ASYMPTOTIC FREEON: AN EXAMPLE OF ELECTROWEAK MAGNETISM*

By J. Ambjørn and P. Olesen

The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

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The classical electroweak theory is unstable for magnetic fields H satisfying $eH > m_W^2$, m_W being the W mass. We discuss the origin of this instability and the peculiar anti-screening property of the W-condensate formed. The anti-screening is closely related to the asymptotic freedom of the $SU(2)_W$-fields, thereby motivating the name "Asymptotic Freeon". For sufficiently large magnetic fields we expect symmetry restoration. This is verified by Bogomol'nyi's method which allows us to reduce the ordinary second order electroweak equations to first order equations in the special case where the Higgs mass m_h is equal to the Z-mass m_Z.

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1. Introduction

QED in principle allows the existence of arbitrary large magnetic fields. This is to be contrasted with the case of electric fields where external electric fields are able to perform work on the virtual electrons and put them on-shell. For a magnetic field we know that the Lorentz force cannot perform any work on charged particles, and the "vacuum" consisting of an external magnetic field is indeed a stable field configuration. The quantum fluctuations of charged scalar and spinor particles will in general try to screen the external field and the resultant effective action for the external field will be non-linear but there will never be a pair production of real particles as is the case if we have strong electric fields.

In this article we will discuss what happens if we couple a spin-one particle to an external magnetic field. We will show that some of the stationary energy eigenvalues in the linearized approximation become imaginary. This implies that large magnetic fields are unstable. As a consequence it is no longer sufficient to consider linear quantum fluctuations around the large constant magnetic field. One has to consider the full non-linear equations for the charged vector particle, which in the real world means the equations coming from the electroweak theory. As we will show the electroweak theory has for such large fields certain similarities with a type II superconductor: One gets a spontaneous break-

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down of translational invariance and the formation of a charged condensate with a periodic structure. However, everything else is turned "upside down" compared to an ordinary type II superconductor: The charged condensate only exists for an average field strength $\bar{f}_{12} > H^{(1)}_{\text{critical}}$ and the condensate currents go in the opposite direction, they *antiscreen*. As we shall show this phenomena is closely related to the asymptotic freedom of the non--Abelian part of the electroweak theory. We show that there exists an upper critical field strength $H^{(2)}_{\text{critical}}$. For $\bar{f}_{12} > H^{(2)}_{\text{critical}}$ we have symmetry restoration in the electroweak theory.

2. *The electroweak transitions*

The Lagrangian of the electroweak theory is (ignoring fermions):

$$\mathscr{L} = -\{\tfrac{1}{2}|\tilde{D}_\mu W_\nu - \tilde{D}_\nu W_\mu|^2 + \tfrac{1}{4} f_{\mu\nu}^2 + \tfrac{1}{4} Z_{\mu\nu}^2 + (\partial_\mu \varphi)^2\}$$

$$-\left\{\frac{g^2\varphi^2}{2} W_\mu^\dagger W_\mu + \tfrac{1}{2}\frac{g^2\varphi^2}{\cos^2\theta}\tfrac{1}{2} Z_\mu^2 - 2\lambda\varphi_0^2\varphi^2\right\} - ig\{(f_{\mu\nu}\sin\theta + \cos\theta Z_{\mu\nu}) W_\mu^\dagger W_\nu\}$$

$$-\{\tfrac{1}{2} g^2((W_\mu^\dagger W_\mu)^2 - W_\mu^2 W_\mu^{\dagger 2}) + \lambda(\varphi^4 + \varphi_0^4)\}, \tag{1}$$

where W_μ and Z_μ are the usual vector boson fields and the covariant derivative is given by:

$$\tilde{D}_\mu = \partial_\mu - ig(A_\mu \sin\theta + Z_\mu \cos\theta). \tag{2}$$

The electromagnetic vector potential is denoted A_μ and the field strength corresponding to electromagnetic currents and neutral currents are

$$f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu. \tag{3}$$

The electromagnetic charge e and the hypercharge g' are related to the $SU(2)_w$ charge g by the standard relations

$$e = g\sin\theta_w, \quad g' = g\tan\theta_w. \tag{4}$$

The Lagrangian (1) is written in the *unitary* gauge where the Higgs field is real.

It is clear that a constant magnetic field (26) is a solution to the classical equations of motion. We simply choose $W_\mu = Z_\mu = 0$, $\varphi = \varphi_0$. More generally we can couple the electromagnetic field to an external electromagnetic current j_μ^{ex} and study the quantum fluctuations of A_μ, W_μ, Z_μ, φ around the classical solution:

$$\partial_\mu f_{\mu\nu}^{\text{ex}} = -j_\nu^{\text{ex}}, \quad \varphi = \varphi_0. \tag{5}$$

We will now argue that $f_{\mu\nu}^{\text{ex}}$ cannot be arbitrary large. Eventually the fluctuations of the charged field W_μ will be so large that the system undergoes a phase transition to a new phase where there is a W_μ (and Z_μ) condensate.

For simplicity we consider the case of a constant magnetic field. The coupling of the W_μ field to A_μ^{ex} is given by

$$\mathscr{L}(W) = -\tfrac{1}{4}(f_{\mu\nu}^{\text{ex}})^2 - \tfrac{1}{2}|D_\mu W_\nu - D_\nu W_\mu|^2 - m_w^2 W_\mu^\dagger W_\mu + ief_{\mu\nu}^{\text{ex}} W_\mu^\dagger W_\nu, \tag{6}$$

where

$$D_\mu = \partial_\mu - ieA_\mu^{ex}, \quad A_\mu(x) = -x_1 H\delta_{\mu 2}, \tag{7}$$

$$m_w^2 = \frac{g^2\varphi_0^2}{2}. \tag{8}$$

The important term in (6) is the "anomalous" magnetic moment term $ief_{\mu\nu}^{ex}W_\mu^\dagger W_\nu$, compared to a minimal coupled theory of charged vector particles. The origin of this term can be traced back to the non-Abelian nature of $SU(2)_{weak}$. The equation of motion corresponding to the linearized Lagrangian (6) is

$$\{(-D^2 g_{\mu\nu} + D_\mu D_\nu) + m_w^2 g_{\mu\nu} - ief_{\mu\nu}^{ex}\}W^\nu = 0 \tag{9}$$

and it follows from (9) that

$$D_\mu W_\mu = 0 \quad \text{for} \quad j_\mu^{ex} = 0. \tag{10}$$

This condition is the natural generalization of the well known relation

$$\partial_\mu W_\mu = 0 \tag{11}$$

valid for a free massive vector particle, and the anomalous term $ief_{\mu\nu}^{ex}W^\nu$ in (9) is necessary in order for (10) to be valid.

The instability of the linearized theory (6) for large magnetic fields is due to the anomalous magnetic moment term. We assume again for simplicity that the electromagnetic field is constant in the $\hat{z}$-direction in space, but the arguments to be given are clearly valid for any field configurations if only the spatial variation is slow compared to the field strength. The assumption implies that f_{12} $(= H)$ is the only field component different from zero and we get an effective mass term

$$(W_1^\dagger, W_2^\dagger)\begin{pmatrix} m_w^2 & ieH \\ -ieH & m_w^2 \end{pmatrix}\begin{pmatrix} W_1 \\ W_2 \end{pmatrix}. \tag{12}$$

The mass eigenvalues are

$$m^2 = m_w^2 \pm eH \tag{13}$$

and it is seen that the lowest mass becomes tachyonic at a critical field strength

$$H_c^{(1)} = m_w^2 \tag{14}$$

and the corresponding eigenvector is

$$(W_1, W_2) = (W, iW) \tag{15}$$

satisfying that the kinetic term in (6) is equal to zero if

$$(D_1 + iD_2)W(x_1, x_2) = 0. \tag{16}$$

The general solution to this equation in the case of a constant magnetic field is

$$W(x_1, x_2) = e^{-\frac{1}{2}eHx_1^2}F(z), \tag{17}$$

$$z = x_1 + ix_2, \tag{18}$$

where F is an arbitrary analytic function. From these considerations it is natural to expect that for external fields larger than or equal to $H_c^{(1)}$ one shall have a W condensate with W_2 and W_1 related by a constant phase $\pi/2$. By analogy with a type II superconductor one would expect $|W(x_1, x_2)|$ to be periodic. It follows that $F(z)$ must be a generalized Jacobi theta function with parameters depending of the unit cell of periodicity in the $x_1 - x_2$ plane (see [1] for details).

The W-condensate differs from the ordinary ψ-condensate of a type II superconductor. The stable solution if $f_{12} = H$, $W = 0$ for low fields and the condensate in only formed for $f_{12} > H_{\text{crit}}^{(1)}$. Contrary for a superconductor where the solution $f_{12} = H$, $\psi = 0$ is stable for $f_{12} > H_{\text{crit}}$ and a ψ-condensate is only formed for the applied field less than H_{crit}. Near H_{crit} we have in both cases that the strength of the condensate is small and it makes sense to study the back reaction of the condensate on the electromagnetic field. We have:

$$\partial_\nu f_{\mu\nu} = -j_\mu^{\text{induced}}, \tag{19}$$

where j_μ^{induced} in the case of a superconductor is the usual convective current

$$j_\mu^{\text{induced}}(\psi) = -ie\psi^\dagger(\vec{D}_\mu - \overleftarrow{D}_\mu^\dagger)\psi. \tag{20}$$

For the electroweak theory the induced current can be derived from the Lagrangian (6) and is given by

$$j_\mu^{\text{induced}}(W_\mu) = j_\mu^{\text{convective}} + j_\mu^{\text{spin}}, \tag{21}$$

where the convective current and the spin current are

$$j_\mu^{\text{conv}} = -ieW_\nu^\dagger(\vec{D}_\mu - \overleftarrow{D}_\mu^\dagger)W_\nu, \tag{22}$$

$$j_\mu^{\text{spin}} = ie(W_\nu^\dagger D_\nu W_\mu - (D_\nu W_\mu)^\dagger W_\nu) - ie\partial_\nu(W_\mu^\dagger W_\nu - W_\mu W_\nu^\dagger). \tag{23}$$

As a consequence of $D_\mu W_\mu = 0$ and the ansatz (15) we get

$$j_\mu^{\text{induced}}(W) = 2ieW^\dagger(\vec{D}_\mu - \overleftarrow{D}_\mu^\dagger)W \tag{24}$$

and we now see a crucial difference between the ordinary ψ-condensate and the W-condensate. It is well known that the ψ-condensate will set up currents to screen the external magnetic field (Lenz law). In the linearized approximation $W = \psi$, and since (24) is identical to (20) *except* for a sign, we conclude the W-condensate set up currents to *enhance* the external magnetic field. We have *anti-screening*.

We can trace the difference to the spin part of the induced current. This part is minus two times the convective current part. It also tells us that this anti-screening is a consequence of asymptotic freedom. The 1-loop vacuum polarization can be calculated from the current-

-current correlation

$$\pi_{\mu\nu}(x-y) = \langle j_\mu^{\text{induced}}(x) j_\nu^{\text{induced}}(y)\rangle \tag{25}$$

and the β-function can be related to the renormalization Z_3 of $\pi_{\mu\nu}$ in the so-called background gauge which is essentially equivalent to (9)–(10). The contributions to β-function from the convective part and from the spin part of the current $j_\mu^{\text{induced}}(x)$ can be explicitly calculated (see [3], [2]) and the negative sign of the β-function is due to the spin-current correlation.

The magnitude of W is not determined by the linear approximation used so far. It is to be expected that the growth in W due to the instability will be stabilized by including the $|W|^4$ terms for the Lagrangian (1). This is what happens in a type II superconductor. The strength of $|\psi|^2$ is eventually determined by including a $\lambda|\psi|^4$ term in the Ginzburg-Landau Lagrangian.

Let us for simplicity still ignore the Z_μ field. In the presence of an electromagnetic field $f_{\mu\nu}$ the "potential" energy involving the Higgs- and the W-field is ($W_\mu = (W, iW, 0, 0)$)

$$\begin{aligned} V(\varphi, W) &= -2ef_{12}|W|^2+g^2\varphi^2|W|^2 \\ &\quad -2\lambda\varphi_0^2\varphi^2+2g^2|W|^4+\lambda(\varphi^4+\varphi_0^4). \end{aligned} \tag{26}$$

We are here ignoring the "kinetic" terms $(\partial_i\varphi)^2$ and $|D_iW_j-D_jW_i|^2$ and the spatial variation of f_{12} in order to present some heuristic arguments for the symmetry restoration for large external fields. If ef_{12} is less than m_w^2 a minimum of $V(\varphi, W)$ is given by $\varphi = \varphi_0$, $W = 0$ and we have no W-condensate. If ef_{12} is above m_w^2 we will get a W-condensate and minimalizing with respect to W gives

$$2g^2|W|_{\max}^2 = ef_{12}-\frac{g^2\varphi^2}{5}\,. \tag{27}$$

As $|W|^2$ increases with ef_{12} the expectation value φ^2 will decrease from φ_0^2 because the term $g^2\varphi^2|W|^2$ will counteract the Higgs term $-2\lambda\varphi_0^2\varphi^2$. One finds

$$\varphi_{\min}^2 = \varphi_0^2\,\frac{m_h^2-ef_{12}}{m_h^2-m_w^2}\,, \tag{28}$$

$$m_h^2 = 4\lambda\varphi_0^2, \quad m_w^2 = \tfrac{1}{2}\,g^2\varphi_0^2. \tag{29}$$

The Higgs- and the W-mass in the ordinary vacuum are denoted m_h and m_w and we see that the expectation value $\langle\varphi^2\rangle$ will approach zero as the average electromagnetic field strength is larger than the Higgs mass *provided* the Higgs mass is larger than the W mass. These heuristic arguments suggest that a W-condensate should exist for

$$H_{\text{crit}}^{(1)} < \bar{f}_{12} < H_{\text{crit}}^{(2)}, \tag{30}$$

where

$$eH_{\text{crit}}^{(1)} = m_w^2, \quad eH_{\text{crit}}^{(2)} = m_h^2 \tag{31}$$

and above $H_{\rm crit}^{(2)}$ the $SU(2)_w \times U_y(1)$ symmetry should be restored although the present simplified arguments do not suggest what happens to the W-condensate above $H_{\rm crit}^{(2)}$.

For $m_h < m_w$ we get by a minimalization of (26) that $\varphi = 0$ as soon as $f_{12} > H_{\rm crit}^{(1)}$ and m_H^{-1} and m_n^{-1} are very much like the coherence length and the penetration length of an ordinary superconductor where $m_h = m_w$ is the borderline between type I ad type II superconductors.

However, the analogy should not be pushed too far.. Contrary to the ψ-condensate in a superconductor the φ field does not couple directly to electromagnetism and it is only through the formation of a W condensate that the symmetry restoration takes place in the electroweak theory. In the next Section we will consider in some detail what happens for $m_h > m_w$ which is presumably the case realized in nature.

3. The condensate solution

We consider again a situation where unspecified extend sources at far away distances produce a homogeneous field H in the $\hat{z}$-direction. For $eH > m_w^2$ we have argued that such a constant field cannot be a stable solution to electroweak field equations. The W-field will develop expectation values in the 1 and 2 direction with $W_1 = iW_2$ $(= iW)$. Making this ansatz and assuming that translational symmetry in the $\hat{z}$-direction is maintained the *static* electroweak energy density of (1) can be written as a sum of squares plus total derivatives:

$$\begin{aligned}\mathscr{E} = {} & |(\tilde{D}_1 + i\tilde{D}_2)W|^2 + \tfrac{1}{2}\left(f_{12} + \frac{g}{2\sin\theta}\varphi_0^2 - 2g\sin\theta|W|^2\right)^2 \\ & + \tfrac{1}{2}\left(Z_{12} - \frac{g}{2\cos\theta}(\varphi^2 - \varphi_0^2) - 2g\cos\theta|W|^2\right)^2 \\ & + \left(\frac{g\varphi}{2\cos\theta}Z_i + \varepsilon_{ij}\partial_i\varphi\right)^2 + \left(\lambda - \frac{g^2}{8\cos^2\theta}\right)(\varphi^2 - \varphi_0^2)^2 - \frac{g^2}{8\sin^2\theta}\varphi_0^4 \\ & + \frac{g\varphi_0^2}{2\sin\theta}f_{12} - \frac{g\varphi_0^2}{2\cos\theta}Z_{12} - \frac{g}{2\cos\theta}\partial_j(\varepsilon_{ij}Z_i\varphi^2).\end{aligned} \tag{32}$$

If we demand

$$\lambda = \frac{g^2}{8\cos^2\theta} \quad \text{or} \quad m_h = m_z \tag{33}$$

the squares in (32) all have positive coefficients and the minimalization of $\int \mathscr{E}dx_1dx_2$ will be obtained if we can solve the *first* order equations [4].

$$(\tilde{D}_1 + i\tilde{D}_2)W = 0, \tag{34}$$

$$f_{12} = \frac{g\varphi_0^2}{2\cos\theta} + 2g\sin|W|^2, \tag{35}$$

$$Z_{12} = \frac{g}{2\cos\theta}(\varphi^2-\varphi_0^2)+2g\cos\theta|W|^2, \tag{36}$$

$$Z_i = -\frac{2\cos\theta}{g}\varepsilon_{ij}\partial_j \ln\varphi. \tag{37}$$

It is seen that the method used is the one of Bogomol'nyi and the relations (33) have a status similar to $e^2 = \lambda/8$, the borderline of type I and type II superconductors, where it is possible to reduce the second order equations of ψ and A_μ to first order equations.

In order to actually solve (34)–(37) it is convenient to eliminate Z_i and A_i in favour of two coupled second order equations in $|W|$ and φ. The phase χ of $W = |W| \exp i\chi$ will have a topological meaning, exactly as the phase of the complex condensate parameter ϕ in a type II superconductor. It determines the number of zeroes of W in a fundamental cell of periodicity. But it can be gauged away by a *singular* gauge transformation:

$$A_i \to A_i + \frac{1}{e}\partial_i\chi. \tag{38}$$

In this singular gauge it is straightforward to derive the following equations for $|W|$ and φ:

$$-\partial^2 \ln|W| = \tfrac{1}{2}g^2\varphi^2+2g^2|W|^2, \tag{39}$$

$$\partial^2 \ln\varphi^2 = \frac{g^2}{2\cos^2\theta}(\varphi^2-\varphi_0^2)+2g^2|W|^2. \tag{40}$$

These equations can be solved numerically imposing the periodic boundary conditions (see [5] for detail). However, assuming that there *is* a solution it is possible to discuss its general structure and the question of symmetry restoration without knowing any details of the solution [6].

If we denote a unit cell of periodicity in the 1–2 plane by $\mathscr{U}$, its boundary by $\partial\mathscr{U}$ and its area $\mathscr{A}$ we have

$$\oint_{\partial\mathscr{U}} A_\mu dx_\mu = \int_{\mathscr{U}} dx_1 dx_2 f_{12} \equiv \Phi_{\mathscr{U}} > 0 \tag{41}$$

and the electromagnetic flux $\Phi_{\mathscr{U}}$ through the unit cell $\mathscr{U}$ will be quantized

$$\Phi_{\mathscr{U}} = \frac{2\pi k}{e}. \tag{42}$$

The flux quantization, which at first sight might be a little surprising, follows in the same way as for a type II superconductor since the line-integral of A_μ around $\partial\mathscr{U}$ can be expressed as the change of the phase χ along this boundary.

By integrating the equations of motion (39) and (40) over $\mathscr{U}$ it follows that

$$2\pi k = \tfrac{1}{2}g^2\int_{\mathscr{U}}\varphi^2+2g^2\int_{\mathscr{U}}|W|^2, \tag{43}$$

$$0 = -m_w^2\mathscr{A}+\tfrac{1}{2}g^2\int_{\mathscr{U}}\varphi^2+2g^2\cos^2\theta\int_{\mathscr{U}}|W|^2. \tag{44}$$

From these two relations we conclude that the area must be bounded by

$$2\pi k/m_{\text{h}}^2 \leqslant \mathscr{A} \leqslant 2\pi k/m_{\text{w}}^2. \tag{45}$$

At the upper limit for $\mathscr{A}$ we find from (35), (42) and (44)

$$\left.\begin{aligned}\langle f_{12}\rangle &= H^{(1)}_{\text{critical}}\\ \langle\varphi^2\rangle &= \varphi_0^2\end{aligned}\right\} \quad \text{for} \quad \mathscr{A} = 2\pi/m_{\text{w}}^2 \tag{46}$$

while the lower limit gives

$$\left.\begin{aligned}\langle f_{12}\rangle &= H^{(2)}_{\text{critical}}\\ \langle\varphi^2\rangle &= 0\end{aligned}\right\} \quad \text{for} \quad \mathscr{A} = 2\pi/m_{\text{h}}^2. \tag{47}$$

We see that (47) exactly agrees with the heuristic limits derived in the last Section, where it was shown that the transition to $\langle\varphi^2\rangle = 0$ when the growth of $g^2|W|^2\varphi^2$ made it comparable to the negative (mass)^2 term $-2\lambda\varphi_0^2\varphi^2$ from the Higgs potential. Here we can calculate the "average coefficients" $\langle g^2|W|^2\rangle$ and $-2\lambda\varphi_0^2$ and we find they are equal at $\mathscr{A} = m_{\text{h}}^2$: *for* $\langle f_{12}\rangle = H^{(2)}_{\text{crit}}$ *on average the Higgs mechanism is cancelled by the coupling between* W *and* φ.

We now turn to a more detailed investigation of symmetry restoration at $\mathscr{A}_{\text{min}} = m_{\text{h}}^2$. It is convenient to rewrite our solution in terms of the variables for $SU(2)_{\text{w}} \times U_{\text{y}}(1)$. If we introduce the vector field B_μ for the group $U_{\text{y}}(1)$ and the non-abelian vector fields A_μ^a for the group $SU(2)_{\text{w}}$ we have

$$B_\mu = A_\mu \cos\theta - Z_\mu \sin\theta, \tag{48}$$

$$A_\mu^3 = A_\mu \sin\theta + Z_\mu \cos\theta, \tag{49}$$

$$A_\mu^1 = \frac{1}{\sqrt{2}}(W_\mu + W_\mu^\dagger), \qquad A_\mu^2 = \frac{i}{\sqrt{2}}(W_\mu - W_\mu^\dagger), \tag{50}$$

and

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{51}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\varepsilon^{abc} A_\mu^a A_\nu^c. \tag{52}$$

From the first order equations of motion (34)–(37) one finds

$$gF_{12}^3 = \tfrac{1}{2}g^2\varphi^2 \tag{53}$$

while all other components of $F_{\mu\nu}^a$ are zero. The result for F_{12}^1 follows in a slightly non--trivial way. *We see that our condensate solution has the property*

$$F_{ij}^a \to 0 \quad \text{for} \quad \varphi \to 0, \tag{54}$$

When $\langle f_{12}\rangle$ is larger than $H^{(2)}_{\text{crit}}$ the only non-vanishing field is thus the boson field $B_{\mu\nu}$ associated with $U_{\text{y}}(1)$. The conclusion is that the solution interpolates sin a self-consistent manner between the broken and the symmetric phase when the magnetic conduction $\langle f_{12}\rangle$ varies between $H^{(1)}_{\text{critical}}$ and $H^{(2)}_{\text{critical}}$.

Is is remarkable that we begin with a "trivial" solution f_{12} = const. and end with a "trivial" solution B_{12} = const., but the condensate solution itself is highly nontrivial.

This is most clearly seen by using the $B_{\mu\nu}$, $F^a_{\mu\nu}$, φ variables and by realizing that $\varphi = \text{const.}$ cannot be a solution to (34)–(37) unless $\varphi = 0$ or $\varphi = \varphi_0$.

4. Discussion

We have shown that the vortex like solutions with W and Z condensates provide us with a classical solution to the field equations which interpolates between the ordinary spontaneously broken symmetric phase and a phase where the $SU(2)_w \times U_y(1)$ symmetry is restored. The condensate has remarkable properties compared to the condensate of an ordinary Type II superconductor. Most notable the currents "anti-screen" a property which motivated us to call the condensate "asymptotic freeon" [7] as this anti-screening was closely related to asymptotic freedom of the non-abelian $SU(2)_w$ part of the electroweak theory.

At this point one can ask whether one can trust classical solutions at such strong fields? Maybe radiative corrections would be large and completely invalidate the above considerations. We can partly answer this question by performing a the one-loop calculation in an external field. The calculations has been done in complete detail for the closely related Georgi-Glashow model where the gauge group is SO(3) and the Higgs fields are in the adjoint representation [see [8]]. The result is

$$\mathscr{E}_{\text{vac}} = \tfrac{1}{2} H^2 \left(1 - \alpha F\left(\frac{eH}{m_w^2}\right) - i\alpha\left(1 - \frac{m_w^2}{eH}\right)\theta\left(1 - \frac{m_w^2}{eH}\right)\right), \tag{55}$$

where $F(x)$ is a regular function for $x \approx 1$. *The smallness of* $\alpha(\approx \frac{1}{137})$ *makes the ordinary radiative corrections small in the region* $m_w^2 \leqslant eH \leqslant m_h^2$. In fact the only serious problem in (55) seems to be the appearance of an imaginary part. This is just the instability we have discussed and the imaginary part tell us that conclusions based on radiative corrections obtained by expanding around this vacuum become physically unreliable. We should clearly, as we have tried to do, find a new (stable) classical solution and expand around that if we want to study quantum corrections.

We have not proven that our condensate solution is stable. A proof would require that we studied the fluctuations around the solution, a difficult task since we only know the solution numerically. However, a simple-minded analysis seems to suggest that the flux tubes could be quite stable and have even macroscopic length [9].

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