# AN APPROXIMATE RELATIVISTIC TWO-BODY (SCALAR-SCALAR AND SCALAR-SPINOR) POTENTIALS DUE TO THE ELECTROMAGNETIC INTERACTION. I*

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Covariant, two-body, relativistic equations containing relevant potentials describing electromagnetic interactions between charged scalar-scalar and scalar-spinor particles, are derived using Barut's method. Only terms proportional to products of the electric charges are considered. Obtained potential can be confining one.

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Our purpose is to derive relativistic equation for wave function describing bound-state of two scalar particles interacting via the electromagnetic field. To this end we shall use the method proposed recently by Barut [1, 2]. It is an approximate method with unknown, as yet, range of its applicability. It would be interesting to find its justification in the framework of one-time relativistic equations based on the quantum field theory [3, 4]. The method can be applied to many-particle system, as well. We consider here only two-particle system, for simplicity. It is not our purpose to compete in accuracy of determining the energy levels of two-body systems. We would like to extend the Barut's method, which was applied to QED only, to the Lagrangians containing higher order terms, including $\phi^4$. We hope that confining potentials may arise because of a non-perturbative character of the method.

We consider here two charged scalar fields $\phi$ and $\tilde{\phi}$ coupled to the electromagnetic

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field $A^\mu$ in the minimal way

$$S = \int dx \left\{ -\tfrac{1}{4} F_{\mu\nu}F^{\mu\nu} + [(\partial_\mu + ieA_\mu)\phi]^*(\partial^\mu + ieA^\mu)\phi - m^2|\phi|^2 - \frac{\lambda}{2}|\phi|^4 \right.$$

$$\left. + [(\partial_\mu + i\tilde{e}A_\mu)\tilde{\phi}]^*(\partial^\mu + i\tilde{e}A^\mu)\tilde{\phi} - \tilde{m}^2|\tilde{\phi}|^2 - \frac{\tilde{\lambda}}{2}|\phi|^4 \right\}$$

$$= \int dx \{L[A^\mu] + L[\phi] + L[\tilde{\phi}] + J_\mu A^\mu + \tfrac{1}{2} BA_\mu A^\mu\}, \tag{1}$$

where we used the notation:

$$L[A^\mu] = -\tfrac{1}{4} F_{\mu\nu}F^{\mu\nu}, \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu, \tag{2a}$$

$$L[\phi] = \partial_\mu \phi^* \partial^\mu \phi - m^2|\phi|^2 - \frac{\lambda}{2}|\phi|^4, \tag{2b}$$

$$L[\tilde{\phi}] = \partial_\mu \tilde{\phi}^* \partial^\mu \tilde{\phi} - \tilde{m}^2|\tilde{\phi}|^2 - \frac{\tilde{\lambda}}{2}|\tilde{\phi}|^4, \tag{2c}$$

$$J_\mu = ie(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi) + i\tilde{e}(\tilde{\phi}\partial_\mu\tilde{\phi}^* - \tilde{\phi}^*\partial_\mu\tilde{\phi}), \tag{2d}$$

$$B = 2(e^2|\phi|^2 + \tilde{e}^2|\tilde{\phi}|^2). \tag{2e}$$

In the Lorentz gauge:

$$\partial_\mu A^\mu = 0, \tag{3}$$

the equations of motion for $A^\mu$ are:

$$\Box A^\mu = J^\mu + BA^\mu, \quad \Box = -\partial_\mu \partial^\mu. \tag{4}$$

According to Barut we elliminate $A^\mu$ by solving these equations using the retarded Green's function

$$A^\mu(x) = A^\mu_{\text{in}}(x) - \int dy D_{\text{ret}}(x-y) J^\mu(y) + O(e^2, \tilde{e}^2). \tag{5}$$

The retarded, zero-mass, Green's function is of the form [5]:

$$D_{\text{ret}}(x) = \begin{cases} \dfrac{1}{4\pi r}\,\delta(x^0 + r), & x^0 > 0 \\ 0, & x^0 < 0, \end{cases} \qquad r = |\vec{x}| \neq 0. \tag{6}$$

Other relevant functions are the advanced Green's function

$$D_{\text{adv}}(x) = D_{\text{ret}}(-x),$$

and their arithmetic mean value:

$$\bar{D}(x) = \tfrac{1}{2}[D_{\text{ret}}(x) + D_{\text{adv}}(x)] = \frac{1}{8\pi r}[\delta(x^0 - r) + \delta(x^0 + r)]. \tag{7}$$

In order to derive the effective action we substitute the solution (5) to the action and perform relevant integrations by parts neglecting surface integrals over the sphere at infinity. We obtain in this way the formula:

$$S = \int dx\{L[\phi]+L[\tilde{\phi}]\} - \tfrac{1}{2}\int dxdyJ^{\mu}(x)D_{\rm ret}(x-y)J_{\mu}(y)+ \ldots \qquad (8)$$

Neglected terms are either higher order than second in $e, \tilde{e}$ or depend on $A^{\mu}_{\rm in}$ fields.

Since the Lagrangian (1) is symmetric with the respect to the global U(1) transformations of the fields $\phi$ and $\tilde{\phi}$ we obtain the relevant conservation laws which we use for imposing the following normalization conditions on $\phi$ and $\tilde{\phi}$:

$$i\int d^3x(\phi^*\partial^0\phi-\phi\partial^0\phi^*) = 1 \qquad (9)$$

and similarly for the field $\tilde{\phi}$. The time arguments in these relations may be arbitrary, which will be used later.

Following Barut's method we introduce the composite, or bilocal fields, describing bound-states of two scalar particles

$$\Phi(x, y) = \phi(x)\tilde{\phi}(y), \quad \Phi^*(x, y) = \phi^*(x)\tilde{\phi}^*(y), \qquad (10)$$

where we assume also that $\phi$, $\tilde{\phi}$ depend on time as the stationary wave functions

$$\phi(x) = \phi(\vec{x})e^{-ix^0E}, \qquad \tilde{\phi}(x) = \tilde{\phi}(x)e^{-ix^0\tilde{E}}. \qquad (11)$$

and $\phi(\vec{x})$, $\tilde{\phi}(\vec{x})$ are fast decreasing functions of their arguments (bound-states). Multiplying single integrals by the normalization conditions (9) one can rewrite the action functional in terms of the bilocal field $\Phi(x, y)$ and its complex conjugate. Varying it with the respect to $\Phi^*(x, y)$ one gets the following equation for the bound-state wave function $\Phi(\vec{x}, \vec{y})$

$$[(E^2+\Delta-m^2)\otimes\tilde{E}+E\otimes(\tilde{E}^2+\Delta-\tilde{m}^2)+V(\vec{x}, \vec{y})]\Phi(x, y) = 0, \qquad (12)$$

where the potential $V(\vec{x}, \vec{y})$ is equal:

$$V(\vec{x}, \vec{y}) = \frac{e\tilde{e}}{4\pi r}(-2E\tilde{E}+2\partial_k\otimes\partial^k)+\tfrac{1}{2}\, e\tilde{e}\delta(\vec{r})$$

$$-\lambda\tilde{E}^2\int d^3z|\Phi(\vec{x}, \vec{z})|^2-\tilde{\lambda}E^2\int d^3z|\Phi(\vec{z}, \vec{y})|^2+\ldots, \qquad \vec{r} = \vec{x}-\vec{y}. \qquad (13)$$

We supplement this equation with the asymptotic condition at the origin in order to regularize the singular terms in the potential

$$\Phi(\vec{x}, \vec{y}) \sim O(r^{2+\varepsilon}), \quad r\to 0, \quad \varepsilon > 0. \qquad (14)$$

We would like to point out the presence in $V$ of integral Hartree-Fock type terms, which are novel, and of a non-perturbative nature [6–14]. Assuming that $\lambda = \tilde{\lambda}$, $E = \tilde{E}$ and that

$$\nabla_x\Phi(\vec{x}, \vec{z})_{|\vec{x}=0} = -\nabla_y\Phi(\vec{z}, \vec{y})_{|\vec{y}=0} \equiv -\nabla\Phi(\vec{z}, 0), \qquad (15)$$

one may find a confining term of the form

$$V_{\text{conf}}(\vec{x}, \vec{y}) = -2E^2 \frac{e\tilde{e}}{4\pi r} - \lambda E^2(\vec{x}-\vec{y}, \vec{C}) + \dots, \tag{16}$$

where the vector $\vec{C}$ is given by

$$\vec{C} \equiv \int d^3z[\Phi^*(0, \vec{z})\nabla\Phi(0, \vec{z}) + \nabla\Phi^*(0, \vec{z})\Phi(0, \vec{z})]. \tag{17}$$

However, the accuracy of the method needs further investigations especially in the light of a recently raised criticism [15] concerning possibility of deriving some quantum effects without the field quantization. In this context it appears desirable to compare the results obtained by Barut's method with those based on the quantum field theory. Before this we shall quote the result of an analogous calculations for the scalar-spinor case corresponding to the Lagrangian

$$L = L[A^\mu] + L[\Phi] + L[\tilde{\phi}] + L[\psi] + L[\tilde{\psi}] + A_\mu J^\mu + \tfrac{1}{2} BA_\mu A^\mu, \tag{18}$$

where the first three terms are as before while $L[\psi]$, $L[\tilde{\psi}]$ are the Dirac Lagrangians with $\gamma^\mu$ taken in the Majorana representation

$$L[\psi] = \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi, \quad L[\tilde{\psi}] = \bar{\tilde{\psi}}(i\gamma^\mu\partial_\mu - \tilde{M})\tilde{\psi}, \tag{19}$$

the current $J^\mu$ contains the part (2d) and the spinor part

$$-e\bar{\psi}\gamma^\mu\psi - \tilde{e}\bar{\tilde{\psi}}\gamma^\mu\tilde{\psi}. \tag{20}$$

In this case apart from the potential (13) we get the terms

$$-\frac{e\tilde{e}}{2\pi r}\,\partial_\mu \otimes i\gamma^\mu, \tag{21}$$

in an equation for $\phi(x)\tilde{\psi}(y)$, a similar term for the $\psi(x)\tilde{\phi}(y)$ wave equation. Besides there are terms found earlier [1, 2] for the $\psi(x)\tilde{\psi}(y)$ bilocal wave function,

$$-\frac{e\tilde{e}}{4\pi r}\gamma^\mu \otimes \gamma_\mu. \tag{22}$$

Equation (12), besides being nonlinear, is also of higher order in $E$, $\tilde{E}$ what is rather inconvenient for a study of the bound-state energy spectrum. We shall remedy this shortcoming in the next paper.

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