NUCLEON PAIRS AS THE BUILDING BLOCKS OF A NUCLEUS*

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The effects induced by the interplay isovector and isoscalar components of the residual nuclear forces were studied in the model based on six bosons: s_μ^+ with $J = 0$, $T = 1$, $\mu = 0, \pm 1$ and p_μ^+ with $J = 1$, $\mu = 0, \pm 1$, $T = 0$. Low-lying energy levels, E2 transitions, p-boson structure of eigenstates, percentage of α-clusters, (p, t) reactions and α elastic scattering were searched in even-even $^{156-166}$Dy and $N = 92$, $Z = 56$–68 nuclei.

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1. Introduction

Two- and four-nucleon dynamical clusters on the nuclear surface are well experimentally and theoretically known [1–5 and references therein]. The correlation energy ($E_2^{\rm cor}$) of the nucleon pairs [1, 4, 5] and the systematics of alpha-transfer and alpha-decay processes throughout the periodic table point to the importance not only of the proton--proton and neutron-neutron but also neutron-proton residual nuclear forces. Because $E_2^{\rm cor}$ is about a few MeV, for low excitation energy, we will treat the nucleon pairs as building blocks of a nucleus. With the above assumption, the nucleon pair with quantum numbers of total angular momentum J, parity π and isospin T corresponds to the boson with the same quantum numbers. Neutron-proton pair has J = even, $T = 1$ or J = odd, $T = 0$. The effective interactions in the $T = 0$ channel are on the average stronger than in $T = 1$ (see for example Table 1 of the paper [6]). In order to have the eigenstates of a system with good quantum numbers J^π, T we choose the model [5] based on six bosons: s_μ^+ with $J = 0, T = 1, \mu = 0, \pm 1$ and p_μ^+ with $J = 1$, $\mu = 0, \pm 1$, $T = 0$. The s^+ boson corresponds to a pair of nucleons coupled by pairing forces and p^+ describes a neutron-proton pair found on single particle shell model levels with $[l_1 - l_2] = 1$.

In this way we take into account the strongest interactions and we overcome difficulties

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appearing in construction of physical basis for a system of interacting bosons with large J [13].

In Sect. 2 we show a sketch of the model, Sect. 3 includes the calculations of low--lying energy levels in some rare-earth nuclei, Sect. 4 presents summary and conclusions.

2. Model

The most general Hamiltonian for a system of interacting s and p bosons is:

$$H = \varepsilon_1 \hat{n}_p + \varepsilon_2 \hat{n}_s + \varepsilon_3 [p^+ p^+]^{J=0,T=0} [\tilde{p}\tilde{p}]^{00} + \varepsilon_4 [s^+ s^+]^{00} [\tilde{s}\tilde{s}]^{00}$$
$$+ \varepsilon_5 [p^+ p^+]^{20} \cdot [\tilde{p}\tilde{p}]^{20} + \varepsilon_6 [s^+ s^+]^{02} \cdot [\tilde{s}\tilde{s}]^{02} + \varepsilon_7 [p^+ s^+]^{11} \cdot [\tilde{s}\tilde{p}]^{11}$$
$$+ \varepsilon_8 ([p^+ p^+]^{00} [\tilde{s}\tilde{s}]^{00} + [s^+ s^+]^{00} [\tilde{p}\tilde{p}]^{00}). \tag{1}$$

Square brackets denote spin and/or isospin coupling and $T^k \cdot T^k = (-1)^k (2k+1)^{1/2} [T^k T^k]^0$; $\tilde{b}_\mu = (-1)^{1-\mu} b_{-\mu}$.

Hamiltonian (1) is invariant under the total angular momentum and isospin rotations independently and it conserves the total number of bosons $N = n_p + n_s$. It can be rewritten in the terms of 36 operators

$$[p^+ \tilde{p}]_\mu^{J=0,1,2;T=0}, \; [s^+ \tilde{s}]_\nu^{J=0;T=0,1,2}, \; [p^+ \tilde{s}]_{\mu\nu}^{11}, \; [s^+ \tilde{p}]_{\mu\nu}^{11}, \tag{2}$$

which are the generators of the unitary group U(6). There are two possible complete chains of subgroups and subalgebras of U(6) which contain the direct product of the rotational total angular momentum and isospin algebras:

$$U(6) \supset U_{n_p}(3) \otimes U_{n_s}(3) \supset SO_J(3) \otimes SO_T(3) \supset SO_J(2) \otimes SO_T(2) \tag{3}$$

and

$$U(6) \supset SO(6) \supset SO_J(3) \otimes SO_T(3) \supset SO_J(2) \otimes SO_T(2). \tag{4}$$

The irreducible representations of the group chains (3) or (4) provide bases

$$|N n_p J M_J T M_T\rangle \tag{5}$$

or

$$|N \omega J M_J T M_T\rangle \tag{6}$$

in which H can be diagonalized.

It is convenient to express the Hamiltonian (1) in terms of the Casimir invariants of each group appearing in the chains (3) and (4)

$$H = H_0(a, \hat{N}, \hat{N}^2) + k_1 \hat{n}_p + k_2 8C_{SO(6)} + k_3 9C_{SU_J(3)} + k_4 9C_{SU_T(3)} + k_5 \hat{J}^2 + k_6 \hat{T}^2, \tag{7}$$

where:

$$k_i = f_i(\varepsilon_1 \ldots \varepsilon_8),$$
$$8C_{SO(6)} = 3\hat{N} + \hat{J}^2 + \hat{T}^2 + 2\hat{n}_p \hat{n}_s + 3([p^+ p^+]^{00} \cdot [\tilde{s}\tilde{s}]^{00} + \text{h.c.}),$$
$$9C_{SU_J}(3) = \hat{n}_p(\hat{n}_p + 3), \quad 9C_{SU_T(3)} = \hat{n}_s(\hat{n} + 3).$$

3. *Results and discussion*

Hamiltonian (7) was diagonalized in basis (5) for the boson numbers equal to 1/2 of the nucleons over the core ${}^{132}_{50}Sn$ (i.e. $N = \frac{1}{2}(A-132)$) and for the isospin numbers $T = T_z$ of the valence nucleons. The admixtures with $T > T_z$ in the ground and low-lying states are smaller than 1% [§2 – 1f in 14].

Fig. 1 presents low-lying energy levels for the $N = 92$, $Z = 56-66$ nuclei obtained with one set of six parameters whereas Fig. 2 shows the same but for the even-even isotopes of Dy. It can be seen that the model reproduces well the values of the levels with $J^\pi = 0^+, 2^+$ and only some with higher J. It is clear because only 0^+ and 1^- bosons were accounted. In both sets of the parameters we have small k_2. It means that $SU_J(3) \otimes SU_T(3)$ dynamical symmetry of the system is slightly distorted by the last term in the Hamiltonian (1) or in

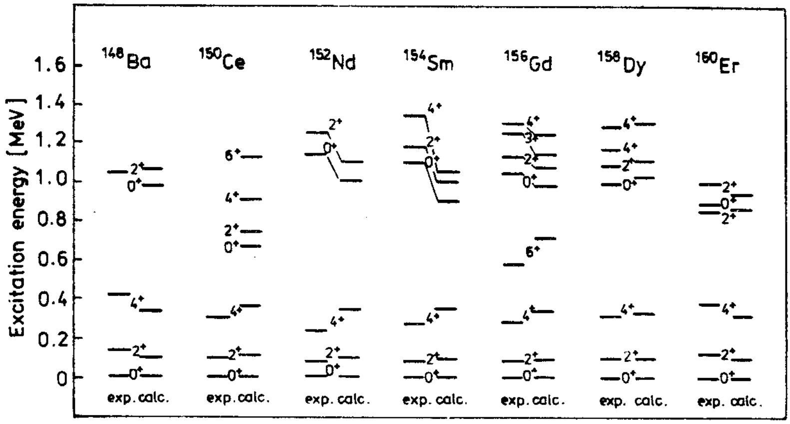

Fig. 1. The experimental [15, 16] and calculated spectra of $N = 92$ isotones with parameters (in MeV): $H_0 = 0$, $k_1 = 2.9954$, $k_2 = 0.0080$, $k_3 = 0.0261$, $k_4 = 0.2848$, $k_5 = 0.0060$, $k_6 = 0.1422$

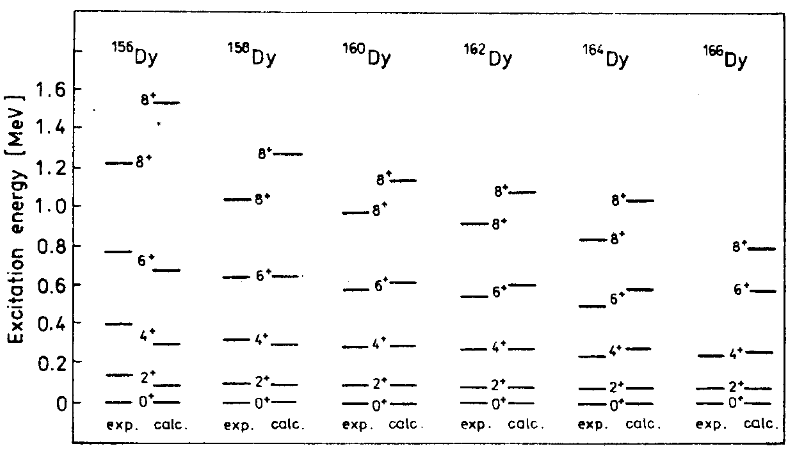

Fig. 2. The experimental [15, 17] and calculated spectra of ${}^{A}Dy$ with: $H_0 = 0$, $k_1 = 1.3485$, $k_2 = 0.0079$, $k_3 = 0.0115$, $k_4 = 0.0954$, $k_5 = 0.0005$, $k_6 = 0.4506$

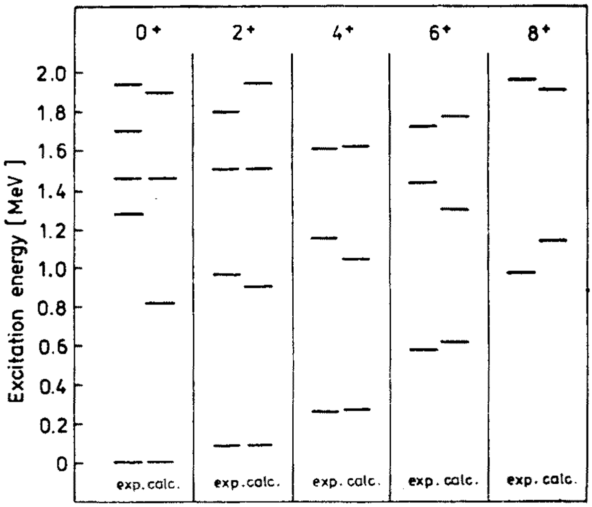

Fig. 3. The same as in Fig. 3 but for ^{160}Dy only

$8C_{SO(6)}$, its strength $\varepsilon_8 = 24$ keV. The interaction between p and s bosons is weak too: $\varepsilon_7 = 16$ keV. This configuration simulates ^{4}H-like cluster. The k_2 and k_5 parameters suggest on the average greater deformations of the different states of Dy isotopes $\left(\frac{\hbar^2}{2\mathscr{I}} = 8.4\text{ keV}\right)$ compared with $N = 92$ isotones $\left(\frac{\hbar^2}{2\mathscr{I}} = 14\text{ keV}\right)$.

Fig. 3 shows the very well reproduced "s-band" [17] whose crossing with the ground--state band is a common explanation of the "backbending" phenomenon.

The eigenstates of H

$$|Nn_pJM_JTM_T, E\rangle = \sum_{n_p=J\text{ step }2}^{N\text{ or }N-1} a_{n_p}(JT, E)\,|n_pJM_J\rangle\,|N-n_pTM_T\rangle \tag{8}$$

make it possible to find the reduced probabilities of E2 transitions, defined as usual [14]

$$B(\text{E2}; J_1 \to J_2) = (2J_1+1)^{-1}|\langle J_2||\,\hat{B}(\text{E2})|\,|J_1\rangle|^2 \tag{9}$$

with

$$\hat{B}_\mu(\text{E2}) = C[p^+\tilde{p}]^2_\mu.$$

Table I shows E2 transitions $2_1^+ \to 0_{gs}^+$ for $C = 0.08\,e^2b^2$. Reduced probabilities of E2 transitions within the ground-state band or "s-band" in ^{160}Dy are about (0.81--1.5) e^2b^2 (experimentally [18]: (1.01—1.84) e^2b^2) and they are more than 10 times larger in comparison with transitions between different bands.

Then we calculated the average number of p and s bosons in any state

$$\bar{n} = \sum_{n_i} |a_{n_i}(JT, E)|^2 n_i. \tag{10}$$

TABLE I

Reduced propabilities of E2 transitions $2_1^+ \to 0_{gs}^+$ in Dy isotopes and $N = 92$ isotones. Experimental values taken from [18]

	$B(E2: 2_1^+ \to 0_{gs}^+)$			$B(E2: 2_1^+ \to 0_{gs}^+)$	
	theory $[e^2b^2]$	experiment $[e^2b^2]$		theory $[e^2b^2]$	experiment $[e^2b^2]$
^{154}Dy	0.34	0.29 ± 0.03	^{148}Ba	0.29	
^{156}Dy	0.49	0.76 ± 0.02	^{150}Ce	0.34	
^{158}Dy	0.64	0.934 ± 0.008	^{152}Nd	0.52	
^{160}Dy	0.81	1.01	^{154}Sm	0.62	
^{162}Dy	0.99	1.03	^{156}Gd	0.86	0.92 ± 0.03
^{164}Dy	1.19	1.08	^{158}Dy	0.97	0.934 ± 0.008
^{166}Dy	1.4		^{160}Er	1.24	1.18 ± 0.02

It is possible to estimate the average number of α-like clusters n_α

$$n_\alpha = 1/2(N - \bar{\omega}), \tag{11}$$

where $\bar{\omega}$ means the average number of bosons not coupled in $J = 0$, $T = 0$ pairs and it can be extracted from a given eigenenergy of (7) using eigenvalue of $8C_{SO(6)}$.

Fig. 4 and Fig. 5 suggest a relatively significant number of α-clusters (or % of the valence nucleons coupled in α-clusters) in any ground state. The same situation occurs in the light nuclei for example in $^{16}_{8}$O and $^{30}_{15}$P [5]. Often n_α decreases when J and/or energy excitation increase, but in any nucleus there are some states 0^+ and 2^+, ... in which n_α is relatively large. For example, α-clustering of 2_1^+ state is, in all examined nuclei, only (4–5)% smaller than in the corresponding ground state. Different shapes of graphs for $N = 92$

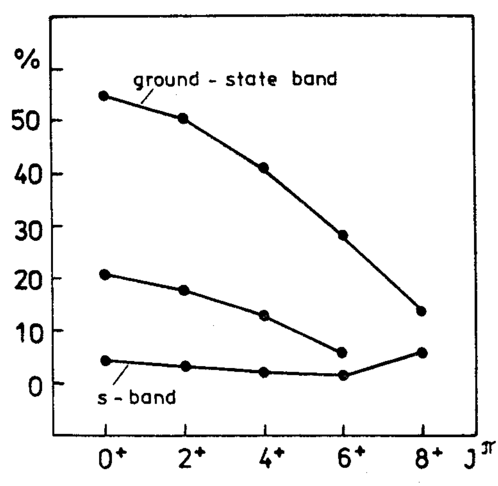

Fig. 4. Percentage of α-clusters in ^{160}Dy

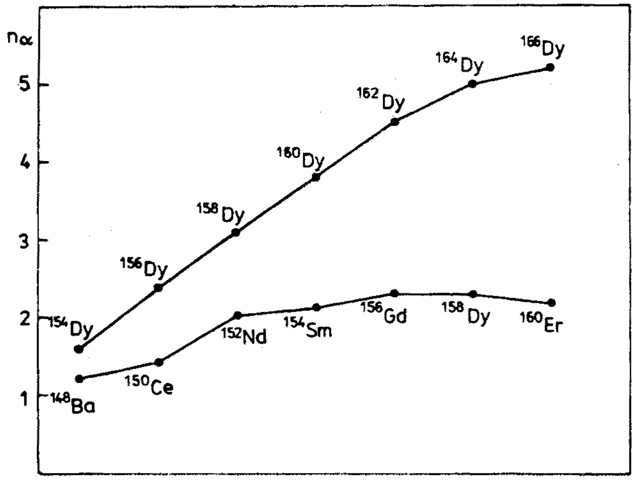

Fig. 5. α-clustering for the ground states in Figs. 1, 2

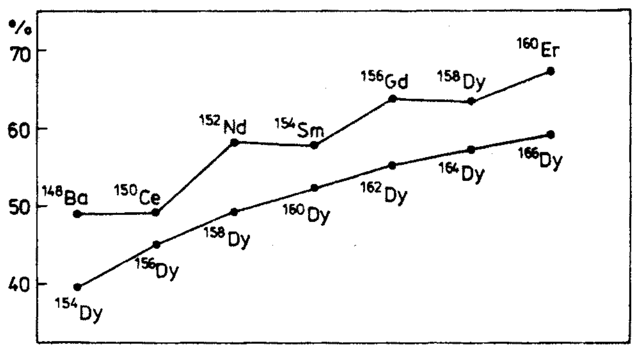

Fig. 6. Percentage of *p*-bosons in each ground state presented in Figs. 1, 2

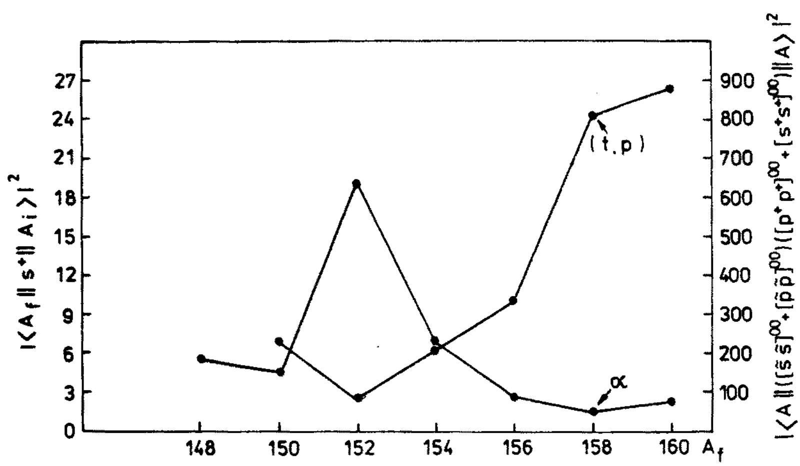

Fig. 7. Intensities of the (t, p) reaction (g.s. → g.s.) and α-elastic scattering on the ground states of $N = 92$ isotones

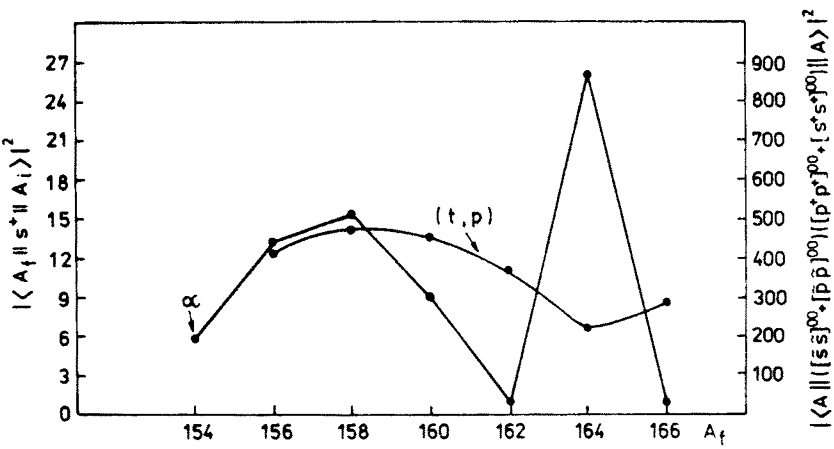

Fig. 8. The same as in Fig. 7 but for ^{A}Dy nuclei

isotones in comparison with those for Dy nuclei result from different single nucleon levels suitable to form J^π, $T = 1^-, 0$ bosons. In spite of larger $\bar{n}_p$ in $N = 92$ nuclei than in Dy isotopes (Fig. 6) we obtain more α-clusters in Dy systematically (Fig. 5). It means that α-cluster

$$\alpha = a[s^+ s^+]^{00} + b[p^+ p^+]^{00} \tag{12}$$

can have $a > b$. In the estimated strength of total cross-section of α-elastic scattering take $a = b = 1$, Figs. 7, 8 also present the intensities $|\langle A_f| |s^+| |A_i\rangle|^2$ of (t, p) reaction from the ground- to the ground-states. A maximum in α-elastic scattering (or minimum (t, p) reaction) for ^{164}Dy and ^{152}Nd can be observed. Both nuclei have $T = 0$ for valence nucleons. Enhancement of α-elastic cross-section towards large angles (ALAS) on these nuclei and their odd neighbours can be expected.

4. *Summary and conclusions*

The basic assumption of the paper: nucleon pairs are "building blocks" of a nucleus for small excitation energies means that a nucleon pair with quantum numbers J^π, T corresponds to a boson with the same quantum numbers. We choose the most interacting pairs with J^π, $T = 0^+, 1$ and $1^-, 0$. The model based on these bosons reproduces well (with fixed six parameters) values of energy of low-lying levels and their E2 transitions for 7 isotopes of Dy and 7 of $N = 92$ isotones. From p-boson structure of states it follows that effective interactions in neutron-proton pairs are not smaller than in proton-proton or neutron-neutron pairs.

In addition, we are able to estimate α-clustering, and intensities of α-elastic scattering and of (t, p) reactions. We believe that this simple model will be supplementary for shell model calculations, especially for nuclei with many valence nucleons.

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