BREMSSTRAHLUNG FROM MAGNETIC MOMENTS OF NEUTRONS*

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To the memory of Jerzy Pniewski

The differential cross-section is calculated for the bremsstrahlung of photons from magnetic moments of protons or *neutrons* decelerated in an external static nuclear (vector) potential. The lowest-order perturbative approximation is used (both with respect to the magnetic moment and external potential). The recent hypothesis of a new magnetic-type interaction of nucleons is also considered in the context of possible bremsstrahlung of a new kind from the corresponding nucleon magnetic-type moments.

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Consider a nucleon N = p, n decelerated in an external static nuclear (vector) potential $V_N(\vec{x})$, where

$$V_N(\vec{x}) = \int \frac{d^3\vec{q}}{(2\pi)^3} V_N(\vec{q}) e^{i\vec{q}\cdot\vec{x}} \tag{1}$$

(for instance, $V_N(\vec{x}) = \mp(g_N^2/4\pi r)\exp(-Mr)$ if $V_N(\vec{q}) = \pm g_N^2/(\vec{q}^2+M^2)$). Then, beside the electric charge e of the proton also the magnetic moment $\mu_N(e/2m_N)$ of the proton or *neutron* will produce the bremsstrahlung of photons according to the *effective* field equations

$$(i\gamma^\mu\partial_\mu - m_N)\Psi_N(x) = [\gamma^0 V_N(\vec{x}) + e_N\gamma^\mu A_\mu(x) + \tfrac{1}{2}\mu_N(e/2m_N)\sigma^{\mu\nu}F_{\mu\nu}(x)]\Psi_N(x) \tag{2}$$

and

$$\Box A^\mu(x) = -e_N\overline{\Psi}_N(x)\gamma^\mu\Psi_N(x) - \mu_N(e/2m_N)\partial_\nu[\overline{\Psi}_N(x)\sigma^{\mu\nu}\Psi_N(x)], \tag{3}$$

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where $\sigma^{\mu\nu} = (i/2)\,[\gamma^\mu, \gamma^\nu]$ with $(\gamma^\mu) = (\beta, \beta\vec{\alpha})$. Here, $e_N = e$ or 0 and $\mu_N = \mu_p$ or μ_n for the proton or neutron, respectively, where $\alpha = e^2/4\pi \simeq 1/137$. The external potential $V_N(\vec{x})$ is here intended to describe nuclear interactions of the nucleon effectively.

In the case of proton, the usual Dirac equation i.e., Eq. (2) without the Pauli term proportional to μ_N, includes implicitly (as a relativistic effect) the coupling of $F_{\mu\nu}(x)$ to the normal magnetic moment $e/2m_p$ (plus eventually the tiny anomalous magnetic moment implied by purely electromagnetic radiative corrections to the point-like proton). Thus, the magnetic moment $\mu_N(e/2m_N)$ in Eq. (3) is to be identified with the large phenomenological *anomalous* magnetic moment which is of a non-purely electromagnetic structural origin. Therefore, $\mu_p \simeq 2.8–1 = 1.8$ and $\mu_n \simeq -1.9$.

The differential cross-section for the bremsstrahlung of a photon $k = (\omega, \vec{k})$ from the magnetic moment of a nucleon $p_i = (E_i, \vec{p}_i) \to p_f = (E_f, \vec{p}_f)$ is given in the *lowest order* (with respect both to $\mu_N(e/2m_N)$ and the external potential) by the formula

$$\frac{d\sigma}{d\omega d\Omega_k d\Omega_{p_f}} = \frac{\mu_N^2\alpha}{(2\pi)^4}\,\frac{\omega|\vec{p}_f|}{4|\vec{p}_i|}\,|M_{fi}|^2, \tag{4}$$

where

$$M_{fi} = iV_N(\vec{q})\bar{u}_f(p_f)\left[i\not{e}(k)\not{k}\,\frac{\not{p}_f+m_N}{2p_f\cdot k}\,\gamma^0-\gamma^0\,\frac{\not{p}_i+m_N}{2p_i\cdot k}\,i\not{e}(k)\not{k}\right]u_i(p_i), \tag{5}$$

with $\omega = E_i-E_f$ and $\vec{k} = \vec{p}_i+\vec{q}-\vec{p}_f$ (note that $\vec{q}^2 = -q^2$, where $k = p_i+q-p_f$). Here, $k^2 = 0$, $k\cdot e = 0$, $e^2 = -1$ and $(\not{p}-m_N)u = 0$, $p^2 = m_N^2$, $\bar{u}u = 1$. Hence, $\not{k}^2 = 0$, $\not{k}\not{e} = -\not{e}\not{k}$, $\not{e}^2 = -1$ and $\not{p}^2 = m_N^2$. Of course, $a = \gamma^\mu\not{e}_\mu$. For the unpolarized photon and nucleon one gets

$$\frac{d\bar{\sigma}}{d\omega d\Omega_k d\Omega_{p_f}} = \tfrac{1}{2}\sum_{e}\sum_{u_f u_i}\frac{d\sigma}{d\omega d\Omega_k d\Omega_{p_f}}, \tag{6}$$

where $\sum_e$ denotes the summation over two physical polarizations of the photon, etc.

A standard but lengthy calculation gives

$$M_{fi} = V_N(\vec{q})\bar{u}_f(p_f)\left(\frac{p_f\cdot e-m_N\not{e}}{p_f\cdot k}\not{k}\gamma^0+\gamma^0\not{k}\,\frac{p_i\cdot e-m_N\not{e}}{p_i\cdot k}\right)u_i(p_i) \tag{7}$$

and

$$\tfrac{1}{2}\sum_{e}\sum_{u_f u_i}|M_{fi}|^2 = \frac{V_N^2(\vec{q})}{(E_f-\vec{p}_f\cdot\hat{k})(E_i-\vec{p}_i\cdot\hat{k})}\Big\{\vec{p}_f^2-(\vec{p}_f\cdot\hat{k})^2+\vec{p}_i^2-(\vec{p}_i\cdot\hat{k})^2-2[\vec{p}_f\cdot\vec{p}_i$$

$$-(\vec{p}_f\cdot\hat{k})(\vec{p}_f\cdot\hat{k})]+\frac{2}{m_N^2}[\vec{p}_f^2-(\vec{p}_f\cdot\hat{k})^2][\vec{p}_i^2-(\vec{p}_i\cdot\hat{k})^2]-\frac{2}{m_N^2}[\vec{p}_f\cdot\vec{p}_i-(\vec{p}_f\cdot\hat{k})(\vec{p}_i\cdot\hat{k})]^2\Big\}, \tag{8}$$

where $\hat{k} = \vec{k}/|\vec{k}|$. In the gauge invariant Eq. (5) the Coulomb gauge $e = (0, \vec{e})$ (where $\gamma^0 \not{e} = -\not{e}\gamma^0$) was used to obtain Eq. (7) and then Eq. (8). Thus, Eqs. (6), (4) and (8) lead to the following lowest-order differential cross-section for the bremsstrahlung from the anomalous magnetic moment of a nucleon:

$$\frac{d\bar{\sigma}}{d\omega d\Omega_k d\Omega_{p_f}} = \frac{\mu_N^2 \alpha}{(2\pi)^4} \frac{\omega E_f v_f}{4E_i v_i} \frac{V_N^2(\vec{q})}{(1-\vec{v}_f \cdot \hat{k})(1-\vec{v}_i \cdot \hat{k})}$$

$$\times \left\{ \frac{E_f}{E_i} [v_f^2 - (\vec{v}_f \cdot \hat{k})^2] + \frac{E_i}{E_f} [v_i^2 - (\vec{v}_i \cdot \hat{k})^2] - 2[\vec{v}_f \cdot \vec{v}_i - (\vec{v}_f \cdot \hat{k})(\vec{v}_i \cdot \hat{k})] \right.$$

$$\left. + \frac{2E_f E_i}{m_N^2} [v_f^2 - (\vec{v}_f \cdot \hat{k})^2][v_i^2 - (\vec{v}_i \cdot \hat{k})^2] - \frac{2E_f E_i}{m_N^2} [\vec{v}_f \cdot \vec{v}_i - (\vec{v}_f \cdot \hat{k})(\vec{v}_i \cdot \hat{k})]^2 \right\}, \tag{9}$$

where $\vec{v} = \vec{p}/E$ and $v = |\vec{v}|$. Introducing the angles θ_i, θ_f between $\vec{k}$ and $\vec{p}_i, \vec{p}_f$, respectively, and the angle ϕ between the planes of $\vec{k}, \vec{p}_i$ and $\vec{k}, \vec{p}_f$, one can write

$$\vec{v}_i \cdot \hat{k} = v_i \cos\theta_i, \quad \vec{v}_f \cdot \hat{k} = v_f \cos\theta_f,$$
$$\vec{v}_f \cdot \vec{v}_i = v_f v_i (\sin\theta_f \sin\theta_i \cos\phi + \cos\theta_f \cos\theta_i) \tag{10}$$

and

$$d\Omega_k = \sin\theta_i d\theta_i d\phi_k, \quad d\Omega_{p_f} = \sin\theta_f d\theta_f d\phi, \tag{11}$$

where ϕ_k is the azimuthal angle of $\vec{k}$ around the axis of $\vec{p}_i$. Then, integrating Eq. (9) (trivially) over ϕ_k one gets the formula

$$\frac{d\bar{\sigma}}{d\omega d\cos\theta_f d\cos\theta_i d\phi} = \frac{\mu_N^2 \alpha}{2\pi} \frac{\omega E_f v_f}{4E_i v_i} \frac{[V_N(\vec{q})/4\pi]^2}{(1-v_f\cos\theta_f)(1-v_i\cos\theta_i)}$$

$$\times \left[\frac{E_f}{E_i} (v_f \sin\theta_f)^2 + \frac{E_i}{E_f} (v_i \sin\theta_i)^2 - 2v_f v_i \sin\theta_f \sin\theta_i \cos\phi \right.$$

$$\left. + \frac{2E_f E_i}{m_N^2} (v_f v_i \sin\theta_f \sin\theta_i \sin\phi)^2 \right]. \tag{12}$$

This is in our case the counterpart of the well-known Bethe-Heitler formula [1] describing in the lowest order (with respect both to e and the external potential) the bremsstrahlung from the electric charge of an electron decelerated in an external Coulomb potential. For a proton decelerated in an external static nuclear (vector) potential the corresponding Bethe-Heitler formula takes the form:

$$\frac{d\bar{\sigma}_{BH}}{d\omega d\cos\theta_f d\cos\theta_i d\phi} = \frac{\alpha}{2\pi} \frac{E_f v_f}{\omega E_i v_i} [V_p(\vec{q})/4\pi]^2$$

$$\times \left[(4E_i^2 - \vec{q}^2) \left(\frac{v_f \sin\theta_f}{1-v_f\cos\theta_f} \right)^2 + (4E_f^2 - \vec{q}^2) \left(\frac{v_i \sin\theta_i}{1-v_i\cos\theta_i} \right)^2 \right.$$

$$-(4E_fE_i-\vec{q}^2+2\omega^2)\frac{2v_fv_i\sin\theta_f\sin\theta_i\cos\phi}{(1-v_f\cos\theta_f)(1-v_i\cos\theta_i)}$$

$$\left.+2\omega^2\frac{(E_f/E_i)(v_f\sin\theta_f)^2+(E_i/E_f)(v_i\sin\theta_i)}{(1-v_f\cos\theta_f)(1-v_i\cos\theta_i)}\right]. \tag{13}$$

Note that

$$\vec{q}^2 = (\vec{p}_f+\vec{k}-\vec{p}_i)^2 = \omega^2+2\omega(E_fv_f\cos\theta_f-E_iv_i\cos\theta_i)$$

$$+E_f^2v_f^2+E_i^2v_i^2-2E_fE_iv_fv_i(\sin\theta_f\sin\theta_i\cos\phi+\cos\theta_f\cos\theta_i). \tag{14}$$

Of course, in the general lowest-order proton bremsstrahlung formula there appears also an interference term of the electric charge and anomalous magnetic moment.

In the nonrelativistic approximation, where $v_i \ll 1$ (and then also $v_f \ll 1$), Eq. (12) assumes the form

$$\frac{d\bar{\sigma}}{d\omega d\cos\theta_f d\cos\theta_i d\phi} \underset{\text{NR}}{=} \frac{\mu_N^2\alpha}{2\pi}\frac{\omega v_f}{4v_i}[V_N(\vec{q})/4\pi)^2$$

$$\times[(v_f\sin\theta_f)^2+(v_i\sin\theta_i)^2-2v_fv_i\sin\theta_f\sin\theta_i\cos\phi], \tag{15}$$

where $\vec{v} \underset{\text{NR}}{=} \vec{p}/m_N$ and $\vec{q} \underset{\text{NR}}{=} \vec{p}_f-\vec{p}_i$, the latter relation following from the inequality $|\vec{k}| = \omega \underset{\text{NR}}{=} (\vec{p}_i^2-\vec{p}_f^2)/2m_N \ll |\vec{p}_i|-|\vec{p}_f|$. It is interesting to observe that in the nonrelativistic approximation the proton Bethe-Heitler formula (13) reduces to

$$\frac{d\bar{\sigma}_{BH}}{d\omega d\cos\theta_f d\cos\theta_i d\phi} \underset{\text{NR}}{=} \frac{\alpha}{2\pi}\frac{4m_p^2v_f}{\omega v_i}[V_p(\vec{q})/4\pi]^2$$

$$\times[(v_f\sin\theta_f)^2+(v_i\sin\theta_i)^2-2v_fv_i\sin\theta_f\sin\theta_i\cos\phi]. \tag{16}$$

Note that in the nonrelativistic approximation Eq. (14) gives

$$\vec{q}^2 \underset{\text{NR}}{=} (\vec{p}_f-\vec{p}_i)^2$$

$$= m_N^2[v_f^2+v_i^2-2v_fv_i(\sin\theta_f\sin\theta_i\cos\phi+\cos\theta_f\cos\theta_i)]. \tag{17}$$

The angular dependence in Eqs. (15) and (16) is the same.

Thus, the ratio of the nonrelativistic cross-sections for the bremsstrahlung from the magnetic moment of a neutron and from the electric charge of a proton is of the order of

$$\mu_n^2\frac{\omega^2}{16m_p^2}\frac{V_n^2(\vec{q})}{V_p^2(\vec{q})} = \mu_n^2\frac{\omega^2}{16m_p^2}O(1), \tag{18}$$

where $\omega^2 \underset{\text{NR}}{=} m_p^2(v_i^2-v_f^2)^2/4$. Of course, in Eq. (18) the same kinematic region is considered for both processes and m_n is put equal to m_p.

It was speculated recently [2, 3] that, in the case of a hypothesis of quarks composed from some more elementary constituents bound by means of a new *Abelian* gauge field, the nucleons should display new magnetic-type moments $\mu_N^{(u)}(e^{(u)}/2m_N)$ coupled to a magnetic-type part of this hypothetical gauge field. Here, the superscript (u) refers to the prefix "ultra", convenient to distinguish $\mu_N^{(u)}$ (the "ultramagnetic moment" in units of $e^{(u)}/2m_N$) and $e^{(u)}$ (the "ultraelectric charge" or "ultracharge") from μ_N and e, respectively, as well as the new Abelian gauge field (the "ultraelectromagnetic field") from the familiar electromagnetic field. In this case a decelerated nucleon, though expected to be neutral with respect to the ultracharge: $e_N^{(u)} = 0$, should emit the "ultrabremsstrahlung" of new gauge vector bosons ("ultraphotons") from its ultramagnetic moment. If both for $\mu_N^{(u)}(e^{(u)}/2m_N)$ and the potential $V_N(\vec{x})$ the lowest-order perturbative approximation could be used to estimate such a process, the corresponding differential cross-section would be given by Eq. (12) (or, in the non-relativistic approximation, by Eq. (15)), where now μ_N and α should be replaced by $\mu_N^{(u)}$ and $\alpha^{(u)} = e^{(u)2}/4\pi$, respectively.

Thus, the ratio of the nonrelativistic cross-sections for the ultrabremsstrahlung from the ultramagnetic moment of a proton and the usual bremsstrahlung from the proton electric charge would be of the order of

$$\mu_p^{(u)2} \frac{\alpha^{(u)}}{\alpha} \frac{\omega^2}{16m_p^2}, \tag{19}$$

where ω^2 is given as in Eq. (18). For instance, if $\mu_p^{(u)} = O(1)$ and $\alpha^{(u)} = O(1)$, the ratio (19) is $8(\omega^2/m_p^2)O(1) = 2(v_i^2 - v_f^2)^2 O(1)$.

Note finally that the ultrabremsstrahlung from ultramagnetic moments of relativistic protons would be a component of the beamstrahlung in proton accelerators.

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APPENDIX

From the phenomenological point of view it may be interesting to consider in the place of the external vector potential $V_N(\vec{x})$ the more general external interaction energy $V_N(\vec{x}) + \gamma^0 S_N(\vec{x})$, where $S_N(\vec{x})$ is an external scalar potential. Both $V_N(\vec{x})$ and $S_N(\vec{x})$ may be complex. Then, the formula (12) transits into the more complicated form

$$\frac{d\bar{\sigma}}{d\omega d\cos\theta_f d\cos\theta_i d\phi} = \frac{\mu_N^2\alpha}{2\pi}\frac{\omega E_f v_f}{4E_i v_i}$$

$$\times \frac{1}{(4\pi)^2}\{|V_N(\vec{q})|^2 A + [V_N(\vec{q})S_N^*(\vec{q}) + V_N^*(\vec{q})S(\vec{q})]B + |S_N(\vec{q})|^2 C\}, \tag{A1}$$

where

$$A = \frac{1}{(1-v_f\cos\theta_f)(1-v_i\cos\theta_i)}\left[\frac{E_f}{E_i}(v_f\cos\theta_f)^2 + \frac{E_i}{E_f}(v_i\cos\theta_i)^2\right.$$

$$\left.-2v_f v_i\sin\theta_f\sin\theta_i\cos\phi + \frac{2E_f E_i}{m_N^2}(v_f v_i\sin\theta_f\sin\theta_i\sin\phi)^2\right] \tag{A2}$$

and

$$B = \frac{E_f}{m_N}\frac{v_f^2(1+\cos^2\theta_f)}{1-v_f\cos\theta_f} + \frac{E_i}{m_N}\frac{v_i^2(1+\cos^2\theta_i)}{1-v_i\cos\theta_i}$$

$$-\frac{1}{m_N}\left(\frac{E_i}{1-v_f\cos\theta_f}+\frac{E_f}{1-v_i\cos\theta_i}\right)v_f v_i(\sin\theta_f\sin\theta_i\cos\phi+2\cos\theta_f\cos\theta_i)$$

$$-2\left(\frac{1}{1-v_f\cos\theta_f}-\frac{1}{1-v_i\cos\theta_i}\right)\frac{1}{m_N}(E_f v_f\cos\theta_f - E_i v_i\cos\theta_i) \tag{A3}$$

and finally

$$C = \frac{2}{(1-v_f\cos\theta_f)(1-v_i\cos\theta_i)}\left[\frac{E_f}{E_i}(v_f\cos\theta_f)^2+\frac{E_i}{E_f}(v_i\cos\theta_i)^2\right.$$

$$\left.-2v_f v_i\sin\theta_f\sin\theta_i\cos\phi-\frac{\vec{q}^2}{E_f E_i}\right]$$

$$-\frac{E_f E_i}{m_N^2}(1-v_f\cos\theta_f)(1-v_i\cos\theta_i)\left[\left(\frac{v_f\sin\theta_f}{1-v_f\cos\theta_f}\right)^2+\left(\frac{v_i\sin\theta_i}{1-v_i\cos\theta_i}\right)^2\right.$$

$$\left.-2\frac{v_f v_i\sin\theta_f\sin\theta_i\cos\phi}{(1-v_f\cos\theta_f)(1-v_i\cos\theta_i)}\right]$$

$$+2\frac{\vec{q}^2}{m_N^2}-4\frac{(E_f-E_i)^2}{m_N^2}-4\frac{E_f-E_i}{m_N}\frac{1}{m_N}(E_f v_f\cos\theta_f-E_i v_i\cos\theta_i). \tag{A4}$$

Note that in the formula (A1) there are no infrared divergences in contrast to the corresponding Bethe-Heitler formula.

In the nonrelativistic approximation, Eqs. (A2)–(A4) give

$$A = B = C \underset{\mathrm{NR}}{=} (v_f\sin\theta_f)^2+(v_i\sin\theta_i)^2-2v_f v_i\sin\theta_f\sin\theta_i\cos\phi. \tag{A5}$$

Then, Eq. (A1) reduces to the form

$$\frac{d\bar{\sigma}}{d\omega\, d\cos\theta_f\, d\cos\theta_i\, d\phi} \underset{\mathrm{NR}}{=} \frac{\mu_N^2\alpha}{2\pi}\frac{\omega v_f}{4v_i}\frac{1}{(4\pi)^2}|V_N(\vec{q})+S_N(\vec{q})|^2 A. \tag{A6}$$

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