DISPERSION RELATION FOR HOT GLUONIC MATTER FROM THE BBGKY HIERARCHY

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We formulate the Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy of kinetic equations for quark-less QCD plasma. Assuming Bose-Einstein distribution for the equilibrium distribution of hot gluons and ghosts, we solve those kinetic equations in the mean-field limit and obtain the dispersion relation in the order g^2. Contrary to this above assumption, the state of noninteracting gluons is unstable, i.e. the damping constant of the colour oscillations is negative. We argue that the non-perturbative effects at the scale $\sim gT$ make the perturbative approximation to the equilibrium distribution of hot gluons inconsistent with the kinetic equations already at the lowest, non-trivial order g^2.

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The thermal properties of gluons have recently been the subject of much controversy. Applying the techniques from the Abelian theories to the non-Abelian case of quantum chromodynamics (QCD), a serious disagreement presently exists concerning the damping of colour oscillations in a quark-gluon plasma. Kajantie et al. have studied the one-loop gluon propagator in the Coulomb gauge and in the temporal axial gauge to derive a positive damping constant [1] γ. In the covariant Lorentz gauge, on the contrary, the same technique yields negative [2] γ. The plasmon decay rate has also been calculated using the background covariant gauges [2–3]. Again, γ is negative and explicitly gauge parameter dependent. This shows that there is at present a severe problem in our understanding of the nature of the plasmon within finite-temperature QCD.

In this letter we do not address the problem of the gauge-dependence [4] of γ or the problem of a correct physical definition [3] of γ. Instead, our aim here is to formulate the relativistic kinetic theory on a quantum field theoretical basis by developing the Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy and truncating it at the mean-field level. This field is largely unexplored in spite of large interest in developing the kinetic equations of quarks and gluons. (For a recent review of the subject see Ref. [6] and refer-

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ences quoted therein.) In addition, we solve those kinetic equations around a postulated equilibrium and derive the dispersion relations in the limit $\omega \gg |\boldsymbol{k}|$.

Let us consider a QCD Lagrangian in a covariant gauge for a system of non-Abelian, self-interacting gluonic fields A_a^μ ($\mu = 0, \ldots, 3$, $a = 1, \ldots, N^2-1$) coupled to the ghost fields c_a, $c_b^\dagger$. The QCD equations for the fields A and c can be replaced by an infinite hierarchy of coupled kinetic equations (the BBGKY hierarchy) for the statistical averages of products of the field operators. Obviously, in order to obtain a meaningful theory one has to propose a truncation scheme. Below, we show the lowest order BBGKY equations for $\langle A \rangle$ in the Feynman gauge ($\alpha = 1$):

$$
\begin{aligned}
-k^2 g_{\mu\nu}\langle A_a^\mu(k)\rangle = {} & ig f_{abc}\int \frac{d^4 l}{(2\pi)^4}\, l^\mu \langle A_{\mu b}(l) A_{\nu c}(k-l)\rangle \\
& +2ig f_{abc}\int \frac{d^4 l}{(2\pi)^4}\, l^\mu \langle A_{\mu b}(k-l) A_{\nu c}(l)\rangle \\
& -ig f_{abc}\int \frac{d^4 l}{(2\pi)^4}\, l_\nu \langle A_{\mu b}(k-l) A_c^\mu(l)\rangle \\
& -g^2 f_{abc} f_{cde}\int \frac{d^4 l_1}{(2\pi)^4}\int \frac{d^4 l_2}{(2\pi)^4} \langle A_{\mu b}(k-l_1) A_d^\mu(l_1-l_2) A_{\nu e}(l_2)\rangle \\
& +ig f_{abc}\int \frac{d^4 l}{(2\pi)^4}\, l_\nu \langle c_b^\dagger(l) c_c(k-l)\rangle,
\end{aligned}
\tag{1}
$$

as well as for the average of the two gluon fields:

$$
\begin{aligned}
k^2\langle A_{\nu a}(k) A_{\sigma h}(p)\rangle = {} & -ig f_{abc}\int \frac{d^4 l}{(2\pi)^4}\, l_\mu \langle A_b^\mu(l) A_{\nu c}(k-l) A_{\sigma h}(p)\rangle \\
& -2ig f_{abc}\int \frac{d^4 l}{(2\pi)^4}\, l^\mu \langle A_b^\mu(k-l) A_{\nu c}(l) A_{\sigma h}(p)\rangle \\
& +ig f_{abc}\int \frac{d^4 l}{(2\pi)^4}\, l_\nu \langle A_{\mu b}(k-l) A_c^\mu(l) A_{\sigma h}(p)\rangle \\
& +g^2 f_{abc} f_{cde}\int \frac{d^4 l_1}{(2\pi)^4}\int \frac{d^4 l_2}{(2\pi)^4} \langle A_{\mu b}(k-l_1) A_d^\mu(l_1-l_2) A_{\nu e}(l_2) A_{\sigma h}(p)\rangle \\
& +ig f_{abc}\int \frac{d^4 l}{(2\pi)^4}\, l_\nu \langle c_b^\dagger(l) c_c(k-l) A_{\sigma h}(p)\rangle,
\end{aligned}
$$

$$
p^2\langle A_{\nu a}(k) A_{\sigma h}(p)\rangle = -ig f_{hbc}\int \frac{d^4 l}{(2\pi)^4}\, l_\mu \langle A_{\nu a}(k) A_b^\mu(l) A_{\sigma c}(p-l)\rangle
$$

$$-2igf_{hbc}\int\frac{d^4l}{(2\pi)^4}\, l_\mu\langle A_{\nu a}(k)A^\mu_b(p-l)A_{\sigma c}(l)\rangle$$

$$+igf_{hbc}\int\frac{d^4l}{(2\pi)^4}\, l_\sigma\langle A_{\nu a}(k)A_{\mu b}(p-l)A^\mu_c(l)\rangle$$

$$+g^2 f_{hbc}f_{cde}\int\frac{d^4l_1}{(2\pi)^4}\int\frac{d^4l_2}{(2\pi)^4}\langle A_{\nu a}(k)A_{\mu b}(p-l_1)A^\mu_d(l_1-l_2)A_{\sigma e}(l_2)\rangle$$

$$+igf_{hbc}\int\frac{d^4l}{(2\pi)^4}\, l_\sigma\langle A_{\nu a}(k)c^\dagger_b(l)c_c(p-l)\rangle, \tag{2}$$

and the two ghost fields:

$$k^2\langle c^\dagger_b(p)c_a(k)\rangle = igf_{adc}k^\mu\int\frac{d^4l}{(2\pi)^4}\langle c^\dagger_b(p)c_d(k-l)A_{\mu c}(l)\rangle,$$

$$p^2\langle c^\dagger_b(p)c_a(k)\rangle = -igf_{dbc}\int\frac{d^4l}{(2\pi)^4}\, l^\mu\langle A_{\mu c}(p-l)c^\dagger_d(l)c_a(k)\rangle. \tag{3}$$

If we neglect the correlation functions of three and more gluon fields and, in addition, if we apply the Vlasov approximation to the statistical averages containing fields A and c: $\langle c^\dagger cA\rangle = \langle Ac^\dagger c\rangle = \langle c^\dagger c\rangle\langle A\rangle$, then we obtain three equations for the three unknown functions: $\langle A\rangle$, $\langle AA\rangle$, $\langle c^\dagger c\rangle$. One should notice that those equations do not result from a perturbative expansion in powers of g.

In equilibrium $\langle A\rangle = 0$ and $\langle A(x)A(y)\rangle \equiv \mathscr{A}(x-y)$, $\langle c^\dagger(x)c(y)\rangle = \mathscr{C}(x-y)$ depend on the relative coordinates. Then Eqs. (1) and (3) are satisfied identically and Eq. (2) reduces to the on-mass shell condition for hot gluons in the mean-field limit:

$$k^2\mathscr{A}_{\nu\sigma}(k) = -g^2N\mathscr{A}_{\nu\sigma}(k)\int\frac{d^4l}{(2\pi)^4}\mathscr{A}^\mu_\mu(k-l)+g^2N\mathscr{A}_{\mu\nu}(k)\int\frac{d^4l}{(2\pi)^4}\mathscr{A}^\mu_\sigma(k-l). \tag{4}$$

Let us now suppose that the system has reached equilibrium. In this case, if one would know the equilibrium distribution functions $\mathscr{A}$ and $\mathscr{C}$, then the above transport equations can be used to study the properties of hot gluons near equilibrium. Let δA be a perturbation of the field A which is induced by the coupling to some external physical system. Assuming that the perturbation is small, we can restrict ourselves to terms linear in δA. Using properties of $\mathscr{A}$ and $\mathscr{C}$, one obtains:

$$-k^2Q_{\nu a}(k) = igf_{abc}\int\frac{d^4l}{(2\pi)^4}\, l^\mu AQ_{\mu\nu bc}(l, k-l)$$

$$+2igf_{abc}\int\frac{d^4l}{(2\pi)^4}\, l^\mu AQ_{\mu\nu bc}(k-l, l)-igf_{abc}\int\frac{d^4l}{(2\pi)^4}\, l_\nu AQ^\mu_{\mu bc}(k-l, l)$$

$$+g^2N\int\frac{d^4l}{(2\pi)^4}\mathscr{A}^{\mu}_{\mu}(k-l)Q_{\nu a}(k)-g^2N\int\frac{d^4l}{(2\pi)^4}\mathscr{A}^{\mu}_{\nu}(k-l)Q_{\mu a}(k)$$

$$+igf_{abc}\int\frac{d^4l}{(2\pi)^4}l_\nu\langle c^\dagger_b(l)c_c(k-l)\rangle, \tag{5}$$

where $\langle\delta A^\mu_a\rangle = Q^\mu_a$ and $AQ_{\mu\nu ab}(k, p) = \langle A_{\mu a}(k)\delta A_{\nu b}(p)\rangle+\langle\delta A_{\mu a}(k)A_{\nu b}(p)\rangle$.

The functions $\langle c^\dagger c\rangle$ and AQ on the right hand side of Eq. (5), are given by:

$$k^2\langle c^\dagger_b(p)c_a(k)\rangle = igf_{abc}k_\mu\int\frac{d^4l}{(2\pi)^4}\langle c^\dagger_b(p)c_d(k-l)\rangle_{\mathrm{R}}Q^\mu_c(l),$$

$$p^2\langle c^\dagger_b(p)c_a(k)\rangle = -igf_{dbc}\int\frac{d^4l}{(2\pi)^4}l_\mu\langle c^\dagger_d(l)c_a(k)\rangle_{\mathrm{R}}Q^\mu_c(p-l), \tag{6}$$

$$k^2AQ_{\nu\sigma ah}(k, p) = -igf_{ahc}\{(p-k)_\lambda\mathscr{A}_{\nu\sigma}(p)+(2k+p)^\mu\mathscr{A}_{\mu\sigma}(p)g_{\nu\lambda}$$
$$-(2p+k)_\nu\mathscr{A}_{\sigma\lambda}(p)\}Q^\lambda_c(k+p),$$

$$g^2N\int\frac{d^4l}{(2\pi)^4}\mathscr{A}^\mu_\mu(k-l)AQ_{\nu\sigma ah}(k, p)-g^2N\int\frac{d^4l}{(2\pi)^4}\mathscr{A}^\mu_\nu(k-l)AQ_{\mu\sigma ah}(k, p)$$

$$-g^2f_{abc}f_{cdh}\mathscr{A}_{\nu\sigma}(p)\int\frac{d^4l}{(2\pi)^4}AQ^\mu_{\mu bd}(k-l, p+l)$$

$$-g^2f_{abc}f_{chd}\mathscr{A}^\mu_\sigma(p)\int\frac{d^4l}{(2\pi)^4}AQ_{\mu\nu bd}(k-l, p+l)$$

$$-g^2f_{ahc}f_{cbd}\mathscr{A}^\mu_\sigma(p)\int\frac{d^4l}{(2\pi)^4}AQ_{\mu\nu bd}(k-l, p+l),$$

$$p^2AQ_{\nu\sigma ah}(k, p) = igf_{ahc}\{(k-p)_\lambda\mathscr{A}_{\nu\sigma}(k)+(2p+k)^\mu\mathscr{A}_{\mu\nu}(k)g_{\sigma\lambda}$$
$$-(2k+p)_\sigma\mathscr{A}_{\nu\lambda}(k)\}Q^\lambda_c(k+p)$$

$$-g^2N\int\frac{d^4l}{(2\pi)^4}\mathscr{A}^\mu_\mu(p-l)AQ_{\nu\sigma ah}(k, p)+g^2N\int\frac{d^4l}{(2\pi)^4}\mathscr{A}^\mu_\sigma(p-l)AQ_{\nu\mu ah}(k, p)$$

$$+g^2f_{hbc}f_{cda}\mathscr{A}_{\nu\sigma}(k)\int\frac{d^4l}{(2\pi)^4}AQ^\mu_{\mu bd}(p-l, k+l)$$

$$+g^2f_{hbc}f_{cad}\mathscr{A}^\mu_\nu(k)\int\frac{d^4l}{(2\pi)^4}AQ_{\mu\sigma bd}(p-l, k+l)$$

$$+g^2f_{hac}f_{cbd}\mathscr{A}^\mu_\nu(k)\int\frac{d^4l}{(2\pi)^4}AQ_{\mu\nu bd}(p-l, k+l), \tag{7}$$

where index R denotes the equilibrium distribution. We note that the zero order terms in Q cancel as a result of Eq. (2). Hence, we have obtained a closed set of nonperturbative integral equations which is complete providing one knows the equilibrium distribution functions $\mathscr{A}$ and $\mathscr{C}$. In the following, we shall approach the solution of those equations perturbatively.

The dispersion relation in order g^2 can be calculated from Eq. (5) after inserting the equilibrium distribution function for noninteracting gluons and ghosts:

$$\mathscr{A}_{\mu\nu}(k) = -\pi g_{\mu\nu}\delta(k^2)\,(n_b(|k_0|)+\theta(-k_0))$$

$$\mathscr{C}(k) = \pi\delta(k^2)\,(n_b(|k_0|)+\theta(-k_0)), \tag{8}$$

into the *rhs* of Eqs. (6), (7) $n_b(k)$ in the above equation is the Bose-Einstein distribution function. The details of this calculation will be published in a separate publication [7]. Here we give only the final result:

$$k^2 Q^a_\nu(k) - \Pi_{\nu\mu}(k) Q^{\mu a}(k) = 0 \tag{9}$$

$$\Pi_{\mu\nu}(k) = ig^2 N \int \frac{d^4 l}{(2\pi)^3}\,\frac{-4l_\mu l_\nu + k_\mu k_\nu - k^2 g_{\mu\nu} - 1/2 l^2 g_{\mu\nu} + 2kl g_{\mu\nu} - 2l_\mu k_\nu - 2l_\nu k_\mu}{(l+k)^2}$$

$$\times\,\delta(l^2)\,(n_b(|l_0|)+\theta(-l_0)) \tag{10}$$

The polarization tensor $\Pi_{\mu\nu}$ in this equation, is the correction of order g^2 to the zero order dispersion relation $k^2 Q^a_\nu(k) = 0$, and satisfies the transversality condition on the mass--shell ($k^2 = 0$), even though expressions for $\Pi_{\mu\nu}$ are not Lorentz covariant. The "vacuum" part of $\Pi_{\mu\nu}$:

$$\Pi_{\mu\nu}(k) = (k^2 g_{\mu\nu} - k_\mu k_\nu)\Pi_{\mathrm{Reg}}(k), \tag{11}$$

where $\Pi_{\mathrm{Reg}}(k) = \frac{1}{3}\,(\Pi^\mu_\mu(k)_{T=0} - \Pi^\mu_\mu(0)_{T=0})$ describes the influence of the vacuum fluctuations on the propagation of the oscillation. The "matter" part of $\Pi_{\mu\nu}$ is connected with the thermal excitations and dominates the dispersion relation at high temperatures. The tensor $\Pi_{\mu\nu}$ is transverse also at finite T.

Let us now separate the polarization tensor into the longitudinal and transverse parts:

$$\Pi_{ij} = \Pi_{\mathrm{T}}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) + \Pi_{\mathrm{L}}\frac{k_i k_j}{k^2}. \tag{12}$$

This tensor has two eigenvalues, one for the longitudinal and another one for the transverse oscillations. Π_{L} and Π_{T} can be calculated explicitly by integrating $\Pi_{\mu\nu}(k)$ on l_0 and angles and keeping only the terms containing the distribution $n_b(l)$. Unfolding then the dispersion relations in the case when the real part of k_0 is much greater than its imaginary part and taking the limit $\omega \gg |\boldsymbol{k}|$ one obtains:

$$\omega^2 = \boldsymbol{k}^2 + \frac{g^2NT^2}{3}\sum_{n=1}^{\infty}\frac{1}{2n+1}\left(\frac{\boldsymbol{k}}{k_0}\right)^{2n-2}$$

for the longitudinal oscillations and

$$\omega^2 = \boldsymbol{k}^2 + \frac{g^2NT^2}{9} + \frac{g^2NT^2}{3}\sum_{n=1}^{\infty}\frac{1}{(2n+1)(2n+3)}\left(\frac{\boldsymbol{k}}{k_0}\right)^{2n}$$

for the transverse oscillations. In the limit $|\boldsymbol{k}| \to 0$ one obtains results of the one-loop calculation [2]:

$$\omega_{\mathrm{L}} = \omega_{\mathrm{T}} = \omega_{\mathrm{p}} = \frac{gT}{3}N^{\frac{1}{2}}; \quad \gamma_{\mathrm{L}} = \gamma_{\mathrm{T}} = -\frac{5g^2NT}{24\pi}.$$

Hence, contrary to our earlier assumption, the state of noninteracting hot gluons (8) is not an equilibrium one, i.e. if one slightly perturbes this state, then the field configuration will begin to oscillate with the growing amplitude, destroying the coloured equilibrium state.

This instability can be caused by a wrong choice of the equilibrium state, i e., the QCD plasma approaches the equilibrium in a state which differs from (8). In this context one should notice the advantage of applying the above proposed method. We have explicitly chosen Bose-Einstein distribution functions for gluons when constructing the dispersion relation. However, contrary to the method of the imaginary-time finite temperature QCD [1–3] one can easily generalize the above equations also for the case of other distributions.

It has been suggested that the interacting gluons, even at high temperatures, may take exotic forms of ordered condensates of colour singlet states [8] and the basic excitations of such a system would be colourless glueballs [9]. If this conjecture is true then the description of the quark-gluon plasma solely in terms of quark and gluonic excitations is inadequate. Consequently, also the predictions concerning properties of the colour charged quasi-particle excitations and, in particular, the colour charged oscillations should be changed.

Actually, the phase transitions in various regions of $(k_0, \boldsymbol{k})$-space between different possible regimes in the condensate remain essentially unknown. Recently, De Tar and Polonyi [9] have suggested nonperturbative effects in the long wavelength limit, i.e. at the scale $\sim g^2T$, due to the existence of the composite colour-neutral objects. According to this idea, the free distribution function at this momentum scale is modified in the nontrivial way, even though, for larger momenta it constitutes the good perturbative approximation. The real part of the polarization tensor is given by an integral of the distribution function from 0 to ∞. Hence, in the leading order the corrections to the distribution function in the finite non-perturbative domain "$\sim gT$" do not change the plasmon frequency ω. On the contrary, the imaginary part of the polarization tensor which determines the damping constant is given by an integral in the domain "$\sim gT$" and is very sensitive to such non-

-pertubative corrections. In that sense both the sign and the magnitude of the plasmon decay rate remain essentially nonperturbative phenomena which cannot be calculated perturbatively at the mean-field limit. This introduces an additional difficulty in the calculation of the plasmon decay rate on the top of the known difficulty with the gauge-dependence of the response function.

Hence, we believe that no quantitative conclusions about the sign and magnitude of the plasmon decay rate can be made before the non-perturbative structure of the equilibrium distribution function is understood, i.e., before the inconsistency between the structure of the mean-field kinetic equations and the chosen equilibrium gluon and ghost distributions is resolved. Solving this problem would largely help in understanding puzzling features of the collective behaviour of the quark-gluon plasma using the finite temperature QCD.

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