU(2)_L \otimes U(1)$ symmetry was broken. As a result, the lepton (e) acquired a mass while the neutrino ν remained massless. It should be noted that with this approach it is necessary to introduce a cutoff momentum associated with the scalar condensate quantity which ia actually the lepton mass reversed in sign. At the present time, however, methods of regularizing four-fermion schemes [24–27] which form the basis for the present treatment have been (and are being) developed. In a recent work [28], it was noted that models of the Nambu-Yona-Lasinio type can be treated as renormalizable theories.

The present paper will generalize the model of [22, 23] to the case of a more general four-fermion interaction of fermions of different generations.

2. *Extended model*

Let us consider an arbitrary number of fermion (lepton) generations n. Then we shall introduce analogs of Higgs doublets for each generation of leptons $\varphi^i = \bar{R}^i L^i$, where

$$\varphi^i = \begin{pmatrix}\varphi_1^i\\ \varphi_2^i\end{pmatrix}, \quad \varphi_1^i = \bar{\psi}_R^i \nu_L^i, \quad \varphi_2^i = \bar{\psi}_R^i \psi_L^i, \quad \nu^i = (\nu^e, \nu^\mu, \nu^\tau, \ldots), \quad \psi^i = (e, \mu, \tau, \ldots).$$

The $SU(2)_L \otimes U(1)$-invariant Lagrangian describing the self-action of the fermions will be taken in the form

$$\mathscr{L} = -\bar{L}^i\gamma_\mu\partial_\mu L^i - \bar{R}^i\gamma_\mu\partial_\mu R^i + \lambda_{ij}\varphi^i\varphi^{j+}, \tag{2.1}$$

where summation over indices $i, j = 1, 2, \ldots n$ is assumed, $\partial_\mu = (\vec{\partial}, \partial_4)$, $\partial_4 = -i\dfrac{\partial}{\partial t}$, γ_μ are Dirac matrices. The passage to a complete invariant Lagrangian describing the interaction of leptons with $W^\pm$, Z, A bosons in achieved by replacing $\partial_\mu \to \mathscr{D}_\mu = \partial_\mu - i(gb_\mu^a(x)t^a - \frac{1}{2}g'Ya_\mu(x))$ and adding a free Lagrangian, where

$$\mathscr{L}_0 = -\tfrac{1}{4}F_{\mu\nu}^2 - \tfrac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a,$$

where

$$F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu, \quad G_{\mu\nu}^a = \partial_\mu b_\nu^a - \partial_\nu b_\mu^a + g\varepsilon^{abc}b_\mu^b b_\mu^c$$

and the fields of the observed bosons are constructed in the conventional manner (see, for example, [29]):

$$W^{\pm}_{\mu} = \frac{1}{\sqrt{2}}(b^1_\mu \pm i b^2_\mu), \quad A_\mu = \frac{g a_\mu - g' b^3_\mu}{\sqrt{g^2+g'^2}}, \quad Z_\mu = \frac{g' a_\mu + g b^3_\mu}{\sqrt{g^2+g'^2}}.$$

In the present work, we shall investigate the dynamic break of the $SU(2)_L \otimes U(1)$ symmetry of the model with Lagrangian (2.1) and the formation of lepton masses. We shall also find the spectrum of collective excitations (composite fields).

From the requirement for Lagrangian (2.1) reality it follows that λ_{ij} is the Hermitian matrix, i.e. $||\lambda||^{\dagger} = ||\lambda||$. The λ_{ij} matrix elements have dimensionality $(m)^{-2}$.

The generating functional for the Green functions will be written as follows:

$$Z[\bar{\eta}, \eta] = N_0 \int \mathscr{D}\bar{\psi}\mathscr{D}\psi\mathscr{D}\bar{v}\mathscr{D}v \exp\left[i\int dx(\mathscr{L} + \bar{L}^i\eta^i_L + \bar{\eta}^i_L L^i + \bar{R}^i\eta^i_R + \bar{\eta}^i_R R^i)\right]. \tag{2.2}$$

To linearize the four-fermion interaction entering into (2.2), we shall use the formula

$$\int \mathscr{D}\Phi \exp\{i[\Phi^i_a\varphi^{i*}_a + \Phi^{i*}_a\varphi^i_a - g_{ij}\Phi^i_a\Phi^{j*}_a]\} = (\det ||g||)^{-1} e^{i\lambda_{ij}\varphi_a{}^i\varphi_a{}^{j*}}, \tag{2.3}$$

where summation with respect to indices $i, j = 1, 2, \ldots n$, $a = 1, 2$ is made and the matrix g_{ij} is reverse to λ_{ij}, i.e., $||g|| = ||\lambda||^{-1}$.

For convenience, we shall introduce the 6-dimensional functions and matrices of [23]:

$$\xi^i = \begin{pmatrix} v^i_L \\ \psi^i \end{pmatrix}, \quad \hat{\partial} = \begin{pmatrix} -i\bar{\sigma}_\mu\partial_\mu & 0 \\ 0 & \gamma_\mu\partial_\mu \end{pmatrix}, \quad \bar{\sigma}_\mu = (-\vec{\sigma}, \; iI),$$

$$M_1 = \begin{pmatrix} 0 & I & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M'_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ I & 0 & 0 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{2.4}$$

$$M'_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} 0 & i\vec{\sigma} \\ -i\vec{\sigma} & 0 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix},$$

where $\vec{\sigma}$ are Pauli matrices, I is a 2×2 unit matrix.

Taking into account the designations of (2.4) and formulae (2.3), expression (2.2) is rewritten as

$$Z[\bar{\eta}, \eta] = N \int \mathscr{D}\bar{\xi}\mathscr{D}\xi\mathscr{D}\Phi \exp\{i\int dx[-\bar{\xi}^i\hat{\partial}\xi^i$$
$$+\bar{\xi}^i(\Phi^i_a M_a + \Phi^{i*}_a M'_a)\xi^i - g_{ij}\Phi^i_a\Phi^{j*}_a + \bar{\xi}^i\eta^i + \bar{\eta}^i\xi^i]\}. \tag{2.5}$$

For the functional integral of (2.5) to exist in Euclidean space-time the integrand at large Φ^i_a must decrease. Therefore, it is necessary to impose the requirement for positive definiteness of the matrix g_{ij}. Due to the fact that the matrix $||g||$ is Hermitian, as is the matrix $||\lambda||$, its eigenvalues are real and, taking into account the foregoing, must be positive.

The complex Bose-fields Φ_a^i introduced above are collective variables [30]. Since the continual integral (2.5) is Gaussian in the fermion fields $\bar{\xi}^i$, ξ^i, it is possible to perform the integration. The result is

$$Z[\bar{\eta}, \eta] = N \int \mathscr{D}\Phi \det [(-\hat{\partial}+\Phi_a^i M_a + \Phi_a^{i*} M_a')(\delta_{ij})]$$
$$\times \exp \{i \int dxdy[-g_{ij}\Phi_a^i(x)\Phi_a^{j*}(y)\delta(x-y)+\bar{\eta}^i(x)K^{ij}(x, y)\eta^j(y)]\}, \tag{2.6}$$

where (δ_{ij}) is the unit $n \times n$ matrix valid in the space of generations and $K^{ij}(x, y)$ is the Green function of fermions in the external collective fields Φ_a^i, satisfying the equation

$$(-\hat{\partial}+\Phi_a^i M_a + \Phi_a^{i*} M_a')K^i(x, y) = -\delta(x-y). \tag{2.7}$$

This equation takes into account that $K^{ij}(x, y)$ is a diagonal matrix, i.e., $K^{ij}(x, y) = \delta_{ij} K^i(x, y)$ and in (2.7) there is no summation over the index i.

Let us assume that in the model with the generating functional (2.6) dynamic break down of the $SU(2)_L \otimes U(1)$ symmetry takes place. In this case, the collective fields Φ_a^i acquire a vacuum mean $\langle \Phi_a^i \rangle = \Phi_{0a}^i$. It will be shown below that this corresponds to the effective potential minimum. Isolating the condensate and making a shift $\Phi_a^i(x) \to \Phi_{0a}^i + \Phi_a^i(x)$ ($\Phi_{0a}^i = \text{const}$) using the formula det = exp tr ln, we express equation (2.6) as

$$Z[\bar{\eta}, \eta] = N \int \mathscr{D}\Phi \exp [iS_{\text{eff}} + i \int dxdy \bar{\eta}^i(x)K^{ij}(x, y)\eta^j(y)], \tag{2.8}$$

where the effective action is

$$S_{\text{eff}} = \int dx[-g_{ij}(\Phi_{0a}^i + \Phi_a^i(x))(\Phi_{0a}^{j*} + \Phi_a^{j*}(x))$$
$$-i \operatorname{tr} \ln [(-\hat{\partial}+(\Phi_{0a}^i + \Phi_a^i(x))M_a + (\Phi_{0a}^{i*} + \Phi_a^{i*}(x))M_a')(\delta_{ij})]. \tag{2.9}$$

The operation tr in (2.9) means taking the trace over matrix and space-time variables. Note that the above shift of the collective fields must be mase in equation (2.7), too.

3. *Mass spectrum*

Operation (2.9) includes the real fields $\Phi_a^i(x)$ which represent quantum excitations over the physical vacuum. The equation of motion of the fields $\Phi_a^i(x)$ can be found by taking a variation of the effective action (and assuming $\Phi_a^i = 0$):

$$\frac{\delta S_{\text{eff}}}{\delta \Phi_a^{j*}(x)} = -g_{ij}\Phi_{0a}^i + i \operatorname{tr} K_0^j M_a' = 0 \tag{3.1}$$

plus the complex-adjoint equation. Here K_0^i — Green's function of free fermion satisfies the equation

$$(-\hat{\partial}+\Phi_{0a}^i M_a + \Phi_{0a}^{i*} M_a')K_0^i(x, y) = -\delta(x-y). \tag{3.2}$$

The condition of (3.1) corresponds to the absence of "tadpole" type diagrams and received the name of the selfconsistency condition. Passing into the momentum space, one can verify that the solution of equation (3.2) is the matrix [23]

$$K_0^i(p) = \frac{1}{p^2(p^2+m_i^2)}\begin{pmatrix} -(p^2+|\Phi_{02}^i|^2)\check{p} & \Phi_{01}^i\Phi_{02}^{i*}\check{p} & -p^2\Phi_{01}^i \\ -p^2\Phi_{01}^{i*} & -p^2\Phi_{02}^{i*} & -p^2\bar{p} \\ \Phi_{02}^i\Phi_{01}^{i*}\check{p} & -(p^2+|\Phi_{01}^i|^2)\check{p} & -p^2\Phi_{02}^i \end{pmatrix}, \tag{3.3}$$

where $\bar{p} = p_\mu\bar{\sigma}_\mu$, $m_i^2 = |\Phi_{01}^i|^2+|\Phi_{02}^i|^2$, $\check{p} = p_\mu\sigma_\mu$, $\sigma_\mu = (\vec{\sigma}, iI)$, $p^2 = \vec{p}^2-p_0^2$. Substituting (3.3) into (3.1) and calculating the trace, we arrive at the equations

$$g_{ij}\Phi_{0a}^i = -\frac{i}{8\pi^4}\int\frac{\Phi_{0a}^j dp}{p^2+m_j^2} \quad (j = 1, 2, \dots n;\ a = 1, 2). \tag{3.4}$$

By virtue of the fact that the matrix λ_{ij} entering into (2.1) is Hermitian the matrix g_{ij} entering into (2.6) is also Hermitian. Therefore, there exists a unitary transformation in the space of generations $\Phi_a \to U\phi_a$, $||g|| \to U||g||U^+$, $U^+U = UU^+ = 1$ leading the matrix $||g||$ to the diagonal form with real eigenvalues g^i. We assume that these eigenvalues g^i are positive and different. Then, doing the above unitary transformation of equations (3.4), we arrive at the conditions

$$g^i\Phi_{0a}^i = -\frac{i}{8\pi^4}\int\frac{\Phi_{0a}^i dp}{p^2+m_i^2}. \tag{3.5}$$

It should be noted that the values of m_i are not invariants of the unitary transformation but they are transformed.

We could have diagonalized the quadratic form in (2.1) from the very beginning, $\varphi^i\lambda_{ji}\varphi^{j*} = \sigma^i\lambda_i\sigma^{i*}$ where $\sigma^i = U_{ij}\varphi^j$, $U^+||\lambda||U = \lambda_i(\delta_{ij})$. Then we could straightforward have arrived at (3.5) where the value of $g^i = (\lambda^i)^{-1}$. In this case the collective fields would have the value $\Phi_{0a}^i = \langle\sigma_a^i\rangle$.

Besides the trivial solutions $\Phi_{0a}^i = 0$, corresponding to the non-breaking of the $SU(2)_L \otimes U(1)$ symmetry, equations (3.5) have nonanalytical nontrivial solutions ($\Phi_{0a}^i \neq 0$) at $0 < 8\pi^2 g^i/\Lambda^2 < 1$, where Λ is the cutoff momentum [16]. Consequently, at $\Lambda^2 > 8\pi^2 g^i$ the phase transition to the state with $m_i^2 \neq 0$ occurs. Equations (3.5) are of the same form as the equations for the energy gap in the superconductivity theory the nontrivial solutions of which correspond to the superconducting state and the trivial solutions correspond to the normal state. Now we shall show that m_i are masses of leptons (e, μ, τ, ...). For this purpose it is convenient to use unitary gauge at which $\Phi_{01}^i = 0$, $\Phi_{02}^i = \Phi_{02}^{i*} \neq 0$. In this case, the fermion Green function of (3.3) will be written as

$$K_0^i(p) = \begin{pmatrix} -\dfrac{\check{p}}{p^2} & 0 \\ 0 & \dfrac{m_i - i\hat{p}}{p^2+m_i^2} \end{pmatrix}. \tag{3.6}$$

Here $m_i = -\Phi_{02}$, $\hat{p} = p_\mu\gamma_\mu$. It follows from the form of (3.6) that as a result of dynamic symmetry breaking, the neutrinos (ν_e, ν_μ, ν_τ, ...) remain massless and the fermions (e, μ, τ, ...) acquire nonzero masses m_i ($i = 1, 2, 3 \ldots n$). To prove the vacuum stability at $\Phi^i_{0a} \neq 0$, let us calculate the effective potential. In a single-loop approximation, when we restrict ourselves to the constant fields Φ^i_{0a}, the effective action related to the effective potential by the formula $S^0_{\text{eff}} = -\int V^0 dx$ has the form

$$S^0_{\text{eff}} = -\int dx g_{ij}\Phi^i_{0a}\Phi^{j*}_{0a} - i\,\text{tr}\ln[(-\hat{\partial}+\Phi^i_{0a}M_a+\Phi^{i*}_{0a}M'_a)(\delta_{ij})]. \tag{3.7}$$

Subtracting from (3.7) the action corresponding to the unbroken symmetry $S^0 = -i\,\text{tr}\ln(-\hat{\partial})$ and using the property tr ln = ln det we find

$$S^0_{\text{eff}} - S^0 = -\int dx\{g_{ij}\Phi^i_{0a}\Phi^{j*}_{0a}$$
$$+i\int\frac{dp}{(2\pi)^4}\sum_i \ln\det[(1+\hat{G}_0(p)(\Phi^i_{0a}M_a+\Phi^{i*}_{0a}M'_a)(\delta_{ij})]\}, \tag{3.8}$$

where

$$\hat{G}_0(p) = \frac{1}{p^2}\begin{pmatrix}\check{p} & 0\\ 0 & i\hat{p}\end{pmatrix}. \tag{3.9}$$

Calculating the determinant entering into (3.8), we obtain

$$V^0_{\text{eff}} = g_{ij}\Phi^i_{0a}\Phi^{j*}_{0a} + \frac{i}{8\pi^4}\int dp\sum_i \ln\left(1+\frac{m_i^2}{p^2}\right). \tag{3.10}$$

The second term in (3.10) determines the one-loop addition to the effective potential. The effective potential minimum of (3.10) yields the condition $\partial V^0_{\text{eff}}/\partial\Phi^{i*}_{0a} = 0$ and a complex-adjoint one. It can easily be verified that this condition exactly gives equation (3.4) of selfconsistency. Thus, the nontrivial solutions of equation (3.4) correspond to the effective potential minimum. It should be noted that we have used only the extremum condition. Actually it is necessary to check also that $\partial^2 V^0_{\text{eff}}/\partial\Phi^{i2}_{02} > 0$ is fulfilled (in the gauge $\Phi^i_{01} = 0$, $\Phi^i_{02} = \Phi^{i*}_{02} \neq 0$), which can be verified immediately.

4. *Expansion in loops*

Let us now transform the effective action (2.9) by expanding it in low fields (excitations) $\Phi^i_a(x)$. Drop in (2.9) the constant S^0_{eff} and take into account the equality $\text{tr}\ln[(-\hat{\partial}+(\Phi^i_{0a}+\Phi^i_a(x))M_a+(\Phi^{i*}_{0a}+\Phi^{i*}_a(x))M'_a)(\delta_{ij})] = \text{tr}\ln[(-\hat{\partial}+\Phi^i_{0a}M_a+\Phi^{i*}_{0a}(M'_a)(\delta_{ij})]+\text{tr}\ln[(1-K^i_0(\Phi^i_a(x)M_a+\Phi^{i*}_a(x)M'_a))(\delta_{ij})]$. Let us also make use of the fact that in (2.9) the terms linear in fields $\Phi^i_a(x)$ are absent by virtue of the condition of (3.1) (or (3.4)). Taking into account the foregoing, expand the action of (2.9) near statistical solutions Φ^i_{0a}:

$$S_{\text{eff}} = S^{(2)}_{\text{eff}} + \sum_{n=3}^{\infty} L_n, \tag{4.1a}$$

$$S^{(2)}_{\rm eff} = -g_{ij}\int dx\Phi^i_a(x)\Phi^{j*}_a(x) + \frac{i}{2}\,{\rm tr}\,[K^i_0(\Phi^i_a M_a + \Phi^{i*}_a M'_a)\,(\delta_{ij})]^2, \qquad (4.1b)$$

$$L_n = \frac{i}{n}\,{\rm tr}\,[K^i_0(\Phi^i_a M_a + \Phi^{i*}_a M'_a)]^n. \qquad (4.1c)$$

The field-quadratic term of (4.1b) which determines the propagation of $\Phi^i_a(x)$ particles can be expressed as

$$S^{(2)}_{\rm eff} = -\tfrac{1}{2}\int dxdy\Phi^i(x)\,[\hat{T}^{ij}(x,y)]^{-1}\Phi^j(y), \qquad (4.2)$$

where the four-component wave function $\Phi^i = (\Phi^i_1, \Phi^{i*}_1, \Phi^i_2, \Phi^{i*}_2)$ has been introduced, and the propagator $\hat{T}^{ij}(x,y)$ is determined by the relations

$$[\hat{T}^{ij}_{AB}(p)]^{-1} = g_{ij}\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}_{AB} - i\int\frac{dk}{(2\pi)^4}\,K^i(k)\Gamma_A K^i(k-p)\Gamma_B\delta_{ij}, \qquad (4.3)$$

where

$$\Gamma_A = (M_1, M'_1, M_2, M'_2).$$

Calculating (4.3) with the use of (3.6), we find the nonzero elements in the basic set where the matrix $\|g\|$ has a diagonal form (i.e., $g_{ij}\to g_i\delta_{ij}$)

$$[\hat{T}^i_{12}(p)]^{-1} = [\hat{T}^i_{21}]^{-1} = g_i + \frac{i}{8\pi^4}\int\frac{q(q-p)dq}{(q-p)^2(q^2+m_i^2)} = p^2 Z_\Phi^{-1} + O, \qquad (4.4)$$

$$[\hat{T}^i_{33}(p)]^{-1} = [\hat{T}^i_{44}(p)]^{-1} = -\frac{i}{8\pi^4}\int\frac{m_i^2 dq}{[(q-p)^2+m_i^2]\,(q^2+m_i^2)} = 2m_i^2 Z_\Phi^{-1} + O,$$

$$[\hat{T}^i_{34}(p)]^{-1} = [\hat{T}^i_{43}(p)]^{-1} = g_i + \frac{i}{8\pi^4}\int\frac{q(q-p)dq}{[(q-p)^2+m_i^2]\,(q^2+m_i^2)} = (p^2+2m_i^2)Z_\Phi^{-1} + O.$$

Here we have used the fact that $[\hat{T}^{ij}(p)]^{-1} = [\hat{T}^i(p)]^{-1}\delta_{ij}$, $Z_\Phi^{-1} = \dfrac{1}{16\pi^2}\left(\ln\dfrac{\Lambda^2}{m^2} - 1\right)$, Λ is the cutoff momentum, m is the point of normalization, O denotes the last part which is independent of the cutoff momentum and determines the radiative corrections. To obtain (4.4), we have used equation (3.5), too.

Substituting (4.4) into formula (4.2) in the momentum space and renormalizing the fields $\Phi^i \to \Phi^i Z_\Phi^{1/2}$ [24–26], we find the field-quadratic action to an accuracy of up to higher radiative corrections

$$S^{(2)}_{\rm eff} = \tfrac{1}{2}\int dx\{\Phi^i_1\,\Box\,\Phi^{i*}_1 + \Phi^{i*}_1\,\Box\,\Phi^i_1 + \Phi^i_2\,\Box\,\Phi^{i*}_2 + \Phi^{i*}_2\,\Box\,\Phi^i_2 - 2m_i^2(\Phi^i_2 + \Phi^{i*}_2)^2\}. \qquad (4.5)$$

In (4.5) summation over the generation index i is implicit and the notation $\Box = \partial_\mu\partial_\mu$ is used. Designating $\Phi^i_2 = \kappa^i + i\chi^i$ we rewrite (4.5) in the form

$$S^{(2)}_{\rm eff} = \int dx\{\tfrac{1}{2}(\Phi^{i*}_1\,\Box\,\Phi^i_1 + \Phi^i_1\,\Box\,\Phi^{i*}_1) + \kappa^i(\Box - 4m_i^2)\kappa^i + \chi^i\,\Box\,\chi^i\}. \qquad (4.6$$

It follows from (4.6) that we have in each generation three massless fields Re Φ_1^i, Im Φ_1^i, χ^i and one massive field κ^i with a mass $2m_i$. In this case, Φ_1^i are charged massless fields and χ^i and κ^i are neutral massless and massive fields, respectively.

So, as a result of the dynamic $SU(2)_L \otimes U(1)$ symmetry breaking, in each generation there appeared three Goldstone fields and one massive field, an analog of the Higgs field. This is in agreement with the general Goldstone theorem [31]. It should be noted that the fields Φ_a^i are composite and represent bound fermion-antifermion states. The fact that the mass of the composite boson field is twice the fermion mass was initially found in an elemental four-fermion model [16]. This phenomenon is analogous to the formation of Cooper pairs in the theory of superconductivity.

The diverging three-point and four-point Green functions that follow from (4.1c) at $n = 3$, 4 will contribute to the effective action (4.1a). The terms of (4.1c) with the number $n = 5$ are converging and determine the higher radiation corrections.

To calculate the remaining contributions to the effective action (4.1a), we find the three-point and four-point Green functions as a one-loop approximation

$$\Gamma^{0i}_{ABC}(x, y, z) = \frac{\delta^3 S_{\text{eff}}}{\delta\Phi_A^i(x)\delta\Phi_B^i(y)\delta\Phi_C^i(z)}, \tag{4.7}$$

$$\Gamma^{0i}_{ABCD}(z, y, z, t) = \frac{\delta^4 S_{\text{eff}}}{\delta\Phi_A^i(x)\delta\Phi_B^i(y)\delta\Phi_C^i(z)\delta\Phi_D^i(t)}$$

in the momentum space

$$\Gamma^{0i}_{ABC}(p, q) = i \operatorname{tr}\left\{\int \frac{dk}{(2\pi)^4}\,[K^i(k+p-q)\Gamma_A K^i(k)\Gamma_B K^i(k+p) + K^i(k-p)\Gamma_B K^i(k)\Gamma_A K^i(k+q-p)]\Gamma_C\right\}, \tag{4.8}$$

$$\begin{aligned}\Gamma^{0i}_{ABCD}(k_1, k_2, k_3) = i \operatorname{tr}\Big\{\int \frac{dp}{(2\pi)^4}\,\{&K^i(p)\Gamma_A K^i(p+k_2)\,[\Gamma_C K^i(p+k_2-k_3)\\ &\times\Gamma_B K^i(p-k_1)\Gamma_A + \Gamma_C K^i(p+k_2-k_3)\Gamma_A K^i(p+k_1+k_2-k_3)\Gamma_B\\ &+\Gamma_B K^i(p-k_1+k_3)\Gamma_C K^i(p-k_1)\Gamma_A + \Gamma_A K^i(p+k_1+k_2)\Gamma_C K^i(p+k_1+k_2-k_3)\Gamma_B\\ &+\Gamma_A K^i(p+k_1+k_2)\Gamma_B K^i(p+k_3) + \Gamma_B K^i(p+k_3-k_1)\Gamma_A K^i(p+k_3)\Gamma_C]\}\Big\},\end{aligned}$$

$$\Gamma^{0i}_{ABC}(x, y, z) = \int \frac{dpdq}{(2\pi)^8}\,\Gamma^{0i}_{ABC}(p, q)e^{ip(y-x)+iq(x-z)}, \tag{4.9}$$

$$\Gamma^{0i}_{ABCD}(x, y, z, t) = \int \frac{dk_1dk_2dk_3}{(2\pi)^{12}}\,\Gamma^{0i}_{ABCD}(k_1, k_2, k_3)e^{ik_1(x-y)+ik_2(t-y)+ik_3(y-z)}.$$

Calculating (4.8), (4.9) taking into account (3.6), on renormalization of the fields and substitution into the formula

$$S_{\text{eff}} = -\tfrac{1}{2}\int dxdy\Phi^i(x)\,(\hat{T}^i)^{-1}\Phi^i(y) + \frac{1}{3!}\int dxdydz\Phi^i_A(x)\Phi^i_B(y)\Phi^i_C(z)\Gamma^i_{ABC}(x, y, z)$$

$$+\frac{1}{4!}\int dxdydzdt\Phi^i_A(x)\Phi^i_B(y)\Phi^i_C(z)\Phi^i_D(t)\Gamma^i_{ABCD}(x, y, z, t) \qquad (4.10)$$

we find (see also [32])

$$S_{\text{eff}} = \int dx\{\tfrac{1}{2}(\Phi^{i*}_1 \Box \Phi^i_1 + \Phi^i_1 \Box \Phi^{i*}_1) + \kappa^i(\Box - 4m_i^2)\kappa^i$$

$$+\chi^i \Box \chi^i + 4m_i\lambda\kappa^i(|\Phi^i_1|^2 + |\Phi^i_2|^2) - \lambda^2(|\Phi^i_1|^2 + |\Phi^i_2|^2)^2. \qquad (4.11)$$

Here a dimensionless coupling constant $\lambda^2 = Z_\Phi$ has been introduced. The action of (4.11) is analogous to the Higgs sector (on field shift) of the standard theory of electro-weak interactions, the only difference being that the Higgs fields are introduced for every generation of fermions.

The fields κ^i have mass which is twice the fermion mass ($2m_i$) of the corresponding generation.

5. Conclusion

Thus, we have shown that if we proceed from a model with Lagrangian (2.1) we find that this model leads, after taking into account the reconstruction of the physical vacuum and the dynamic symmetry breaking, to the same conclusions as in the Glashow–Weinberg–Salam theory. The difference is in the composite nature of the Higgs particles and their mass.

On introduction of interaction with gauge vector bosons, some of them ($W^\pm$, Z) will acquire nonzero masses while the electromagnetic field A_μ will remain massless, as the calculations of [33] show. It should be noted that the complete effective Lagrangian is analogous to the standard one.

The present treatment is only an analog, but not the substitute of the Glashow–Weingerg–Salam theory, since there has been, so far, no interpretation of κ^i states with masses $2m_i$ (for example, parapositronium, etc.) and the quark sector has not been taken into account. Tempting is the fact that there is no ambiguity here in regard to the Higgs particle mass which exists in the standard theory.

We also believe that the investigation of dynamic symmetry breaking in the $SU(2)_L \otimes U(1)$ four-fermion model may itself be of interest without appealing to the GWS theory. The model considered In this paper can find application in considering strong interactions at low energies when nonperturbative effects manifest themselves and the QCD passes to the four-quark model [34–36].

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