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THE DECOUPLING THEOREM AND THE PAULI-VILLARS REGULARIZATION*

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It is shown how the Pauli-Villars regularization method is affected by the violation of the Appelquist-Carazzone decoupling theorem.

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It is well known [2, 3, 10] that in a certain class of gauge theories, including the standard Glashow-Salam-Weinberg (GSW) one, heavy particles can manifest themselves in experiments performed at energy scales much lower than their masses. This effect, which is obviously a violation of the Appelquist-Carazzone [1] decoupling theorem, can arise in practical one-loop calculations from finite heavy-particle loops that do not vanish in the formal limit $m_{\text{heavy}} \to \infty$. The authors of Ref. [10] observed that, in the case of fermion loops, these finite parts do not appear if one uses the Pauli-Villars (PV) regularization [5, 9]. Their conclusion was that the PV regularization is inappropriate for studying decoupling. The status of other than dimensional regularizations is quite important because of the problems in dimensional extension of the γ_5 matrix (Appendix C). Therefore, it seems worthwhile to give an explicit example of how the PV regularization works in a theory with nondecoupling.

As our example we choose the fermion induced four-point Z boson interaction in the GSW model (with possible additional heavy families). This example is especially simple because the $(Z_\mu Z^\mu)^2$ counter term in the GSW action is not generated by the usual multiplicative renormalization [6]. Thus, we will not have to refer to any particular renormalization scheme.

Let us consider a sum of diagrams shown in Fig. 1. The index i numerates all the N fermions of the model. These diagrams give a contribution to the Z boson effective interaction Lagrangian. The contribution calculated with help of the dimensional regularization

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reads:

$$\frac{N}{8}\,\frac{1}{384\pi^2}\,(g^2+g'^2)^2\,(Z_\mu Z^\mu)^2+R[Z,\,m_i,\,i=1,\,\dots,\,N],\tag{1}$$

where $R[Z, m_i] \xrightarrow[i=1\ \dots,\,N]{m_i \to \infty} > 0$. If the PV regularization is used the contribution equals to $R[Z, m_i]$ only. The PV heavy regulator particles "do not decouple", and they produce the same finite local Z boson interaction but with the opposite sign. Therefore, the local terms cancel out.

How can we generate the local part of the Z boson effective interaction in the case of the PV regularization? It cannot be obtained by redefinitions of fields and constants in the original Lagrangian, as it has been already mentioned. It is neither possible to get it from any other diagrams. One can check it by thinking of what diagrams can give a contribution to the four Z boson interaction in the fourth order in the coupling constants. All such diagrams, besides these of Fig. 1, fall into 2 groups[1]. The diagrams of the first group are presented in Fig. 2. The loopy lines denote there vector boson, higgson or ghost propagators. Such diagrams are not proportional to N. The second group contains diagrams with the physical higgson line not in a loop — such diagrams depend on m_φ and vanish in the formal limit $m_\varphi \to \infty$.

The thing we should do is just adding the local $(Z_\mu Z^\mu)^2$ counter term as an additional counter term to the GSW Lagrangian. It can be justified in the following way: The PV

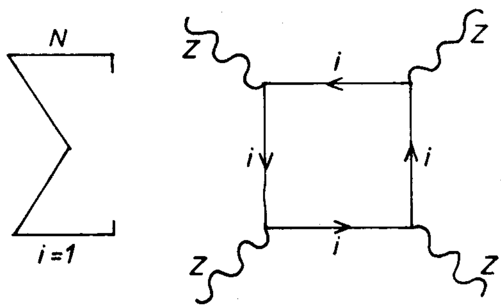

Fig. 1

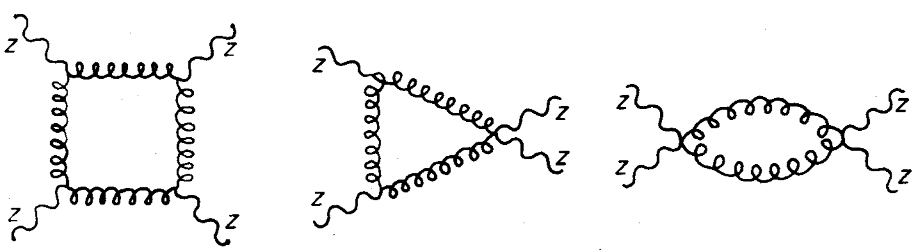

Fig. 2

[1] The Feynman rules for the GSW model given in Ref. [6] are used.

regularization of divergent fermion loops is equivalent to replacing an original action S of a model by a new one S' where additional "infinitely" heavy particles appear [9]. They ought to have the same couplings as the physical ones but may have nonphysical statistics. In the case of the GSW theory the new action is no longer $SU(2) \times U(1)$ symmetric[2] because the Yukawa couplings of the regulator particles are not proportional to *their* masses but to the masses of the corresponding physical fermions. Consequently, the regularized effective action

$$S_{\text{eff}} = -i \ln \int D\psi D\bar{\psi} e^{iS} \tag{2}$$

arising from the GSW action S after integrating out all the fermions ψ *could be* $SU(2) \times U(1)$ symmetric only if the regulator nonsymmetric contributions vanished in the limit $M_{\text{regulator}} \to \infty$. Unfortunately, this is not the case, which is strongly related to the nondecoupling effect. Thus, the PV regularization breaks the gauge symmetry. We are then allowed to introduce not only gauge symmetric counter terms obtainable from multiplicative renormalization, but also all the other Lorentz-invariant, local counter terms of dimension $\leqslant 4$. Once the usual renormalization constants are determined by some renormalization conditions, the coefficients of all the additional counter terms are determined uniquely by the gauge symmetry requirement [4].

Thus, if we want to use the PV regularization, we have to add, among others, the $\kappa(Z_\mu Z^\mu)^2$ auxiliary counter term to the GSW Lagrangian. In the following, we shall find the coefficient κ.

If we write the gauge transformation of the U(1) and SU(2) gauge fields, respectively, in the form:

$$\begin{aligned} \delta B_\mu &= (-1/g')\partial_\mu \alpha, \\ \delta W^a_\mu &= (-1/g)\partial_\mu \beta^a - \varepsilon^{abc}\beta^b W^c_\mu \end{aligned} \tag{3}$$

then the transformation of the Z_μ field is:

$$\delta Z_\mu = (g^2+g'^2)^{-1/2}\partial_\mu(\alpha-\beta^3) + \cos\theta_{\text{w}}(\beta^2 W^1_\mu - \beta^1 W^2_\mu) \tag{4}$$

and, consequently:

$$\begin{aligned} \delta[\kappa(Z_\mu Z^\mu)^2] &= 4\kappa(g^2+g'^2)^{-1/2}\partial_\mu(\alpha-\beta^3)Z^\mu Z_\nu Z^\mu \\ &\quad +(\text{quartic gauge boson terms}). \end{aligned} \tag{5}$$

Some other local term built out of fields present in S_{eff} (2) must behave under the gauge trasformation so that it cancels the first term in the r.h.s. of (5). Looking at the transformation rules for these fields (Appendix A) it is easy to convince oneself that the only possible term is

$$X = (\text{const}) \int dx Z_\mu Z^\mu Z^\nu \partial_\nu \chi_3 \tag{6}$$

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[2] Let us remind that the GSW Lagrangian posesses the $SU(2) \times U(1)$ gauge symmetry also after taking the spontaneous symmetry breakdown into account and expressing everything in terms of fields with vanishing vacuum expectation values.

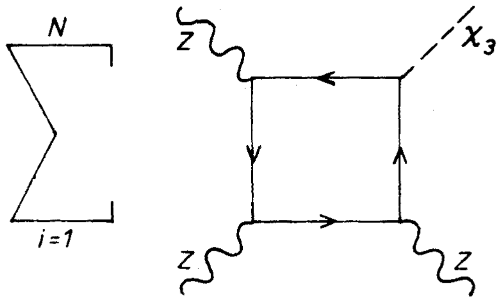

Fig. 3

Its dimension is 5, so it cannot be a counter term — it has to come from S_{eff} (2). The coefficient at the term (6) in S_{eff} (2) is given by the sum of diagrams shown on Fig. 3, and it comes out to be $N(g^2+g'^2)^{3/2}/384\pi^2 v$ [3], where $2^{-1/2}v$ is the higgson vacuum expectation value. The diagrams of Fig. 3 are superficially divergent but the considered coefficient is not affected by both the PV or dimensional regularizations. In the latter case it is due to the actual finitness of the diagrams, and in the first — to the independence of the PV regulator masses and their Yukawa couplings. Gauge transformation applied to X (6) gives

$$\delta X = -\frac{N}{2}\frac{(g^2+g'^2)^{3/2}}{384\pi^2 v}\int dx \partial_\mu(\alpha-\beta^3)Z^\mu Z_\nu Z^\nu$$
$$+\text{(terms involving scalar fields).} \qquad (7)$$

Comparing (5) and (7) we see that only for

$$\kappa = \frac{N}{8}\frac{(g^2+g'^2)}{384\pi^2} + \dots \qquad (8)$$

the sum of the PV regularized effective action (2) and the counter terms can be gauge invariant. The dots in (8) stand for terms not proportional to the number of fermions that may be neccessary to cancel possible gauge dependence of the PV regularized diagrams of Fig. 2, and higher order terms. Thus, we have recovered the result (1) of the dimensional regularization. This ends our exercise with the $(Z_\mu Z^\mu)^2$ effective interaction. One more thing that may be worth emphasizing is that the 4Z-vertex is a simple example of the heavy fermion nondecoupling in the standard model. Unfortunately, it is experimentally irrelevant at present — the contribution to the ZZ → ZZ amplitude from the local part of (2) can be at most 10^{-3} correction to the still unmeasured leading term.

The necessity of looking for additional, auxiliary counter terms makes the PV regularization quite troublesome in theories with nondecoupling. In the Appendix B we present the effective action (2) for a more general gauge theory, which shows what additional counter terms should be expected in the pure gauge boson sector.

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[3] The final expression for the diagrams of Fig. 3 has been expanded in external momenta and the relations between fermion masses and their Yukawa couplings have been used.

In conclusion, we have given an example how the PV regularization works in a theory with nondecoupling.

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APPENDIX A

Fields present in S_{eff} (2) (for the GSW theory) are the gauge bosons $W^{1,2}_\mu$, Z_μ, A_μ, the physical higgson φ and the nonphysical ones χ_i, $i = 1, 2, 3$ (the notation of Ref. [6] is used). If the SU(2) and U(1) gauge transformations parameters are defined as in Eq. (3), the fields transform as follows:

$$\begin{aligned}
\delta W^1_\mu &= -(1/g)\partial_\mu\beta^1 + \beta^2(\cos\theta_w Z_\mu + \sin\theta_w A_\mu) - \beta^3 W^2_\mu,\\
\delta W^2_\mu &= -(1/g)\partial_\mu\beta^2 + \beta^3 W^1_\mu - \beta^1(\cos\theta_w Z_\mu + \sin\theta_w A_\mu),\\
\delta Z_\mu &= (g^2+g'^2)^{-1/2}\partial_\mu(\alpha-\beta^3) + \cos\theta_w(\beta^2 W^1_\mu - \beta^1 W^2_\mu),\\
\delta A_\mu &= -(1/e)\partial_\mu(\alpha+\beta^3) + \sin\theta_w(\beta^2 W^1_\mu - \beta^1 W^2_\mu),\\
\delta\varphi &= \tfrac{1}{2}[-\beta^1\chi_1 - \beta^2\chi_2 + (\alpha-\beta^3)\chi_3],\\
\delta\chi_1 &= \tfrac{1}{2}[\beta^1(v+\varphi) + (\alpha+\beta^3)\chi_2 - \beta^2\chi_3],\\
\delta\chi_2 &= \tfrac{1}{2}[\beta^2(v+\varphi) - (\alpha+\beta^3)\chi_1 + \beta^1\chi_3],\\
\delta\chi_3 &= \tfrac{1}{2}[\beta^2\chi_1 - \beta^1\chi_2 + (\alpha-\beta^3)(v+\varphi)].
\end{aligned}$$

APPENDIX B

Here we present the effective action (2) for a model described by the action

$$S = \int dx\bar{\psi}(i\hat{\partial} - \hat{V} - \hat{A}\gamma_5 - m)\psi + \tilde{S}. \tag{B1}$$

V and A are gauge fields (not necessarily independent) interacting with N fermions ψ. The gauge group does not need to be simple or its representation — irreducible. We assume the fermions have equal masses m. The coupling constants are absorbed in the fields V and A. $\tilde{S}$ stands for terms not involving ψ. S_{eff} defined as in Eq. (2) reads:

$$\begin{aligned}
\Delta S^{(V,A)} &= \frac{1}{4\pi^2} C_2 \int dx\, \mathrm{tr}\,(A^\mu A_\mu) - \frac{1}{96\pi^2}(C_1 - \tfrac{1}{2}) \int dx\, \mathrm{tr}\,[F^{\mathrm{L}}_{\mu\nu}F^{\mathrm{L}\mu\nu}\\
&\quad + F^{\mathrm{R}}_{\mu\nu}F^{\mathrm{R}\mu\nu}] + \frac{1}{48\pi^2}\int dx\, \mathrm{tr}\,\{(\partial_\mu A^\mu + i[V_\mu, A^\mu])^2\\
&\quad -\tfrac{1}{2}F^{\mathrm{L}}_{\mu\nu}F^{\mathrm{R}\mu\nu} + 2i(F^{\mathrm{L}}_{\mu\nu} + F^{\mathrm{R}}_{\mu\nu})[A^\mu, A^\nu] + 4A_\mu A_\nu A^\mu A^\nu\} + R[A, V, m]
\end{aligned} \tag{B2}$$

where

$$F_{\mu\nu}^{\mathrm{L,R}} = \partial_\mu A_\nu^{\mathrm{L,R}} - \partial_\nu A_\mu^{\mathrm{L,R}} + i[A_\mu^{\mathrm{L,R}}, A_\nu^{\mathrm{L,R}}],$$

$$A_\mu^{\mathrm{L,R}} = V_\mu \mp A_\mu,$$

and the functional $R[A, V, m]$ vanishes when m goes to the infinity. The constants C_1 and C_2 depend on the regularization parameters, and diverge when the regularization is taken off. The applied dimensional extension of γ_5 is described in the Appendix C. The part underlined by the dashed line appears only in the case of the dimensional regularization. In the case of the PV regularization these terms should be expected as the additional counter terms.

APPENDIX C

The evaluation of the effective action (B2) and the expression (1) with help of the dimensional regularization required a proper "analytic continuation" of the γ_5 matrix to n dimensions. There are two commonly used "analytic continuations" of γ_5: in one of them (due to t'Hooft and Veltman) γ_5 is assumed to anticommute with the first four γ_μ's and commute with the "additional" γ_μ's, in the other — it anticommutes with all the γ_μ's. None of the prescriptions is free of ambiguities [7, 8], and the proper way of dealing with this ambiguities in multi-loop calculations is still unknown. In this paper the second prescription was used. The ambiguities appear then in the calculation of Dirac traces involving an odd number of γ_5's: the expansion of such a trace in $\varepsilon = n-4$ depends on how the trace is calculated [8]. For the purposes of this paper the following generalization of the procedure suggested by Ovrut [7] was used:

1) The traces were written in the form with only 4 free Lorentz indices (this was possible for all of them).

2) They were transformed into the form (const)$\mathrm{tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\varrho\gamma_\sigma)$ with the help of the equations $\{\gamma_5, \gamma_\mu\} = 0$, $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$, $\gamma_5^2 = 1$, $g_\alpha^\alpha = n$, *but in such a way that* γ_5 *was anticommuted with the contracted* γ_μ*'s as few times as possible.*

3) The equation $\mathrm{tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\varrho\gamma_\sigma) = 4i\varepsilon_{\mu\nu\varrho\sigma}$ was used.

This procedure unambiguously leads to the results for Feynman diagrams in which the chiral anomaly appears in the divergence of the axial current. If γ_5's were not anticommuted in the way described above, the anomaly could appear in the divergence of the vector current. This cannot be allowed[4] because, once we have set the masses of the fermions ψ in the action (B1) equal, the vector gauge symmetry must hold. It should be pointed out that the $(Z^\mu Z_\mu)^2$ term in (1) would appear also if the t'Hooft-Veltman "continuation" of γ_5 was applied.

[4] As long as we want the dimensional regularization to preserve the symmetries under which both the classical action and the functional measure are invariant.

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