INCOMPRESSIBILITY OF ISOSPIN-ASYMMETRIC NUCLEAR MATTER

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A Hartree-Fock approach with interaction of Skyrme type is used to investigate properties of warm and isospin-asymmetric nuclear matter. The incompressibility turns out to depend sensitively on the neutron excess and temperature. The interaction enhances considerably the proton admixture in beta-stable neutron star matter.

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1. *Introduction*

Thermodynamic properties of nuclear matter are of current interest. In particular the stiffness is at present under serious consideration because it contains essential information on the nuclear equation of state (EOS). The EOS determines the flow properties of colliding heavy nuclei as well as the characteristics of the supernova bounce and the neutron star gross properties. One goal of the theoretic research in this field is the construction of better founded approximations of the EOS and the comparison with experimental data. Of particular interest are the phase transitions in nuclear matter, such as the liquid-gas transition and the superfluid/superconducting state and pion/kaon condensation and the deconfinement.

While heavy ion collisions point to a rather stiff EOS (cf. [1]), the models for prompt supernova II explosions seem to require a soft EOS (cf. [2]). It has been suggested to resolve this apparent puzzle by taking into account not only the momentum dependence of the nuclear forces (cf. [2, 3]) but also the proper isospin dependence of the stiffness at high density (cf. [4]). Both ideas stimulated a considerable number of theoretical studies in this line.

Already in 1981 Blaizot and Haensel [5] calculated the nuclear incompressibility coefficient of isospin-asymmetric nuclear matter in Hartree-Fock approximation by using effective interactions. Treiner et al. [6] tried to extrapolate the experimental data of the

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giant monopole resonance to the incompressibility coefficient of infinite nuclear matter. For a rather long time the work of Blaizot [7] served as standard reference for the nuclear incompressibility $K \cong 210$ MeV. More recently Kolehmainen et al. [8] used the Thomas--Fermi approximation and effective interactions of Skyrme type for the determination of the incompressibility of nuclei in coexistence with a neutron fluid. Lopez-Quelle et al. [9] applied the relativistic Dirac-Hartree-Fock approach to treat isospin-asymmetric nuclear matter. They found results in agreement with nonrelativistic calculations relying on the Skyrme interaction. A critical review on most reliable non-relativistic [10] and relativistic [11] calculations can be found in Ref. [2]. For the completeness' sake we recall also the relativistic mean field approaches [12] and the series of derivations of operative EOS for simulation of the stellar core collapse [13].

In the present work we study the nuclear EOS at finite temperature and arbitrary isospin asymmetry within the Hartree-Fock (HF) approximation with effective interactions of Skyrme type. A similar approach has been employed by Kuo and collaborators [14] and Vinas et al. [4]. The method allows for a consistent determination of the proton and neutron chemical potentials and thus for the evaluation of the neutron excess in beta-stable nuclear matter. By using this approach one can estimate also the isospin dependence of the incompressibility coefficient. We present our results merely in view of the application to the structure and properties of neutron star matter which is, due to the long time scales, in beta equilibrium.

In Chapter 2 we recapitulate the necessary basic formulae for the evaluation of properties of warm isospin-asymmetric nuclear matter. The resulting numerical findings are presented in Chapter 3. The discussion and summary can be found in Chapter 4.

2. *Basic formulae*

In the calculation of the properties of finite nuclei and infinite nuclear matter as well a commonly used method is the HF approximation with effective interactions (cf. [15] and further Refs. therein). These interactions describe also short-range correlations and therefore, represent a kind of phenomenologic G matrix. Often a suitable parametrization is chosen to facilitate explicit calculations; the parameters are fixed by the requirements of reproducing known nuclear matter properties, such as binding energy, saturation density and so on.

We use here the famous Skyrme force in the form [5]

$$\begin{aligned} \langle \boldsymbol{k}, \boldsymbol{k}'|V|\boldsymbol{k}, \boldsymbol{k}'\rangle &= \Omega^{-1}[t_0(1+x_0\delta_{\sigma\sigma'})+\tfrac{1}{4}(t_1+t_2)(\boldsymbol{k}-\boldsymbol{k}')^2 \\ &\quad +\tfrac{1}{6}t_3(1+x_3\delta_{\sigma\sigma'})\varrho^d]. \\ \langle \boldsymbol{k}, \boldsymbol{k}'|V|\boldsymbol{k}', \boldsymbol{k}\rangle &= \Omega^{-1}[t_0+\tfrac{1}{4}(t_1-t_2)(\boldsymbol{k}-\boldsymbol{k}')^2+\tfrac{1}{6}t_3\varrho^d]\delta_{\sigma\sigma'}\delta_{\mathrm{II}} \\ &\quad +\Omega^{-1}[t_0x_0+\tfrac{1}{6}t_3x_3\varrho^d]\delta_{\mathrm{II}}, \end{aligned} \tag{2.1}$$

with the parameter set $t_0 = -1057.3$ MeV fm^3, $t_1 = 235.9$ MeV fm^5, $t_2 = -100$ MeV fm^5, $t_3 = 14463$ MeV fm^{3+3d}, $x_0 = 0.56$, $x_3 = 0$, $d = 1$.

Inserting the matrix elements (2.1) into the definition of the HF single-particle (sp.) energy E, which is determined by the kinetic energy, the chemical potential, μ, and the HF shift Σ^{HF},

$$E(k) = \hbar^2 k^2 (2m)^{-1} - \mu + \Sigma^{\mathrm{HF}}(k), \tag{2.2}$$

$$\Sigma^{\mathrm{HF}}(k) = \sum_{\sigma,\mathrm{I}} \Omega \int d^3\boldsymbol{k}'(2\pi)^{-3} f(E[k'])\,[\langle \boldsymbol{k}, \boldsymbol{k}'|V|\boldsymbol{k}, \boldsymbol{k}'\rangle - \langle \boldsymbol{k}', \boldsymbol{k}|V|\boldsymbol{k}, \boldsymbol{k}'\rangle]$$

one arrives at

$$E_{\mathrm{p}}(k) = \hbar^2 k^2 (2m_{\mathrm{p}})^{-1} - \mu_{\mathrm{p}} \tag{2.3}$$

$$+\int d^3k'(2\pi)^{-3}[t_0(1-x_0)+\tfrac{1}{6}\,t_3(1-x_3)\varrho^d+\tfrac{1}{4}\,(t_1+3t_2)\,(k^2+k'^2)]n_{\mathrm{p}}(k')$$

$$+\int d^3k'(2\pi)^{-3}[t_0(2+x_0(+\tfrac{1}{6}\,t_3(2+x_3)\varrho^d+\tfrac{1}{2}\,(t_1+t_2)\,(k^2+k'^2)]n_{\mathrm{n}}(k')$$

for the protons; for neutrons the replacement $\mathrm{n} \Leftrightarrow \mathrm{p}$ holds. The nucleon distribution functions $n_{\mathrm{I}}(k)$, $I = \mathrm{n, p}$, depend on the temperature T and read

$$n_{\mathrm{I}}(k) = [1+\exp(E_{\mathrm{I}}/T)]^{-1}. \tag{2.4}$$

Since we are interested in isospin-asymmetric nuclear matter, one has to solve the system of four coupled equations for the respective sp. energies and the distribution functions for proton and neutrons self consistently. From the sp. distribution function the particle densities follow as usual

$$\varrho_{\mathrm{I}} = 2\int d^3k(2\pi)^{-3} n_{\mathrm{I}}(k) \tag{2.5}$$

(we consider not too high temperatures; therefore the antiparticle admixtures are negligible). Following Ref. [14] we introduce an effective mass approach in the form

$$E_{\mathrm{I}}(k) = \hbar^2 k^2 (2m_{\mathrm{I}}^*)^{-1} + \varepsilon_{0\mathrm{I}} - \mu_{\mathrm{I}}, \tag{2.6}$$

$$\varepsilon_{0\mathrm{I}} = [t_0(2+x_0)+t_3\,\tfrac{1}{6}\,(2+x_3)\varrho^d]\,\tfrac{1}{2}\,\varrho$$

$$-[t_0(1+2x_0)+t_3\,\tfrac{1}{6}\,(1+2x_3)\varrho^d]\,\tfrac{1}{2}\,\varrho_{\mathrm{I}}$$

$$-3T[(m_{\mathrm{p}}^*)^{-1}(\partial m_{\mathrm{p}}^*/\partial\varrho_{\mathrm{I}})\lambda_{\mathrm{p}}^{-3} f_{5/2}(z_{\mathrm{p}})+(m_{\mathrm{n}}^*)^{-1}(\partial m_{\mathrm{n}}^*/\partial\varrho_{\mathrm{I}})\lambda_n^{-3} f_{5/2}(z_{\mathrm{n}})], \tag{2.7}$$

$$m_{\mathrm{I}}^* = m_{\mathrm{I}}(1+\tfrac{1}{4}\,m_{\mathrm{I}}[2(t_1+t_2)\varrho+(t_2-t_1)\varrho_{\mathrm{I}}])^{-1}, \tag{2.8}$$

$$f_{5/2}(z) = 4\pi^{-1/2}\int dx x^2 \log(1+z\exp[-x^2]),$$

$$z_{\mathrm{I}} = \exp([\mu_{\mathrm{I}}-\varepsilon_{0\mathrm{I}}]/T). \tag{2.9}$$

By inversion of Eq. (2.5) one can explicitly evaluate the $f_{5/2}$ integrals in Eq. (2.7) and thence calculate the chemical potentials of the nucleons.

For our purposes it is useful to have the EOS in the form $p(\varrho, \alpha, T)$, where α describes the neutron excess

$$\alpha = (\varrho_{\mathrm{n}}-\varrho_{\mathrm{p}})\,(\varrho_{\mathrm{n}}+\varrho_{\mathrm{p}})^{-1}, \quad \varrho = \varrho_{\mathrm{n}}+\varrho_{\mathrm{p}}, \tag{2.10}$$

and p is the thermodynamic pressure. To this end we extent a method used by Jaquaman et al. [16] to the generic case of arbitrary values of α in the range 0...1. To do this a virial expansion of the $f_{5/2}$ integrals is exploited, which enables one to use the thermodynamic relation

$$\mu_{\mathrm{I}} = (\partial f/\partial \varrho_{\mathrm{I}})|_{T,\alpha} \tag{2.11}$$

for the explicit determination of the free energy density f. The pressure then follows from (cf. [14] for computational details)

$$p = \varrho_{\mathrm{n}}\mu_{\mathrm{n}} + \varrho_{\mathrm{p}}\mu_{\mathrm{p}} - f \tag{2.12}$$

in the form

$$\begin{aligned} p = {} & \tfrac{1}{8} t_0(3-\alpha^2-2\alpha^2 x_0)\varrho^2 + \tfrac{1}{48} t_3(3-\alpha^2-2\alpha^2 x_3)(1+d)\varrho^{2+d} \\ & +(1+\tfrac{3}{16} m_{\mathrm{p}}^*[3t_1+5t_2-\alpha(t_2-t_1)]\varrho)I_{\mathrm{p}} \\ & +(1+\tfrac{3}{16} m_{\mathrm{n}}^*[3t_1+5t_2+\alpha(t_2-t_1)]\varrho)I_{\mathrm{n}}, \end{aligned} \tag{2.13}$$

where

$$I_{\mathrm{I}} = 2\lambda_{\mathrm{I}}^{-3} f_{5/2}(z_{\mathrm{I}}), \qquad \lambda_{\mathrm{I}} = (2\pi h^2/m_{\mathrm{I}}^* T)^{1/2}.$$

The incompressibility coefficient is defined as

$$K = 9(\partial p/\partial \varrho), \tag{2.14}$$

where again the virial expansion is used for the explicit calculation. A useful characteristics of the stiffness of matter is the adiabatic coefficient Γ

$$\Gamma = (\varrho/p)(\partial p/\partial \varrho). \tag{2.15}$$

3. *Numerical results*

Using the EOS (2.13), the pressure as function of the total baryon density ϱ and the neutron excess α can be easily calculated for various temperatures. In Fig. 1 the isotherms for symmetric nuclear matter are displayed. The curves show the known occurrence of the liquid-gas phase instability (cf. [17]) for more detailed discussions). The critical temperature is in the order of 20 MeV. To get an impression of the isospin dependence in Fig. 2 the isotherm $T = 10$ MeV is displayed for various values of the neutron excess α as function of the density. One observes that the pressure increases with increasing neutronization. This is a consequence of the increasing Fermi energy of the neutrons. Note that increasing values of the neutron excess act as the increase of the temperature [18]: the liquid-gas instability vanishes at sufficiently high neutron excess or temperature or both. This is in accordance with relativistic HF calculations of Weber and Weigel [12]. They have shown that for certain parameters in Hartree approximation (= mean field theory) there is the phase instability in neutron matter which vanishes when including the Fock term. For the given temperature $T = 10$ MeV we find the critical value of the neutron excess $\alpha_c = 0.6$. This example also demonstrates the strong isospin dependence of nuclear matter properties. Therefore, the phase diagram of nuclear matter must be considered in a three dimensional state space, e.g., in the variables T-n-α. The line of critical points ϱ_0 and T_c as function of α and the instability region are considered in more detail in Ref. [18].

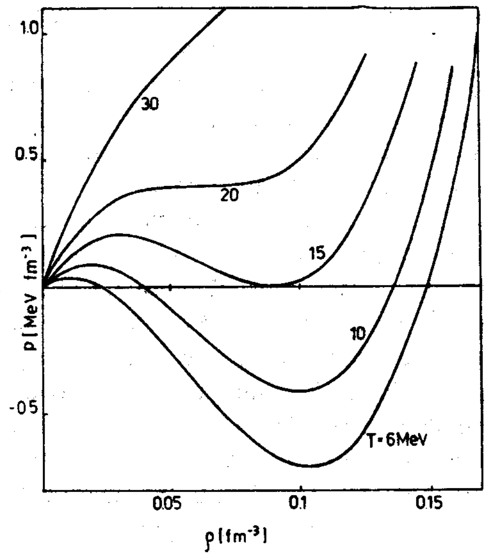

Fig. 1. Isotherms for the pressure as functions of the density ϱ for symmetric nuclear matter

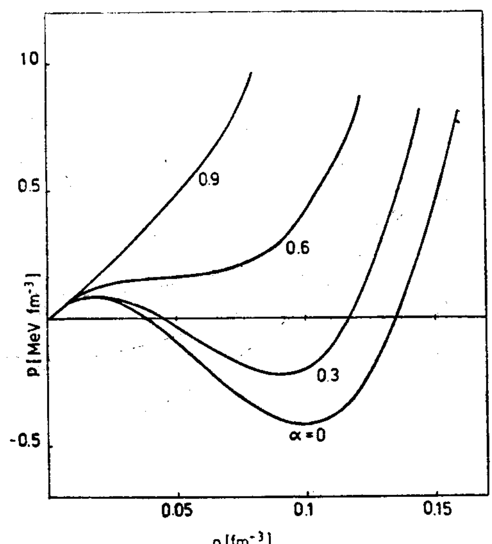

Fig. 2. The pressure as function of the density for isospin-asymmetric nuclear matter at temperature $T = 10$ MeV

The chemical potentials of protons and neutrons as function of the total density for various values of the neutron excess are displayed in Fig. 3 in case of vanishing temperature. Using the expression (2.5) for the chemical potentials one can easily calculate the neutron excess in beta-stable neutron star matter. The relation

$$\mu_n = \mu_p + \mu_e + \mu_\nu \tag{3.1}$$

holds for equilibrium with respect to non-strangeness changing weak interactions. For the electron chemical potential μ_e the usual formula for ideal relativistic electrons is used (we discard here muons). In stable neutron star matter the neutrinos diffused away, i.e., the neutrino chemical potential μ_ν vanishes ($n_\nu = 0$). Electric charge neutrality means equal densities of protons and electrons. In Fig. 4 the resulting neutron excess is displayed as function of the density for various temperatures. At very low densities the ideal gas approximation appears as rather accurate. In comparison with the ideal gas approximation for the nucleons one observes however strong deviations in the region of nuclear matter saturation density. This demonstrates the insanity of the ideal gas approximation. At nuclear matter saturation density the difference is rather large ($\alpha = 0.83$ in contrast to $\alpha = 0.92$ [19] in ideal gas approximation at vanishing temperature). This effect is, of course, related to the liquid-gas instability, and cannot be described in any ideal gas approximation. Increasing temperature results in an overall decrease of the neutron excess. For larger densities (say, above two times the nuclear matter saturation density), the results must be taken with caution, because the Skyrme parametrization becomes too repulsive in this density region. Also the non-relativistic approach is not longer valid. Using the ultra-relativistic ideal gas approximation for nucleons one finds the limiting value

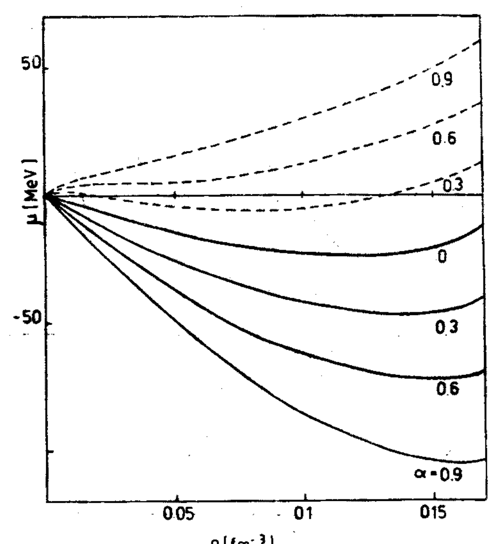

Fig. 3. Neutron (full lines) and proton (dashed lines) chemical potentials as function of the density at vanishing temperature for several values of the neutron excess α. The heavy full line is for symmetric nuclear matter

$\alpha_{\lim} = 0.77$ [19]. Even though the proton concentration in interacting beta-stable neutron star matter is considerably larger than in case of non-interacting (ideal) nucleons, the neutron excess is still too large to allow for the liquid-gas instability (see Figs. 2, 4).

In Fig. 5 the incompressibility coefficient ($K_0 \equiv K(\varrho_0)$ as function of the neutron excess is displayed for different temperatures. The saturation point ϱ_0 is defined by the

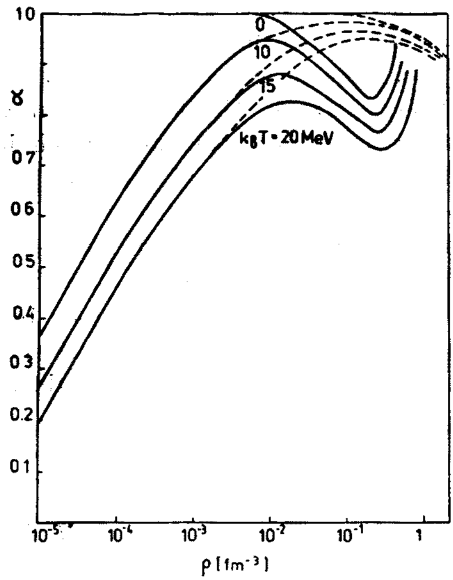

Fig. 4. The neutron excess α as function of the density for beta-stable neutron star matter in HF (full lines) and in ideal gas (dashed lines) approximations for several temperatures

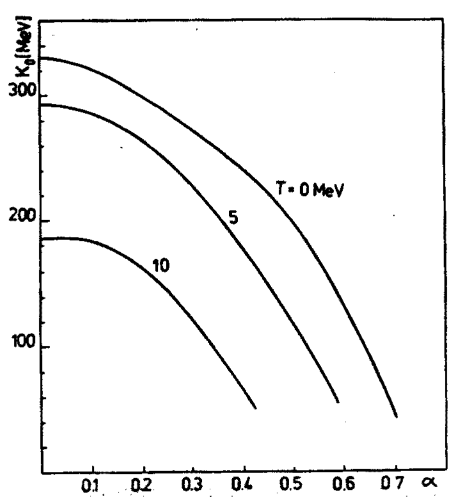

Fig. 5. The incompressibility coefficient K_0 at saturation density as function of the neutron excess α for several temperatures

minimum of the internal energy or $p = 0$; hence ϱ_0 depends on α and T. One observes a drastic decrease with increasing values of α. This is a wanted effect for resolving the apparent discrepancy of the stiff EOS needed for interpreting the flow effects in high--energy nuclear collisions and the needed softness of the EOS in modeling the supernova bounce [2]. Observe also the softening of the EOS with increasing temperature. The Fig. 5 shows that the chosen parameters of the Skyrme force (Eq. (2.1)) result in a rather stiff EOS at vanishing temperature.

In Fig. 6 the adiabatic index Γ as function of the density is displayed for vanishing temperature. For high values of the density, Γ increases rapidly because the short-range repulsive correlations become operative. This behavior is in contrast with the ideal gas approximation of the neutron-proton-electron mixture, which shows a slightly decreasing Γ. The differences between pure neutron matter and beta-stable neutron star matter are rather negligible. That is, the use of the EOS of pure neutron matter may serve as good approximation in neutron star calculations. Note the large difference to the results of the EOS II of Baym, Bethe and Pethick [20], who considered the clustering of nucleons in the region of sub-nuclear densities, not included here. The dip in their EOS is caused by the neutron dripping-out at densities slightly above 3×10^{11} g cm^{-3}. At higher densities (i.e., slightly below nuclear saturation density) the nuclei dissolve, and the results of Ref. [20] and ours

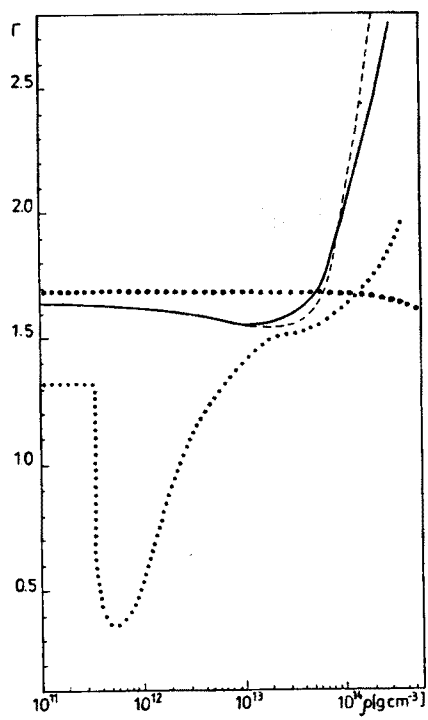

Fig. 6. The adiabatic index Γ as function of the density at vanishing temperature (dashed line: beta-stable neutron star matter in HF approximation, heavy dotted line: mixture of ideal neutron-proton-electron gases, full line: pure neutron matter in HF approximation, dotted line: results of Ref. [20])

are rather similar. Although the EOS of Ref. [20] shows the steep increase of Γ in the supranuclear density region as our EOS, there is some quantitative difference which may be traced back to the too strong repulsive Skyrme force at high density.

4. *Discussion and summary*

We consider here warm isospin-asymmetric nuclear matter within the HF approximation with an effective and density dependent interaction of Skyrme type. Using a suitable representation of the EOS we calculate the chemical potentials of the nucleons, and with this at disposal the neutron excess in beta-stable neutron matter. Therefore, the nuclear incompressibility coefficient as function of density and neutron excess and temperature is accessible. We also consider the adiabatic index at vanishing temperature.

The EOS is affected by the growing neutron excess in two ways: first, the pressure increases, and second, as consequence, there is a critical neutron excess above which the liquid-gas instability does not longer appear. The neutron excess for very dilute beta-stable neutron star matter is found to agree fairly well with the ideal gas predictions (note however, that we do not include here clustering effects). In the density range $10^{-2} < \varrho < 3\times10^{-1}\ \mathrm{fm}^{-3}$ we find a strong influence of the interaction which results in a larger concentration of protons. This is of importance for the suprafluidity/supraconductivity which are considered elsewhere.

In agreement with the findings of other authors we also get a very strong decrease of the incompressibility with increasing neutron excess. This can be considered as support of the idea to resolve the conflict of the stiff EOS as "observed" in relativistic heavy-ion collisions and the need of a soft EOS to run successful supernova models with prompt explosion by bounce-off. Growing temperature also considerably softens the EOS.

The adiabatic index as global measure of the stiffness of the EOS shows in a clear manner the need of including repulsive correlations caused in the high-density region by the hard core of the nucleon-nucleon interaction; at low densities the ideal gas approximation turns out to agree with the results of the HF approximation. Because of the lack of bound states and clusters in the HF approach our EOS differs in the low-density region from them of Ref. [20].

Altogether we state that the HF approximation allows one to parametrize the nuclear EOS in an appropriate way. The parameters of the underlying effective density dependent force are fixed in order to reproduce known nuclear properties. With this at disposal one can extrapolate to beta-stable neutron star matter and fix some important values needed for further investigations. Unfortunately, the present calculations are restricted to densities not being too high (due to non-relativistic treatment) and not being too small (due to the lack of including clustering effects).

In summary we present here an investigation of the nuclear EOS in a HF approach. In agreement with earlier findings we confirm the very sensitive dependence of the incompressibility coefficient on the neutron excess and the temperature. The proton admixture in beta-stable neutron star matter is considerably enhanced by the interaction.

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