BLACK HOLE EVAPORATION AND INFLATIONARY COSMOLOGY*

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Evolution of evaporating black hole in inflationary universe is investigated for a Bardeen–Vaidya–deSitter line element. The Raychaudhuri equation is examined up to second order terms in the luminosity in the vicinity of the black hole event horizon and the equations governing the evolution of a black hole are given. A back reaction programme in a simplified Vaidya–deSitter background is examined and the luminosity formula is found. The effective temperature assigned to the Bardeen–Vaidya–deSitter black hole is proposed.

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1. Introduction

It has been known for some time that on a basis of quantum field theory black hole emits radiation at a temperature proportional to its surface gravity [1]. The original Hawking calculations of this effect in the Schwarzschild background were performed in terms of particles observed near null infinities, J^+ and J^-, where they can be unambiguously defined. In a general spacetime, however, the concept of "particle" loses its universal character so other concepts to describe radiation are needed.

Perhaps the most important consequence of the Hawking result is indication that a black hole may evaporate which is classically forbidden by the area theorem. The profound analysis of the black hole evolution, especially as the Planck regime is approached, certainly requires incorporation of the quantum gravity effects into the picture of evaporation. Although at present time there is no quantum theory of gravity satisfactorily describing interaction of gravitational field with itself, the theory in which classical gravitational field interacts with quantized fields is far better known [2].

It seems that the consensus has been achieved that the physical content of the quantum field theory in curved background is carried by the regularized mean value of the stress

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energy tensor $\langle T_{\alpha\beta}\rangle_{\mathrm{REG}}$ in appropriate vacuum state. The stress energy tensor may serve as the constituent of the source term of the Einstein field equations allowing in principle to determine the further evolution of the system. The back reaction programme as understood here is to solve the semi-classical field equations

$$G_{\alpha\beta}[g]+\lambda g_{\alpha\beta} = 8\pi\langle T_{\alpha\beta}[g]\rangle_{\mathrm{REG}}, \qquad (1.1)$$

where $G_{\alpha\beta}$ is the Einstein tensor and λ the cosmological constant, for a classical (i.e. totally ignoring quantum fluctuations) metric. Unfortunately any attempt to find an exact solution to the semi-classical back reaction equations is invalidated besides their nonlinearity by the necessity of having at one's disposal the detailed knowledge of the dependence of $\langle T_{\alpha\beta}\rangle_{\mathrm{REG}}$ on a wide class of metrics. Such information provided by approximate analytical methods are restricted to Einstein spaces [3–6] to which class the dynamical spacetime of evaporating black hole obviously does not belong. To overcome this difficulty partially a method based on simplified two dimensional models was invented [7]. Due to conformal triviality in such models it is possible to evaluate $\langle T_{\alpha\beta}\rangle_{\mathrm{REG}}$ and one of the advantages of this approach is the possibility to investigate fairly complicated spaces. It is argued that since in a two dimensional space information concerning scattering of field modes is lost, the expectation value of the stress energy tensor for these lower dimensional models presents the geometrical optics limit of $\langle T_{\alpha\beta}\rangle_{\mathrm{REG}}$ in a physical four dimensional spacetime.

The black hole spacetime modeled by the line element of the Vaidya type were extensively studied in the variety of contexts by a number of authors. The analysis of the evolution of the event horizon by means of the Raychaudhuri equations have found its most elaborated treatment in the article of York [8], though his investigation did not appeal to $\langle T_{\alpha\beta}\rangle_{\mathrm{REG}}$ being quantal in spirit only. Studying the Raychaudhuri equations for the event horizon generators one concludes that the advanced time derivative of the expansion $\frac{d}{dv}\theta$ is negligible in $O(L)$, where L is the observed luminosity. Hence $O(L^2)$ behaviour is required. The line of attack originated by Balbinot [9] makes use of the regularized stress energy tensor and provides the approximate formula for the luminosity of the black hole.

Recently there has been considerable interest in the effects of nonvanishing cosmological term. The possibility of adding a cosmological vacuum density λ to the Einstein field equations raised the question of empirical justification of such a step. Observational data indicate that the cosmological constant, if nonzero, is smaller than $10^{-122}M_{\mathrm{Pl}}^2$, and consequently the present universe in large scale is accurately described by the standard Friedman–Robertson–Walker line element. However, since anything that contributes to the vacuum energy acts as a cosmological constant it cannot just be dropped.

Studying the evolution of the standard model "backwardly" in time one faces the problem that very unnatural initial conditions have to be imposed for a model to fulfill observational cosmology, and that these very conditions are connected with the famous puzzles, most prominent of which are the horizon, flatness and monopole problems. At the beginning of this decade Guth [10] proposed a scenario of the evolution of the universe capable of avoiding the above mentioned problems. The most striking feature of the model is the appearance of a inflationary phase described by the deSitter lihe element

$$ds^2 = -dt^2 + \exp\left(2\sqrt{\frac{\lambda}{3}}\,t\right)(dr^2 + r^2 d\Omega^2). \tag{1.2}$$

The cosmological constant λ is proportional to T^4_{GUT}, where T_{GUT} is a critical temperature of the phase transition in a grand unification scheme.

It was pointed out that black holes could have formed at very early stages of the evolution of universe as a result of initial inhomogeneities [11]. Some of them may survive to our times. The similar problem has been undertaken in the context of inflationary model [12]. Though the results obtained so far seem not to be conclusive it was indicated that it is possible for a sufficiently large fluctuation to produce a black hole. Such the black hole evaporates and consequently loses its mass, though its evolution, as compared with the asymptotically flat models is certainly modified by the inflationary environment.

In this paper we shall consider the evolution of the black hole in the spacetime with the positive cosmological constant. Throughout the paper the dimensionless units are used in which $c = G = k = \hbar = 1$.

2. *The model*

We shall start with the generalization of the Bardeen–Vaidya line element to the case of the positive cosmological constant, which accurately describes, at least in the vicinity of the black hole, the process of the emission of energy into the inflationary environment. It takes the following form:

$$ds^2 = -e^{2\psi}\left(1 - \frac{2m}{r} - \frac{\lambda r^2}{3}\right)dv^2 + 2e^{\psi}dvdr + r^2 d\Omega^2, \tag{2.1}$$

where m and ψ are functions of the advanced time v and the radial coordinate r. The Einstein field equations relate functions m and ψ to the components of the energy momentum tensor. It could be verified by a direct calculations that the relevant equations are

$$4\pi r^2 T^r_v = \frac{\partial m}{\partial v}, \tag{2.2}$$

$$4\pi r T_{rr} = 4\pi r T^v_r = \frac{\partial \psi}{\partial r}, \tag{2.3}$$

$$4\pi r^2 T^v_v = -\frac{\partial m}{\partial r}. \tag{2.4}$$

The spherically symmetric spacetime of an evaporating black hole described by the line element (2.1) with m and ψ constant allows the existence of horizons of two types: the event horizon and its cosmological companion. In a dynamical case however one has to consider another surface of principal importance namely an apparent horizon.

Since the choice of the vector field l and β in the form given by Carter [13] has proved to be useful in studying the properties of the Bardeen–Vaidya spacetime we adopt this convention and define

$$l^{\alpha} = [1, \tfrac{1}{2}\, e^{\psi}F, 0, 0] \tag{2.5}$$

and

$$\beta^{\alpha} = [0, -e^{-\psi}, 0, 0], \tag{2.6}$$

where $F = g_{vv}$. In terms of these vectors an expansion θ is defined as

$$\theta = \frac{2}{r}\, l^{r}. \tag{2.7}$$

It follows that the position of the apparent horizon being the outermost marginally trapped surface is given by the smallest positive solution, if any exist, $r = r_{AH}$, of the equation

$$\lambda r^3 - 3r + 6m(r, v) = 0. \tag{2.8}$$

Since we have been interested in the neighbourhood of the black hole, the presumably existing cosmological horizon will not be considered in this section. Defining the black hole mass $M(v)$ at a given instant of the advanced time as the value of $m(v, r)$ such that

$$M(v) = m(v, r = r_{AH}), \tag{2.9}$$

one can also define luminosity $L = -\dfrac{dM}{dv}$, measured in the region where $\dfrac{d}{dv}$ is timelike.

Because of its definition $\partial I^{-}(J^{+})$, valid in the asymptotically flat space, which roughly means that the outgoing null rays can never reach large distances, the evaluation of the position of the event horizon requires knowledge of the whole history of the black hole. However working only with the approximate expressions York [8] pointed out that the question which null rays generate the event horizon is a matter of qualitative degree and he assumed that it is practically defined by the following condition: $\dfrac{d^2r}{dv^2} = 0$. To be exact this condition describes locus of inflexion points and hence slightly overestimates the true position of the event horizon. Since

$$\frac{d^2r}{dv^2} = \frac{\partial\psi}{\partial r}\left(\frac{dr}{dv}\right)^2 + \frac{dr}{dv}\left(\frac{\partial\psi}{dv} - \frac{e^{\psi}}{r}\frac{\partial m}{\partial r} + e^{\psi}\frac{m}{r^2} - e^{\psi}\frac{\lambda r}{3}\right) - \frac{e^{\psi}}{r}\frac{\partial m}{\partial v} \tag{2.10}$$

the event horizon to $O(L)$ is given by

$$r_{EH} = r_{AH}(1 - L\beta^{-2}), \tag{2.11}$$

where $\beta = r_{AH}\kappa$, and $\kappa = \dfrac{M}{r_{AH}^2} - \dfrac{\lambda}{3}\, r_{AH}$. The quantity defined as

$$\kappa = \frac{\partial\psi}{\partial v} + e^{\psi}\frac{\partial\psi}{\partial r}F + \tfrac{1}{2}\, e^{\psi}\frac{\partial F}{\partial r}, \tag{2.12}$$

when evaluated on the event horizon is the surface gravity. Inserting κ, θ and $R_{\alpha\beta}l^{\alpha}l^{\beta}$ estimated at the event horizon into the Raychaudhuri equation

$$\frac{d\theta}{dv} = \kappa\theta - \tfrac{1}{2}\theta^2 - R_{\alpha\beta}l^{\alpha}l^{\beta} \tag{2.13}$$

it could be easily shown that to $O(L)$ one has

$$\frac{d\theta}{dv} \cong 0. \tag{2.14}$$

It follows that to describe satisfactorily the dynamics of the evaporating black hole higher terms in L should be included. Ingredients of Eq. (2.13) to $O(L^2)$ can be written as

$$\kappa = \tfrac{1}{2}\frac{\partial F}{\partial r}, \tag{2.15}$$

$$\theta = \frac{1}{r} F \tag{2.16}$$

and

$$R_{\alpha\beta}l^{\alpha}l^{\beta} = \frac{2}{r^2}\frac{\partial m}{\partial v}. \tag{2.17}$$

Combining (2.15), (2.16) and (2.14) and making use of (2.2) and (2.4) we have on the event horizon

$$\kappa\theta = R_{\alpha\beta}l^{\alpha}l^{\beta}. \tag{2.18}$$

Therefore we can write the Raychaudhuri equation (2.13) in the form

$$\frac{d\theta}{dv} = -\tfrac{1}{2}\theta^2. \tag{2.19}$$

In order to determine the evolution of the event horizon of the Bardeen–Vaidya–deSitter black hole it is necessary to evaluate the expansion up to the L^2 terms. Inspection of Eq. (2.16) shows that the estimates of the partial derivatives of the mass functions are needed since

$$m(v, r = r_{EH}) \cong M(v) - \frac{L}{\beta}\frac{\partial m}{\partial r}\Big|_{r=r_{AH}}. \tag{2.20}$$

Here we apply the method proposed by Arai and Shimoda [14] which is based on the observation that imposing regularity conditions on the form of the Bardeen tetrad [15] components of the stress tensor near the event horizon it is possible to extract information concerning its tensor components. Thus we have

$$T^{vv} = \frac{L}{16\pi r_{AH}^2}\left(3 + \frac{3L}{\beta} + \frac{4L}{\beta^2}\right), \tag{2.21}$$

$$T^{vr} = -\frac{L}{8\pi r_{AH}^2}\left(1+\frac{3L}{\beta}\right), \tag{2.22}$$

$$T^{rr} = -\frac{L}{4\pi r_{AH}^2}\left(1-\frac{2L}{\beta}+\frac{4L}{\beta^2}\right). \tag{2.23}$$

Using (2.10), (2.16) and (2.21)–(2.23) we have

$$\theta = -\frac{2L}{\beta r_{AH}}\left[1+L\left(\frac{4}{\beta^2}-\frac{1}{2\beta}-\frac{1}{\beta^3}\right)\right]. \tag{2.24}$$

The parameter β may be written in the form

$$\beta = r_{AH}^{-1}(3M-r_{AH}). \tag{2.25}$$

Equations (2.19) and (2.24) govern the evolution of the black hole. Now we invoke the exact value of the position of the apparent horizon. If the condition $9M^2\lambda < 1$ holds r_{AH} is given by

$$r_{AH} = 2\lambda^{-1/2}\cos\left(\frac{\eta}{3}+\frac{4\pi}{3}\right), \tag{2.26}$$

where $\cos\eta = -3M\lambda^{1/2}$, with $\pi < \eta < 3\pi/2$. Hence $2M < r_{AH} < 3M$. Equations (2.24) and (2.26) together with the solution of Eq. (2.19)

$$\theta = -\frac{2}{v-v_f}, \tag{2.27}$$

where v_f is given by the condition $M(v_f) = 0$, present rather complicated system and probably require numerical calculation that is beyond the scope of this paper. We observe however, that when the cosmological constant is small $r_{AH} \cong 2M$ and $\beta \cong 1/2$, and hence our formulae reduce to the analogous expressions given in Arai and Shimoda paper [14], with the luminosity

$$L = \frac{M_i}{v_f}\left(1-\frac{7M_i}{v_f}+\frac{14M_i}{v_f}\frac{v}{v_f}\right), \tag{2.28}$$

where M_i is the initial mass of the black hole.

3. *The back reaction*

In this Section we consider the influence of the quantized massless scalar field obeying the conformally invariant Klein–Gordon equation on the black hole evolution in the inflationary universe. To be more definite let us consider a simple but somewhat unphysical picture in which imploding thin shell of null fluid forms a black hole in the spacetime with the effective cosmological constant. After the black hole formation negative null fluid modeling the process of evaporation falls down the hole reducing its mass. Incoming negative flux is accompanied with the outgoing positive one. Assuming the line element in the form given in Eq. (2.1) to be valid in the neighbourhood of the black hole and confining attention to $O(L)$ effects one can further simplify the metric [16]

$$ds^2 = -\left(1-\frac{2M}{r}-\frac{\lambda r^2}{3}\right)dv^2+2dvdr+r^2d\Omega^2. \tag{3.1}$$

Since the spacetime of the region to the past of the outgoing positive flux is static the metric of this region is described by the Schwarzschild–deSitter line element characterized by the initial mass of the shell M_i. It follows that the larger positive root of the equation

$$\lambda r^3-3r+6M_i = 0 \tag{3.2}$$

may be regarded as a position of the static past cosmological horizon.

As is well known any attempt to evaluate the stress energy tensor of the quantized field in a such background encounters the enormous mathematical complexity. In order to make calculations tractable we follow a prescription in which simplified two dimensional slice of the four dimensional space is taken to model evaporating black hole. It could be shown [7] that due to transparent conformal triviality of the two dimensional metric written in a double null form

$$ds^2 = C(U, V)dUdV \tag{3.3}$$

the components of the mean value of the stress energy operator evaluated in the vacuum state generated by the normal mode functions η could be written as

$$\langle T_{uu}\rangle^{\eta}_{\mathrm{REG}} = -\frac{1}{12\pi}C^{1/2}C^{-1/2}_{,uu}, \tag{3.4}$$

$$\langle T_{vv}\rangle^{\eta}_{\mathrm{REG}} = -\frac{1}{12\pi}C^{1/2}C^{-1/2}_{,vv}, \tag{3.5}$$

$$\langle T_{uv}\rangle^{\eta}_{\mathrm{REG}} = \langle T_{vu}\rangle^{\eta}_{\mathrm{REG}} = -(48\pi)^{-1}RC, \tag{3.6}$$

where R is the curvature scalar and a colon denotes partial differentiation. The normal mode vacuum is inappropriate in the study of the evolution of the black hole. The exact form of the base functions defining the relevant vacuum state is unknown. It is possible, however, to extract definite information concerning $\langle T_{\alpha\beta}\rangle_{\mathrm{REG}}$ imposing on its form some general requirements. The stress energy tensor should be covariantly conserved

$$\nabla_\alpha\langle T^\alpha{}_\beta\rangle_{\mathrm{REG}} = 0 \tag{3.7}$$

and lead to the standard form of the trace anomaly

$$\langle T^\alpha{}_\alpha\rangle_{\mathrm{REG}} = \frac{R}{24\pi}\cdot \tag{3.8}$$

The most general tensor satisfying (3.7) and (3.8) may be written as [17]

$$\langle T_{\alpha\beta}\rangle_{\text{REG}} = \langle T_{\alpha\beta}\rangle^{\eta}_{\text{REG}} + t_{\alpha\beta}, \tag{3.9}$$

where $t_{\alpha\beta}$ is any conserved and traceless tensor obeying

$$t_{uv} = t_{vu} = 0 \tag{3.10}$$

and

$$\frac{\partial}{\partial v} t_{uu} = \frac{\partial}{\partial u} t_{vv} = 0. \tag{3.11}$$

To select further the $\langle T_{\alpha\beta}\rangle_{\text{REG}}$ one has to impose appropriate boundary conditions. Since our principal interest lies in the investigation of the process of evaporation it is natural to look for an analog of the Unruh vacuum state. The requirement that invariants of the stress-energy tensor be non singular on the future event horizon [18] seems obvious, what means that when $r \to r_{\text{EH}}$

$$C^{-2}\langle T_{uu}\rangle_{\text{REG}} \doteq 0. \tag{3.12}$$

The above discussion indicates that the central role in evaluating $\langle T_{\alpha\beta}\rangle_{\text{REG}}$ is played by $C(u, v)$ function. The double null form of the Vaidya–deSitter line element is achieved by means of the transformation

$$\begin{cases} du = g\left[dv - \left(1 - \dfrac{2M}{r} - \dfrac{\lambda r^2}{3}\right)^{-1} dr\right], \\ v = v \end{cases} \tag{3.13}$$

where g satisfies the following equation

$$\frac{\partial}{\partial r} g + 2\frac{\partial}{\partial v}\left[\left(1 - \frac{2M}{r} - \frac{\lambda r^2}{3}\right)^{-1} g\right] = 0. \tag{3.14}$$

The major difficulty with Eq. (3.14) and hence with the integrating factor g consists in the fact that it contains the unknown mass function which is to be determined by solving in a self-consistent way of the back reaction problem. It could be shown, however, that the incoming component of the stress tensor $\langle T_{\alpha\beta}\rangle_{\text{REG}}$ is independent of the integrating factor that is the remarkable feature of the two dimensional models. Eqs (3.5) and (3.9) applied to the Vaidya–deSitter geometry yield [19]

$$\langle T_{vv}\rangle_{\text{REG}} = \frac{1}{24\pi}\left(\frac{3M^2}{2r^4} - \frac{M}{r^3} + \frac{\dot{M}}{r^2} + \frac{M\lambda}{r} - \frac{\lambda}{6}\right) + t_{vv}, \tag{3.15}$$

where a dot denotes differentiation with respect to the advanced time coordinate. To determine t_{vv} we observe that when $M = 0$ the Unruh vacuum state preserves the symmetries of underlying manifold. It follows then that

$$t_{vv} = -\langle T_{vv}\rangle^{\eta}_{\text{REG}}. \tag{3.16}$$

Taking M to be constant, that results in the Schwarzschild–deSitter geometry, one can recover the incoming component of the stress energy tensor in the Unruh vacuum by demanding that

$$\langle T_{vv}\rangle_{\text{REG}} = 0 \tag{3.17}$$

near the past cosmological horizon. This procedure can be easily justified by the analysis of the exact form of the Unruh base functions given in the form [20]

$$\phi_{\omega} = [8\pi|\omega| \sinh(\pi\omega/\kappa_{\text{CH}})]^{-1/2} \exp(\pi\omega/2\kappa_{\text{CH}}) \exp(-i\omega v). \tag{3.18}$$

Assuming the line element describing the spacetime between the past cosmological horizon and the future event horizon of the evaporating black hole to be smooth and covered by (u, v) co-ordinates we have

$$\langle T_{vv}\rangle_{\text{REG}} = \frac{1}{24\pi}\left(\frac{3M^2}{2r^4} - \frac{M}{r^3} + \frac{\dot{M}}{r^2} + \frac{M\lambda}{r} - \frac{\lambda}{6}\right) + \frac{1}{48\pi}\kappa^2_{\text{CH}}. \tag{3.19}$$

The foregoing considerations deserve some comment: the stress energy tensor consists of term describing its local behaviour and the boundary dependent reminder, bearing information of the global structure of the manifold, and thus the evolution of the black hole is determined by the physics in the vicinity of the event horizon and the cosmological flux. The actual form of the latter is obscure, since we have no detailed knowledge of the geometry of the whole spacetime. (If the spacetime is constructed by topological glueing of the segments with different line elements another term appears that we discard). Here we accept the widespread point of view that the origin of Hawking radiation lies in the quantum ergosphere.

Making use of (2.11) one can rewrite expression (3.19) in a more transparent form

$$\langle T_{vv}\rangle_{\text{REG}} = -\frac{\kappa^2_{\text{EH}}}{48\pi}(1-2\mathscr{L}) + \frac{\kappa^2_{\text{CH}}}{48\pi}, \tag{3.20}$$

where $\mathscr{L} = -L\beta^{-2}$. Unfortunately other components of the stress tensor cannot be worked out so easily.

As it is well-known in the case of a static black hole the temperature T is connected with the surface gravity through the famous formula

$$T = \frac{\kappa}{2\pi}. \tag{3.21}$$

Making use of (3.21) in the Schwarzschild–deSitter geometry one has

$$\langle T_{vv}\rangle_{\text{REG}} = -\frac{\pi}{12}(T^2_{\text{BH}} - T^2_{\text{CH}}). \tag{3.22}$$

Extending validity of the equation (3.22) to the dynamical Vaidya–deSitter background the following expression for temperature may be written

$$T_{\mathrm{EH}} = \frac{\kappa_{\mathrm{EH}}}{2\pi}(1-2\mathscr{L})^{1/2}. \tag{3.23}$$

Now the Raychaudhuri equation gives

$$L = \frac{\pi^2 r_{\mathrm{AH}}^2}{30r^2} A_{\mathrm{E}}(T_{\mathrm{BH}}^4 - T_{\mathrm{CH}}^4) \tag{3.24}$$

what may be written, in virtue of (3.21) in the form

$$L = \frac{r_{\mathrm{AH}}^2(\kappa_{\mathrm{EH}}^4 - \kappa_{\mathrm{CH}}^4)}{4(30\pi - \kappa_{\mathrm{EH}}^2)}. \tag{3.25}$$

We have approximated the right hand side of Eq. (2.18) by the Stephan–Boltzmann law regarding T_v^r to be

$$T_v^r = -\frac{\pi}{120r^2} A_{\mathrm{E}}(T_{\mathrm{EH}}^4 - T_{\mathrm{CH}}^4), \tag{3.26}$$

with $A_{\mathrm{E}} = 4\pi r_{\mathrm{AH}}^2$. Equation (3.26) together with the luminosity definition is difficult to solve. It should be noted, however, that for a small cosmological constant Eq. (3.25) is reduced to the form obtained by Balbinot [21].

$$L = \frac{\alpha}{M^2 - 16\alpha}, \tag{3.27}$$

where $\alpha = (7680\pi)^{-1}$. The only relevant back reaction equation for the line element (3.1) is

$$\frac{dM}{dv} = -4\pi r^2 T_v^r \tag{3.28}$$

and hence (3.28) together with (3.26) yield no more information.

As we have assumed the process of evaporation starts with the black hole mass M_i and since $9M_i^2\lambda < 1$ the surface gravity of the event horizon is always greater than the surface gravity of the cosmological horizon. Assuming the effective cosmological constant of the inflationary phase as $\chi \sim 10^{-11}$ [22], where $\lambda = 3\chi^2$ one concludes that κ_{EH} reaches the values comparable with $(30\pi)^{-1}$ for the mass of order $10^{-2}M_{\mathrm{PL}}$. Because of presumed quantum fluctuation of the metric in the Planck regime the applicability of the formula (3.25) is restricted to the masses above the Planck mass. Therefore any analysis of the last stages of the black hole evolution cannot be performed without incorporation of the still unknown quantum theory of gravity. In order to gain more detailed information more sophisticated models are required.

A qualitative analysis indicates that it is possible to produce a black hole of a mass of order of 10^9 M_{PL} if fluctuation region has the size of cosmological horizon [12]. According to rough estimates (based on the static Schwarzschild model) such black holes are probably to small to survive to our times. Taking typical temperature $T_{GUT} \sim 10^{14}$ GeV, the surface gravities of the event and cosmological horizon are comparable in magnitude and therefore such black holes remain practically stable during the inflationary phase. Their further evolution may be hampered during the reheating of the Universe.

So far we have considered the simplified version of the Bardeen–Vaidya–deSitter line element in the context of the quantum evaporation of the black hole in the inflationary universe. Following a similar method which leads to equation (3.23) one easily obtains the formula describing the effective temperature in $O(L)$ in the Bardeen–Vaidya–deSitter background

$$T_{EH} = \frac{\kappa_{EH}}{2\pi}\left(1 + \frac{L}{\beta^2} - \frac{\partial m}{\partial r}\bigg|_{r=r_{EH}}\right)^{1/2}. \tag{3.29}$$

Equation (3.29) differs from the expression describing the temperature connected with the Vaidya–deSitter black hole by the $\frac{\partial m}{\partial r}$ term. Unfortunately we have no information concerning this term. Recent analysis, however, carried out in the Vaidya black hole spacetime indicates that this term may not be negative in the context of the spin 1 fields [23] that is rather unexpected feature of the vector bosons. It follows then that if a similar situation holds in the presence of the nonzero cosmological constant the effective temperature will be shifted toward its smaller values and it could be made small by allowing the existence of N spin 1 fields.

4. Conclusions

We have investigated the dynamical behaviour of the evaporating black hole in the inflationary universe. The latter were modeled by the deSitter spacetime with the effective cosmological constant $\lambda = 3\chi^2$. By means of the Raychaudhuri equation we derived the formulas describing the black hole evolution in the second order in the luminosity.

Studying the impact of the quantized conformally invariant massless scalar field we showed that the luminosity is altered by the presence of the nonzero cosmological flux at a rate proportional to the fourth power of the surface gravity of the cosmological horizon. Since κ_{EH} is always greater than κ_{CH} the evaporation process cannot be stopped by the accretion of the positive cosmological fluid. Inspection of Eq. (3.25) shows that as κ_{EH} tends to $(30\pi)^{1/2}$ the luminosity unboundedly grows. It is argued however that the extension of the validity of the luminosity formula below the Planck mass is highly speculative.

It should be emphasized, however, that we have incorporated only one temperature scale into the picture of evaporation. Besides T_{CH} there exists the ambient temperature of the universe which is expected to fall exponentially in time.

Our results are critically dependent on the thermal hypothesis and seem to be valid for the black hole mass comfortably above the Planck mass. One cannot, however, exclude the possibility that in the last stages of its evolution the black hole undergoes explosion with unboundedly growing luminosity.

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