SOME REMARKS ON THE EFFECTIVE ACTION IN CURVED SPACE-TIME WITH TORSION

By S. D. Odintsov*

Department of Physics, The University Newcastle upon Tyne NE 1 7 RU, U.K.

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The Vilkovisky–De Witt effective action for scalar electrodynamics and SU(5) GUT, nonminimally interacting with torsion, is calculated on de Sitter background with torsion. The same calculation on the same background for quantum gravity with torsion is discussed. A short discussion of Green functions in curved space-time with torsion is also presented.

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1. Introduction

The natural extension of quantum field theory in curved space-time (for a general review, see [1]) is a quantum field theory in curved space-time with torsion (for a review, see, for example, [2]). The investigation of the interacting fields in curved space-time with torsion [3] shows that it is necessary to introduce additional terms (non-minimal interaction with torsion) to Lagrangian to provide multiplicative renormalizability. It leads to a theory which contains nonminimal coupling of matter with curvature and torsion. It is also necessary to introduce non-minimal interaction with torsion for the scalar field which does not interact minimally with torsion at all!

In the present paper we want to discuss the quantum effective action in curved space-time with torsion, practically, using the de Sitter space with torsion as a background. The attention to de Sitter space is explained by many reasons (see, [1]) connected mostly with black-hole physics (for recent review see [4]). The effective action for some gauge theories and (super)gravity in Euclidean de Sitter space (without torsion) has been obtained in Refs. [6–8] (see also [17, 18]) using ζ-regularization [5]. (The effective action for $\lambda\phi^4$-theory in de Sitter space with torsion has been calculated in Ref. [9]).

However, it is well-known now that the off-shell effective action in gauge theories depends on the gauge conditions. It seems that using gauge and parametrization invariant off-shell, Vilkovisky–De Witt effective action [10–12] (for a recent review, see [13, 14]) as true off-shell effective action is more acceptable in off-shell calculations in gauge theories.

* Permanent address: Pedagogical Institute, 634041 Tomsk, USSR.

The calculation of the Vilkovisky–De Witt EA in quantum gravity on de Sitter background has been done in Ref. [15] and in quantum gravity with SU(5) GUT on de Sitter background — in Ref. [16].

In the present short paper we calculate the Vilkovisky–De Witt effective action for scalar electrodynamics and minimal SU(5) GUT in de Sitter space with torsion. The attempt of the same calculation for quantum gravity in de Sitter space with torsion is also presented. But the appearance of some technical problems demanding the development of ζ-regularization technique for the calculation of non-minimal operators determinants in de Sitter space with torsion does not allow us to finish this calculation. In conclusion we discuss topics concerning the Green functions in curved space-time with torsion.

2. The Vilkovisky–De Witt effective action in de Sitter space with torsion

2.1. Let us choose the background geometry in the following form

$$L = -k^{-2}(R+h_1 S^2-2\Lambda_0), \quad k^2 = 16\pi G, \tag{1}$$

where $R = 4\Lambda$, $S^2 = S_\mu S^\mu$, $\int d^4x\sqrt{g} = \frac{24\pi^2}{\Lambda^2}$, for $h_1 = -\frac{1}{24}$. Lagrangian (1) corresponds to the Einstein-Cartan theory of the gravitation. For simplicity we assume the torsion tensor $T_{\alpha\beta\gamma}$ to be entirely antisymmetric so that its independent components are determined by the pseudovector $S_\mu = \varepsilon_{\alpha\beta\mu\nu}T^{\alpha\beta\nu}$. The classical equations of motion for Lagrangian (1) have the following solution: $\Lambda = \Lambda_0$, $S^2 = 0$ corresponding to Euclidean de Sitter space. We assume non-zero constant torsion background (after taking into account quantum corrections, solutions for Λ and S^2 are changed).

Let us write the Lagrangian of scalar electrodynamics in de Sitter space with torsion:

$$L_1 = \tfrac{1}{4} F_{\mu\nu}^2+\tfrac{1}{2}(\nabla_\mu\varphi^a+g\varepsilon^{ab}\varphi^b A_\mu)^2+\tfrac{1}{2}\xi_1 R\varphi^a\varphi^a$$
$$+\tfrac{1}{2}\xi_2 S^2\varphi^a\varphi^a+\frac{\lambda}{4!}(\varphi^a\varphi^a)^2. \tag{2}$$

Here $a = 1, 2$, ∇_μ is the covariant derivative without torsion. This theory does not interact with torsion in the minimal way ($\xi_2 = 0$). However, it is useful to investigate properties of matter interacting with torsion in the frame of such simple model before going to more complicated theories of matter interacting with torsion in a minimal way [3]. It is interesting also that one can introduce the gauge non-invariant vector torsion interactions: S^2A^2 (cf. with the terms: $R_{\mu\nu}A^\mu A^\nu$ [19]).

Let us find now gauge-invariant Vilkovisky–De Witt effective action for the theory with the Lagrangian (2) in de Sitter space with torsion. According to results of Ref. [15] it coincides with the appropriate off-shell effective action calculated in Landau–De Witt gauge (presence of torsion does not change this result). The Landau–De Witt gauge for the scalar electrodynamics is:

$$\nabla^\mu A_\mu - g\varepsilon^{ab}\phi^a\varphi^b = 0, \tag{3}$$

where ϕ^a is background scalar, A_μ, φ^a are the quantum fields. Thus, the gauge-fixing Lagrangian is:

$$L_{\mathrm{GF}} = \frac{1}{2\alpha}(\nabla^\mu A_\mu - g\varepsilon^{ab}\phi^a\varphi^b)^2, \tag{4}$$

where gauge parameter $\alpha \to 0$ at the end of calculations.

Working with non-minimal operators we must represent [15, 17, 18]

$$\begin{aligned} A_\mu &= A_\mu^\perp + A_\mu^L, \quad \nabla^\mu A_\mu^\perp = 0, \quad A_\mu^L = \nabla_\mu \chi, \\ h_{\mu\nu} &= \bar{h}_{\mu\nu} + \tfrac{1}{4} g_{\mu\nu} h, \quad \bar{h}_{\mu\nu} g^{\mu\nu} = 0, \\ \bar{h}_{\mu\nu} &= \bar{h}_{\mu\nu}^\perp + \nabla_\mu \xi_\nu^\perp + \nabla_\nu \xi_\mu^\perp + \nabla_\mu \nabla_\nu \sigma - \tfrac{1}{4} g_{\mu\nu} \nabla_\sigma. \end{aligned} \tag{5}$$

Here χ, σ are the scalars, and for generality we write the decompositions also for quantum field $h_{\mu\nu}$ with spin 2. Let us define the operators Δ_s corresponding to the irreducible decompositions of SO(5) group [15, 17, 18]:

$$\Delta_0 \phi = (-\Box + X)\phi, \tag{6}$$

$$\Delta_1 A_\nu^\perp = (-g^{\mu\nu}\Box + g^{\mu\nu} X) A_\nu^\perp, \tag{7}$$

$$\Delta_{2\alpha\beta}^{\mu\nu} \bar{h}_{\mu\nu}^\perp = (-\delta_{\alpha\beta}^{\mu\nu}\Box + \delta_\alpha^\mu \delta_\beta^\nu X)\bar{h}_{\mu\nu}^\perp. \tag{8}$$

Integrating over the fields A_μ in Eq. (2)+(4) with account of Jacobian (see Eq. (5)) and ghost contribution we have

$$\Gamma_{A\mu} = \tfrac{1}{2}\,\mathrm{Sp}\ln \Delta_1(\Lambda + g^2\phi^2)\Delta_0(\alpha g^2\phi^2) - \mathrm{Sp}\ln \Delta_0(g^2\phi^2) \tag{9}$$

where $\phi^2 = \phi^a\phi^a$. Besides, the additional contribution to Lagrangian of quantum fields φ^a appears:

$$\Delta L_\varphi = \frac{g^2}{2}\left(1 + \frac{1}{\alpha}\right)^2 \varepsilon^{ab}\phi^a\varphi^b \frac{\Box\, \varepsilon^{cd}\phi^c\varphi^d}{\left(-\frac{1}{\alpha}\Box + g^2\phi^2\right)}. \tag{10}$$

Introduce the operators $\Pi^{\|ab} = \dfrac{\phi^a\phi^b}{\phi^2}$ and $\Pi^{\perp ab} = \delta^{ab} - \Pi^{\|ab}$.

Then, integrating over the fields φ^a (as in Refs. [15, 16]) and getting $\alpha = 0$, we get

$$\begin{aligned} \Gamma_\varphi &= \tfrac{1}{2}\,\mathrm{Sp}\ln \Delta_0\left(4\xi_1\Lambda + \xi_2 S^2 + \frac{\lambda\phi^2}{2}\right) \\ &+ \tfrac{1}{2}\,\mathrm{Sp}\ln\left(-\Box^2 + (4\xi_1\Lambda + \xi_2 S^2)\Box + \frac{\lambda\phi^2}{2}\Box + 2g^2\phi^2\Box - g^4\phi^4\right) \\ &- \tfrac{1}{2}\,\mathrm{Sp}\ln \Delta_0(0). \end{aligned} \tag{11}$$

Thus, the one-loop Vilkovisky–De Witt effective action for scalar electrodynamics in de Sitter space with torsion has the form:

$$\begin{aligned}\Gamma &= \tfrac{1}{2}\,\mathrm{Sp}\ln \Delta_1(\Lambda+g^2\phi^2)-\mathrm{Sp}\ln \Delta_0(g^2\phi^2)\\ &\quad+\tfrac{1}{2}\,\mathrm{Sp}\ln \Delta_0\left(4\xi_1\Lambda+\xi_2S^2+\frac{\lambda\phi^2}{2}\right)+\tfrac{1}{2}\,\mathrm{Sp}\ln\\ &\quad\times\left[-\Box^2+\left(4\xi_1\Lambda+\xi_2S^2+\frac{\lambda\phi^2}{6}+2g^2\phi^2\right)\Box-g^4\phi^4\right]. \end{aligned}\tag{12}$$

$\tfrac{1}{2}\,\mathrm{Sp}\ln\Delta_s(X)$ has been calculated in Ref. [15]:

$$\tfrac{1}{2}\,\mathrm{Sp}\ln\Delta_s(X) = \tfrac{1}{2}B_4^{(s)}\ln\frac{\Lambda}{3\mu^2}-\tfrac{1}{6}(2s+1)F_s'(b_s),\tag{13}$$

where $s = 0, 1, 2$, $b_s = \frac{9}{4}+s-3X/\Lambda$, $B_4^{(s)}$ is well-known B_4-Schwinger–De Witt coefficient in expansion of corresponding propagator on proper time, $F_s'(b_s)$ is given by Eq. (2.36) of Ref. [15], μ is normalization point. Using Eq. (13) in Eq. (12) one can easily find the explicit expression for different values of theory parameters. We do not make this analysis here.

The second example in minimal SU(5) GUT:

$$\begin{aligned}L &= \tfrac{1}{4}F^i_{\mu\nu}F^{i\mu\nu}+\mathrm{tr}\,[D_\mu\phi D^\mu\phi]-a[\mathrm{tr}\,\phi^2]^2\\ &\quad-2b\,\mathrm{tr}\,\phi^4+\xi_1R\,\mathrm{tr}\,\phi^2+\xi_2S^2\,\mathrm{tr}\,\phi^2.\end{aligned}\tag{14}$$

Here $i = 1, \ldots, 24$, $\phi = \phi^iF^i$, $D_\mu\phi = \nabla_\mu\phi-ig[A_\mu,\phi]$, F^i are the SU(5) generators in fundamental representation; one can show that Landau–De Witt gauge for this theory has the form

$$\begin{aligned}&(\delta A^i_\mu = D^{ij}_\mu\varepsilon^j, \quad \delta\phi^i = gf^{ikj}\phi^k\varepsilon^j)\\ &\nabla_\mu A^{i\mu}+gf^{ikj}\phi^k\varphi^j = 0,\end{aligned}\tag{15}$$

where ϕ^k is a background scalar. Then the same calculation as above gives the Vilkovisky–De Witt effective action (EA) for SU(5) GUT in the following form:

$$\Gamma = \tfrac{1}{2}[\mathrm{Sp}\ln\Delta_1(\delta^{ij}\Lambda+M^{2ij})-2\,\mathrm{Sp}\ln(-\delta^{ij}\Box+M^{2ij})+\mathrm{Sp}\ln K^{mj}].\tag{16}$$

Here $K^{mj} = \left(-\delta^{mj}\Box^2+\left(\left.\frac{\partial^2V_0}{\partial\phi^m\partial\phi^j}\right|_\phi+2M^{2mj}\right)\Box-M^{2ml}M^{2lj}\right)$, $V_0 = -a[\mathrm{tr}\,\phi^2]^2-2b \times \mathrm{tr}\,\phi^4+(4\xi_1\Lambda+\xi_2S^2)\,\mathrm{tr}\,\phi^2$. The Eq. (16) is only in symbolic form, for explicit calculation one must diagonalize the matrices M^{2ij}, K^{mj}.

2.2. In the Sect. 2.1 we considered the quantum fields in external classical gravitational field with torsion. Let us try to consider the question of possibility of Vilkovisky–De Witt EA calculation in quantum gravity with torsion.

Let us represent $g_{\mu\nu}\to g_{\mu\nu}+h_{\mu\nu}$, $S_\mu\to S_\mu+t_\mu$ in Eq. (1), where $h_{\mu\nu}$, t_μ are the quantum fields, $S^2 = S_\mu S^\mu = \mathrm{const.}$ and $g_{\mu\nu}$ is de Sitter space metric. The torsion field is not dynamical one and one can integrate over the field t_μ from the very beginning. Then, we have additional term to bilinear part of Einstein action:

$$\Delta L_{t_\mu} = \frac{1}{2\kappa^2}\left[-\frac{h_1}{2} h h^{\mu\nu} S_\mu S_\nu + \frac{h_1}{4} h^2 S^2\right] \tag{17}$$

Here $h = h^\mu_\mu$.

To calculate Vilkovisky–De Witt EA we must use the Landau–De Witt gauge [15] in the appropriate EA:

$$L_{\mathrm{GF}} = \frac{\gamma}{2\kappa^2}(\nabla^\mu \bar{h}_{\mu\nu} - \tfrac{1}{4}\nabla_\nu h)^2 \tag{18}$$

where after calculations $\gamma \to \infty$ (Landau–De Witt gauge).

Let us write the bilinear part of the Einstein action (1) with account of additional term (17) and Eq. (18)

$$\begin{aligned} L_2 = \frac{1}{2\kappa^2}(&\tfrac{1}{2}\bar{h}^\perp \Delta_2(\tfrac{8}{3}\Lambda - 2\tilde{\Lambda}_0)\bar{h}^\perp + 2(\Lambda - \tilde{\Lambda}_0)\xi^\perp \\ &\times \Delta_1(-\Lambda)\xi^\perp - \tfrac{3}{16}[\sigma \Box(\Box + \tfrac{4}{3}\Lambda)(-\Box + 4\tilde{\Lambda}_0 - 4\Lambda + 3\gamma\Box \\ &+ 4\gamma\Lambda)\sigma + 2(1-\gamma)\sigma\Box(\Box + \tfrac{4}{3}\Lambda)h + h(-\Box - \tfrac{4}{3}\tilde{\Lambda}_0 \\ &+ \tfrac{1}{3}\gamma\Box)h]) + hA_1 h - \frac{h_1}{4\kappa^2}[h^{\perp\mu\nu} S_\mu S_\nu h \\ &+ 2\nabla^\mu \xi^{\nu\perp} S_\mu S_\nu h + h(\nabla^\mu \nabla^\nu \sigma) S_\mu S_\nu - \tfrac{1}{4} S^2 h \Box \sigma]. \end{aligned} \tag{19}$$

Here $\tilde{\Lambda}_0 = \Lambda_0 - \frac{1}{2} h_1 S^2$, $A_1 = \dfrac{h_1 S^2}{16k^2}$ and we used Eqs. (5) to write the Eq. (19).

As we see, two technical problems appear in comparison with the same calculation in quantum gravity without torsion [15]:

a) Mixing between $h^{\perp\mu\nu}$ and h, $\xi^{\nu\perp}$ and h. It means probably, that we must modify the decompositions (5) with account of torsion.

b) Appearance of new kind of non-minimal operators, connected with the terms $S_\nu S_\mu \nabla^\mu \xi^{\nu\perp} h$, $S_\mu S_\nu h \nabla^\mu \nabla^\nu \sigma$. One must develop the technique of calculation of such operators in de Sitter space. It is possible that one can use the method of Ref. [20] of calculation of non-minimal operator divergences for this purpose.

We can find the Vilkovisky–De Witt EA in quantum gravity with torsion only in very restricted unnatural situation, when $S_\mu S_\nu \sim g_{\mu\nu}$. Then the last term in Eq. (19) disapears and in the same way as in Sect. 2.1 we get:

$$\begin{aligned} \Gamma = &\tfrac{1}{2}\,\mathrm{Sp}\ln \Delta_2(\tfrac{8}{3}\Lambda - 2\tilde{\Lambda}_0) - \tfrac{1}{2}\,\mathrm{Sp}\ln \Delta_1(\Lambda -) \\ &- \mathrm{Sp}\ln \Delta_0(-2\Lambda) + \tfrac{1}{2}\,\mathrm{Sp}\ln[\Box^2 + 2(\Lambda + \tilde{\Lambda}_0)\Box + 4\tilde{\Lambda}_0\Lambda + 8\kappa^2 A_1(3\Box + 4\Lambda)]. \end{aligned} \tag{20}$$

If $S^2 = 0$, then Γ is Vilkovisky–De Witt EA for quantum gravity in de Sitter space [15].

3. The Green functions in curved space-time with torsion

In Sect. 2 we discussed the Euclidean EA in de Sitter space with torsion. It is well--known now [1] that EA is closely connected with corresponding Green function. For example, in some cases the knowledge of Green function gives the possibility to find the EA (or EA divergences) [1].

However, the characteristic feature of quantum field theory in curved space-time with torsion is the existence of different Green functions (for simplicity, we consider only free scalar fields φ):

$$G(x, x') = -i_{\text{out}}\langle 0|0\rangle_{\text{in}}^{-1}\ {}_{\text{out}}\langle 0|T\varphi(x)\varphi(x')\,|0\rangle_{\text{in}}, \tag{21}$$

$$\tilde{G}(x, x') = -i_{\text{in}}\langle 0|T\varphi(x)\varphi(x')\,|0\rangle_{\text{in}}. \tag{22}$$

The Green functions $G(x, x')$, $\tilde{G}(x, x')$ satisfy the same equation but different boundary condition [21]. The function $G(x, x')$ is connected with the effective action (discussed above for de Sitter background). There is a convenient integral representation by Schwinger and De Witt for $G(x, x')$:

$$G(x, x') = -i\int_0^\infty ds \exp\left[-is(m^2 - i\varepsilon)\right]\langle xs|x'0\rangle. \tag{23}$$

For $\tilde{G}(x, x')$ such a representation is absent and one must use the definition:

$$\tilde{G}(x, x') = i(\theta(x^0 - x^{0'})\sum_n \varphi_n(x)\varphi_n^*(x') + \theta(x'^0 - x^0)\sum_n \varphi_n(x')\varphi_n^*(x')), \tag{24}$$

where $\varphi_n(x)$ is positive frequency as $x^0 \to -\infty$ (see also [1, 21]). However, it is evident that the Green function $\tilde{G}(x, x')$ is connected with the mean-field EA. In the absence of natural definition of mean-field EA (see [21]) we can discuss the properties of $\tilde{G}(x, x')$. For example, one can assume the existence of a representation for $\tilde{G}$ in the form (23) with the same kernel $\langle xs \mid x'0\rangle$ but with the integral over some contour in a complex s-plane (see, Ref. [22]). We present below a simple example of such representation for Green function $\tilde{G}$ in space with some special metric and torsion (cf. with the Ref. [22]).

We consider a real massless conformally-invariant free scalar field $\varphi(\xi_1 = \frac{1}{6}$, ξ_2 is arbitrary):

$$\left(\frac{1}{\sqrt{-g}}\,\partial_\mu \sqrt{-g}\; g^{\mu\nu}\partial_\nu + \tfrac{1}{6}R + \xi_2 g^{\mu\nu}S_\mu S_\nu\right)\varphi = 0. \tag{25}$$

Assume that the gravitational field is described by a quasi-Euclidean Friedman-Robertson--Walker metric [23]: $ds^2 = (b^2\eta^2 + a^2)(d\eta^2 - d\vec{x}^2)$, $-\infty < \eta < \infty$ and $g^{\mu\nu}S_\mu S_\nu \equiv F^2 = \text{const}$ (it means that $\eta^{\mu\nu}S_\mu S_\nu = (b^2\eta^2 + a^2)F^2$). In this case using the results of Ref. [22] we get

$$\tilde{G}(x, x') = -i\int_0^\infty ds e^{-is(M^2 - i\varepsilon)}\langle xs|x'0\rangle$$
$$+ i\int_{\varepsilon - \frac{\pi i}{2Mb}}^{\infty - \frac{\pi i}{2Mb}} ds e^{-isM^2}\langle xs|x'0\rangle + i\int_{\Gamma_1} ds e^{-isM^2}\langle xs|x'0\rangle. \tag{26}$$

Here

$$\langle xs|x'0\rangle = -i(16\pi^2)^{-1}(2Mbs)^{1/2}(\mathrm{sh}\,2Mbs)^{-1/2}$$

$$\exp\left[iM^2s(1-a^2)-\frac{i(\vec{x}-\vec{x}')^2}{4s}-\frac{iMb}{4}(\mathrm{ch}\,bMs(\eta-\eta')^2 +\mathrm{sh}\,bMs(\eta+\eta')^2)\right].$$

Γ_1 is the following contour: $-\frac{\pi i}{2Mb}+\varepsilon e^{i\varphi}$, $-\pi \leqslant \varphi \leqslant 0$ with the limit $\varepsilon \to 0$ taken after integration, $M^2 \equiv \xi_2 F^2$ plays a role of mass. In the same way we can write $\tilde{G}(x, x')$ for other metrics with some variants of torsion, using the results of calculation of $\tilde{G}(x, x')$ for spaces without torsion (for example, Bianchi spaces or Chitre-Hartle space [22]). However, the unique prescription for calculation $\tilde{G}(x, x')$ in integral form (such as for $G(x, x')$) is absent.

4. Conclusion

In this paper we calculated the one-loop Vilkovisky–De Witt EA for scalar electrodynamics and SU(5) GUT in de Sitter space with torsion. We also discussed the problem of the same calculation for quantum gravity with torsion. We showed that Landau–De Witt gauge does not change in de Sitter space with torsion in comparison with de Sitter space without torsion. This fact is very important to make the calculation simpler. In the last Section we also discussed the existence of different Green functions in curved space-time with torsion.

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