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HOW DOES THE GLUON PROPAGATE?*

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In the present paper we would like to emphasize that the terms *gauge invariance* and *independence of the gauge choice* have different meaning and to discuss the natural choice of the gauge in QCD when the rising potential in included. To demonstrate the situation we consider the infrared behaviour of the gluon propagator and the running coupling constant.

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1. *Introduction*

The discovery of the dependence of the β-function on the choice of the gauge [1, 2] again attracts attention to the infrared behaviour of the QCD coupling constant and of the gluon propagator, and in the literature statements arise of the type of *approximate gauge invariance* of physical quantities in this connection [3]. Of course, the gauge non-invariant propagator of gluons is the central problem of this talk; but before trying to answer the question *how does the gluon propagate?* some details of the considered framework must be given.

The trivial fact that the quarks and gluons are not free particles in hadrons is the reason for considering QCD for hadrons as bound states and develop the bound state ideology for gauge theories.

The main difference between the parton QCD and the hadron QCD is the gauge dependence of the bound state physics [4] which radically changes the definition of the perturbation theory [5].

2. *Gauge dependence of the bound state physics*

First, let us recall such notions as "gauge invariance", "choice of a gauge", and "change of a gauge" (for simplicity we used some examples of QED to illustrate the main ideas).

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The gauge invariance of Lagrangian $\mathscr{L}(A)$ means that it does not vary under gauge transformations of the fields $A : \mathscr{L}(A^g) = \mathscr{L}(A)$, where $A^g = g(A+\partial)g^{-1}$.

The choice of the gauge is a specific gauge transformation g^f depending on the field A, so that the new field $A^f[A] = g^f[A]\,(A+\partial)\,[g^f[A]]^{-1}$ satisfies the additional condition

$$f(A^f[A]) = 0. \tag{1}$$

The quantization of the fields and the Feynman rules are always formulated in terms of a certain gauge: $f_1 = 0, f_2 = 0, \ldots$. I would like to draw your attention to some not well known consequences of these definitions.

(*i*) The explicit solution of gauge condition (1) gives the physical variables A^f as a functional on the initial fields A_i [6]. In QED this is the axial field

$$A^{(3)}_\mu[A] = \left(\delta_{\mu\nu} - \partial_\mu \frac{1}{\partial_3}\delta_{3\nu}\right) A_\nu, \quad (A^{(3)}_3[A] = 0),$$

or the transversal field

$$A^{\mathrm{T}}_i[A] = \left(\delta_{ij} - \partial_i \frac{1}{\partial^2}\partial_j\right) A_j, \quad (\partial_i A^{\mathrm{T}}[A] = 0)$$

and so on. These functionals are invariant under gauge transformations of the initial fields in the sense of the equation $\mathscr{L}(A^g) = \mathscr{L}(A)$. So, *any gauge choice is a transition from the initial fields to the gauge invariant physical variables.*

(*ii*) The change of the gauge (from A^{f_1} to A^{f_2}) is fulfilled by the substitution [7]

$$A^{f_2}[A^{f_1}] = V[A^{f_1}]\,(A^{f_1}+\partial)V^{-1}[A^{f_1}]; \quad \psi^{f_2} = V[A^{f_1}]\psi^{f_1}. \tag{2}$$

All Green functions are invariant under operation (2)

$$\langle \psi^{f_2} \ldots \bar\psi^{f_2} \rangle \equiv \langle V[A^{f_1}]\psi^{f_1} \ldots \bar\psi^{f_1} V^{-1}[A^{f_1}] \rangle$$

(if anomalies are absent). This substituton contains not only the modification of the Feynman rules (i.e. the gauge change) but also the spurious diagrams induced by the factor $V[A^{f_1}]$ (which do not follow from the initial Lagrangian).

On the mass-shell these additional diagrams do not contribute, and the invariance under the gauge change takes place. But off the mass-shell the dependence on the gauge takes place and this does not mean the gauge noninvariance (any "gauge" f is gauge invariant as we have seen above).

For example, it is easy to see that the sum of the Coulomb field and transversal photon propagator

$$K^{\mathrm{R}}(J) = J^{(1)}_0 \frac{1}{\boldsymbol{q}^2} J^{(2)}_0 + J^{(1)}_i \left(\delta_{ij} - q_i \frac{1}{\boldsymbol{q}^2} q_j\right) \frac{1}{q_0^2 - \boldsymbol{q}^2} J^{(2)}_j$$

coincides with the Feynman gauge propagator K^F

$$K^R(J) = -[J_0^{(1)}J_0^{(2)} - J_i^{(1)}J_i^{(2)}]\frac{1}{q_0^2-\boldsymbol{q}^2}$$

$$+((q_0J_0^{(1)})(q_0J_0^{(2)}) - (q_iJ_i^{(1)})(q_jJ_j^{(2)}))\frac{1}{\boldsymbol{q}^2(q_0^2-\boldsymbol{q}^2)}$$

$$\equiv K^F(J) + K^L(J)$$

up to the longitudinal term $K^L \sim (q_0J_0^{(1)})(q_0J_0^{(2)}) - (q_iJ_i^{(1)})(q_jJ_j^{(2)})$ that disappears only on the mass-shell (because of the current conservation law $J_0^{(1,2)}q_0 = J_i^{(1,2)}q_i$). But off the mass-shell for the Bethe-Salpeter equation the currents (J) turn into the vertices (Γ) which do not satisfy the conservation law

$$\Gamma_0^{(1,2)}q_0 \neq \Gamma_i^{(1,2)}q_i; \quad K^R(\Gamma) \neq K^F(\Gamma).$$

There are several papers [8, 9] devoted to the proof of gauge independence of an atom spectrum. In these treatments, the Coulomb interaction is used in the rest frame with the choice of the time-axis $\eta_\mu = (1, 0, 0, 0)$. However, all the authors have not taken into account that the vector η_μ (contained in the Coulomb part of the interaction) indeed can be arbitrary, and that a transition from one vector η_μ to another η'_μ ($\eta_\mu'^2 = 1$) is realized by means of a special change of the gauge.

It is easy to check that the usual Lorentz transformation ($P \to P'$) or the special gauge change ($\eta \to \eta'$) break the dispersion law (i.e. $P'^2 \neq M_B^2$) [10]. The dispersion law is invariant only under a combination of the usual Lorentz transformation ($P \to P'$) and a special gauge change ($\eta \to \eta'$) satisfying the parallelism of the time-axis to the total momentum ($\eta_\mu \sim P_\mu$; $\eta'_\mu \sim P'_\mu$). This combined transformation has been pointed out first by Heisenberg and Pauli [11]. Thus, we have seen that the dependence of bound state calculations on a gauge not only exists, but even is necessary to provide the relativistic covariance (unlike the dominating belief that the relativistic covariance of bound states is realized by transition to the covariant gauge [8, 9]). The parallelism of the time-axis and the total momentum is equivalent to the Markov-Yukawa relativistic description of bilocal fields [12]. According to Markov and Yukawa the bound state as a bilocal field $M(x, y)$ is a Lorentz group representation if there is the "redundance" of relative time [13]

$$z_\mu \frac{\partial}{\partial X_\mu} M(x, y) = 0: \quad \left(z_\mu = x_\mu - y_\mu;\, X = \frac{x+y}{2}\right). \tag{3}$$

On the level of atom description in QED the relative time is implicitly identified with the time of quantization of gauge fields, i.e. the temporal component of the gauge field $A_0 = (\eta \cdot A)$ where η_μ is defined as an eigen-vector of the bound state total momentum operator

$$\eta_\mu M(x, y) \sim \frac{\partial}{\partial X_\mu} M(x, y) = P_\mu M(x, y). \tag{4}$$

This means that the Coulomb potential moves with the relativistic atom, the wave function of which satifies the Schrödinger equation

$$\left[\left(\frac{m_1+m_2}{m_1 m_2}\right)(p^{\perp})^2+(M_A-m_1-m_2)\right]\varphi(p^{\perp}) = \int \frac{d^3 q^{\perp}}{2\pi^2} \frac{\alpha}{|p^{\perp}-q^{\perp}|^2}\varphi(q^{\perp}) \tag{5}$$

with respect to the transversal relative momentum

$$q_\mu^{\perp} = q_\mu - \eta_\mu (q\cdot\eta); \quad (P_\mu = M_A \eta_\mu; \eta_\mu^2 = 1). \tag{6}$$

3. *The minimal quantization scheme*

The Feynman rules in the radiative gauge applied to the atom physics and the Heisenberg–Pauli relativistic group can be justified by the minimal quantization scheme of gauge field theories which has been formulated in Ref. [14] (see also [4]) as the following two axioms:

(*i*) The axiom of the choice of physical variables by the projection of the Belinfante energy-momentum tensor

$$T_{\mu\nu} = F_{\mu\lambda}F_{\nu}^{\lambda}+\bar\psi\gamma_\mu[i\partial_\nu+eA_\nu]\psi-g_{\mu\nu}L+\frac{i}{4}\partial_\lambda[\bar\psi\Gamma_{\mu\nu}^{\lambda}\psi],$$

$$\Gamma_{\mu\nu}^{\lambda} = \tfrac{1}{2}[\gamma^{\lambda}\gamma_\mu]\gamma_\nu - g_{\mu\nu}\gamma^{\lambda}-g_{\nu}^{\lambda}\gamma_\mu \tag{7}$$

upon the Gauss equation solution for the temporal component $A_0 = (\eta\cdot A)$: $\partial L/\partial A_0 = 0$.

(*ii*) The axiom of quantization of the minimal set of the physical variables by the diagonalization of the Belinfante Hamiltonian T_{00}.

In *QED* the first axiom expresses the tensor (7) straigtforward in terms of the transversal variables A^T, ψ^T as a nonlocal gauge invariant functional on the initial fields

$$T_{\mu\nu}\left[A_i, A_0 = \left(\frac{1}{\partial^2}\partial_i\partial_0 A_i + j_0\right)\right] = T_{\mu\nu}[A_i^{\mathrm{T}}[A_i], \psi^{\mathrm{T}}[A,\psi]],$$

$$\hat A_i^{\mathrm{T}}[A] = V[A](\hat A_i+\partial_i)V^{-1}[A],$$

$$\psi^{\mathrm{T}}[A,\psi] = V[A]\psi,$$

$$V[A] = \exp\left(\frac{1}{\partial^2}\partial_i \hat A_i\right),$$

$$\hat A = ieA. \tag{8}$$

The usual Lorentz transformation of the initial fields in the Gauss equation leads to the Heisenberg-Pauli transformaton of the transversal functional

$$\psi[A_i+\delta_{\mathrm{L}}^0\psi, \psi+\delta_{\mathrm{L}}^0\psi]-\psi[A_i,\psi] = \delta_{\mathrm{L}}^0\psi^{\mathrm{T}}+ie\Lambda\psi^{\mathrm{T}} \tag{9}$$

where ε_k are the transformation parameters

$$\delta^0_{\rm L} = [\varepsilon_i(x_i\partial_t - t\partial_i) + \varepsilon_k\gamma_0\gamma_k]; \qquad \Lambda = \varepsilon_k \frac{1}{\partial^2}\left[\partial_0 A^{\rm T}_k + \frac{\partial_k}{\partial^2} j_0\right]. \tag{10}$$

The second axiom leads to the same transformation law (9) for the quantum fields

$$i\varepsilon_k[\int dx(T_{00}x_k - T_{0k}t),\, \psi^{\rm T}] = \delta^0_{\rm L}\psi^{\rm T} + ie\Lambda\psi^{\rm T}.$$

In the minimal quantization scheme the relativistic transformation of the classical variables (8) coincides with the quantum ones on the operator level.

This coincidence is the main difference between the minimal quantization and the one in the usual radiative gauge. Another difference is the phase physics due to the infrared zero modes in the exponent of the factor $V[A]$ in Eq. (8).

The same explicit construction of the physical variables for non-Abelian theory [6, 14, 15] leads to the topological degeneration of these phase factors and to a confinement mechanism as a destructive phase interference.

The third difference from the conventional approch is the recognition of the dependence of the bound state physics on the time-axis of quantization η_μ and of the importance of one more empirical bound state principle — the Markov–Yukawa choice of the time-axis (3), (4).

The minimal quantization with the Markov–Yukawa choice of the time-axis η_μ does not change the S-matrix with the asymptotic free states of elementary particles (as this S-matrix does not depend on gauge and η_μ) but these empirical axioms are necessary and really are used in the atom physics independently of the validity of perturbation theory.

4. The gluon propagator in hadron QCD

The minimal quantization of chromodynamics [4, 14] up to the phase (confinement) phenomenon [6, 15] is reduced to the explicit gauge invariant construction of the Schwinger operator quantization of the non-Abelian theory [16] with the Hamiltonian

$$\begin{aligned}\mathscr{H}(g^2D) &= \int dx\,[\tfrac{1}{2}(E^a_i(x))^2 + \tfrac{1}{4}(F^a_{ij}(x))^2 + \bar{q}(x)(i\gamma_k\nabla_k + m^0)q(x)] \\ &+ \tfrac{1}{2}\int dx dy J^a_{\rm tot}(x)\,[g^2 D^{ab}(x-y|A)] J^b_{\rm tot}(y) + \text{nonlocal Schwinger terms}.\end{aligned} \tag{11}$$

Here

$$\nabla_k = \partial_k + gA^a_k\frac{\lambda^a}{2}; \qquad F^a_{ij} = \partial_i A^a_j - \partial_j A^a_i + gf^{abc}A^b_i A^c_j;$$

$$J^a_{\rm tot} = q^+\frac{\lambda^a}{2}q + f^{abc}E^b_i A^c_i; \qquad \partial_i A^a_i = \partial_i E^a_i = 0; \tag{12}$$

g is the coupling constant and the function $D^{ab}(x-y|A)$ satisfies the equation

$$\left[(\nabla_i\partial_i)\frac{1}{\partial^2}(\nabla_j\partial_j)\right]^{ac} D^{cb}(x-y|A) = \delta(x-y)\delta^{ab} \tag{13}$$

(where $\nabla_i^{ab} = \delta^{ab}\partial_i + g f^{abc} A_i^c$). The Schwinger terms are defined from the Lorentz covariance condition [4, 14, 16].

We shall consider the Hamiltonian (11) as a basis for constructing of QCD for hadrons. Just this Hamiltonian (unlike the QED one) contains a new type of the infrared divergences at zero three-dimensional momenta $k^2 = 0$.

The asymptotic freedom formula

$$\alpha(Q^2) = \left[\beta \log\left(\frac{Q^2}{\Lambda^2}\right)\right]^{-1} \tag{14}$$

cannot remove these static divergences and becomes the phenomenological supposition. The removal of these divergences has not only a purely mathematical (theoretical) character. (Recall that in QED the solution of the infrared problem is accompanied by including the phenomenological parameter of the type of the dimension of a device).

One thing is known: these static divergences are related to the modification of the static Coulomb potential at ling-distances (or at $k^2 \sim 0$) and to the physical dimensional transmutation. Instead of the asymptotic freedom phenomenology let us take the form and the parameter of the modification from the experiment: i.e. the heavy quarkonium spectroscopy that definitely points out the rising potential [17]. (This potential can be forced by the nontrivial boundary condition of the Gauss equation [15] like Λ_{QCD} appeared in the boundary condition of the renormalization group equations).

We would like to draw your attention to the fact that the rising potential ansatz

$$\mathscr{H}(g^2 D(\boldsymbol{x}|A)) \to \mathscr{H}(V_{\mathrm{R}}(\boldsymbol{x}) + g^2 D(\boldsymbol{x}|A)) \tag{15}$$

removes all infrared divergences in a perturbation theory in the coupling constant g^2 [5, 18]. This hadron QCD perturbation theory contains in particular the old parton QCD, the nonlocal chiral Lagrangian for light quarks, and the potential model for J/ψ spectroscopy.

We should like to comment on some details of the hadron QCD ($\mathrm{QCD_h}$). We choose as a test potential the oscillator one with the dimension parameter $\sim$ 300 MeV. In the

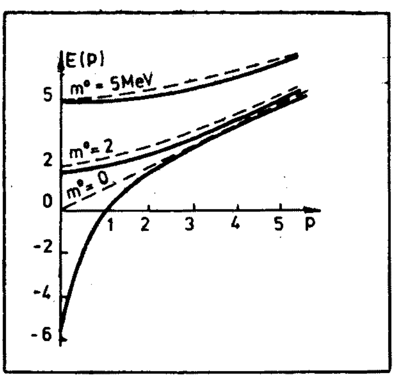

Fig. 1. The solid lines are the numerical solution $E(p)$ to the Schwinger–Dyson equation [5] for different bare quark masses m^0. Here dashed lines described the free case $E_0(p) = \sqrt{p^2+(m^0)^2}$

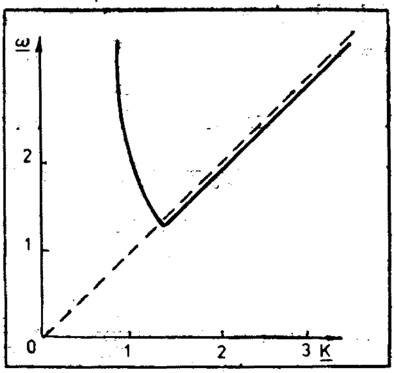

Fig. 2. The solution for the gluon spectrum, where the dashed and solid lines correspond to the free and bounded (in a hadron) gluon, respectively

lowest order in coupling constant the rising potential leads to the constituent masses of light quarks and gluons and does not change the heavy quark masses [18, 19] (see Figs. 1, 2). The QCD_h perturbation theory is formulated in terms of the modified gluon and quark propagators which in the explicit form depend on the total hadron momentum P_μ. For large transversal momenta $|q^\perp| \geqslant 300$ MeV these modified propagators turn into the parton ones of the usual QCD without confinement properties.

Let us illustrate the above remarks only in the gluonic sector and try to give an answer to the question which is the title of our talk, of course, in our understanding of gluons as constituent particles of hadrons.

In the lowest order in g^2, we obtain the following Hamiltonian

$$\mathscr{H} = \int d\boldsymbol{x}\, [\tfrac{1}{2}(E_i^a(\boldsymbol{x}))^2 + \tfrac{1}{2}(\partial_i A_j^a(\boldsymbol{x}))^2]$$
$$+\tfrac{1}{2} f^{abe} f^{ecd} \int d\boldsymbol{x} d\boldsymbol{y} E_i^a(\boldsymbol{x}) A_i^b(\boldsymbol{x}) V_R(\boldsymbol{x}-\boldsymbol{y}) E_j^c(\boldsymbol{y}) A_j^d(\boldsymbol{y}). \tag{16}$$

For simplicity we consider the oscillator potential [10]

$$V_R(\boldsymbol{r}) = V_0 \boldsymbol{r}^2, \quad V_0 = (234\ \text{MeV})^3. \tag{17}$$

The fields E_i^a and A_i^a have the following decomposition over creation and annihilation operators $a_k^{b(\pm)}$

$$E_j^b = i \int \frac{d\boldsymbol{k}}{(2\pi)^{\frac{3}{2}}} \sqrt{\frac{\omega(\boldsymbol{k})}{2}} \{\exp[i(\omega(\boldsymbol{k})t - \boldsymbol{k}\boldsymbol{x})] e_j^r a_r^{b(+)}(\boldsymbol{k})$$
$$- \exp[-i(\omega(\boldsymbol{k})t + \boldsymbol{k}\boldsymbol{x})] e_j^r a_r^{b(-)}(\boldsymbol{k})\},$$
$$A_j^b = \int \frac{d\boldsymbol{k}}{(2\pi)^{\frac{3}{2}}} \sqrt{\frac{1}{2\omega(\boldsymbol{k})}} \{\exp[i(\omega(\boldsymbol{k})t - \boldsymbol{k}\boldsymbol{x})] e_j^r a_r^{b(+)}(\boldsymbol{k})$$
$$+ \exp[-i(\omega(\boldsymbol{k})t + \boldsymbol{k}\boldsymbol{x})] e_j^r a_r^{b(-)}(\boldsymbol{k})\}. \tag{18}$$

Here $k_j e_j^r = 0$, $e_i^r e_j^r = \delta_{ij} - (k_i k_j/k^2)$, $k = |\boldsymbol{k}|$, the operators $a^{(\pm)}$ satisfy the commutator relations $[a_r^{b(-)}(\boldsymbol{k}),\ a_{r'}^{c(+)}(\boldsymbol{q})] = \delta^{bc}\delta_{rr'}\delta(\boldsymbol{k}-\boldsymbol{q})$, $[a_r^{b(\pm)}(\boldsymbol{k}),\ a_{r'}^{c(\pm)}(\boldsymbol{q})] = 0$, and the single-particle energy $\omega(\boldsymbol{k})$ is defined as the matrix element of the Hamiltonian (16) over the one-gluon states $|b, r, \boldsymbol{k}\rangle_g$ with the quantum numbers b, r and momentum $\boldsymbol{k}$

$$_g\langle b', r', \boldsymbol{k}'|H|b, r, \boldsymbol{k}\rangle_g = \omega(\boldsymbol{k})\delta(\boldsymbol{k}-\boldsymbol{k}')\delta^{bb'}\delta^{rr'} \tag{19}$$

with $|b, r, \boldsymbol{k}\rangle_g = a_r^{b(+)}(\boldsymbol{k})\ |0\rangle$. After the substitution of (18) into (16) expression (19) can be rewritten as the following equation for $\omega(\boldsymbol{k})$

$$\frac{\omega(\boldsymbol{k})}{2} + \frac{\boldsymbol{k}^2}{2\omega(\boldsymbol{k})} - V_0 N_c \left\{\left[\frac{1}{2\omega(\boldsymbol{k})}\frac{d\omega(\boldsymbol{k})}{d\boldsymbol{k}}\right]^2 - \frac{1}{k^2}\right\} = \omega(\boldsymbol{k}), \tag{20}$$

where the left-hand side corresponds to three terms of Hamiltonian (16). To obtain the solution of (20) two numerical methods are used: the "Shooting" [20] and the Runge–Kutta–Gill [21] methods Both give similar results (the solution is shown in Fig. 2).

In dimensionless variables the asymptotic behaviour is the following

$$\bar{\omega}(k) \to \frac{2}{\bar{k}^2} \qquad (\bar{k} \to 0);$$

$$\bar{\omega}(k) \to \bar{k} \qquad (\bar{k} \to \infty) \tag{21}$$

with

$$\begin{Bmatrix}\bar{\omega}\\ \bar{k}\end{Bmatrix} = (N_c V_0)^{-\frac{1}{3}} \begin{Bmatrix}\omega\\ k\end{Bmatrix}.$$

Thus the gluons effectively acquire the constituent mass depending on the momentum $(m_g(k^2) = \sqrt{\omega^2(k) - k^2})$ and $m(0) = \infty$.

We see that the rising potential leads to the appearance of a mass for massless color particles, i.e. it has infrared regularizing properties. And it is easy to see that the Green function of the transversal gluon

$$D_{ij}^{\mathrm{mod}}(k_0, \boldsymbol{k}) = \left(\delta_{ij} - \frac{k_i k_j}{\boldsymbol{k}^2}\right)\frac{1}{k_0^2 - \omega^2(\boldsymbol{k}) - i\varepsilon}, \tag{22}$$

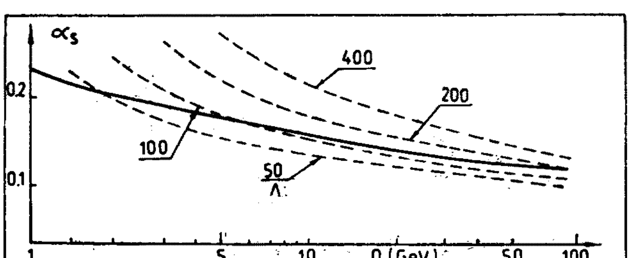

Fig. 3. The dependence of α_s on the momentum Q and parameter Λ. Dashed lines correspond to the asymptotical freedom formula. The solid line corresponds to the modified formula with $\alpha_s^{\mathrm{mod}}(0) = 0.24$ when $N_f = 0$, $N_c = 3$ and $\Lambda = 110$ MeV

vanishes in the region of small k and changes into the standard parton Green function for large momenta k ($k \geqslant 300$ MeV).

The modified gluon propagator also modifies the running coupling constant (see Fig. 3) in the region of small transfer momenta [5, 18]. The new running coupling constant has no singularities in the whole region of transfer momenta and is smaller than $\alpha_s^{\mathrm{mod}} \sim 0.2$. At large momenta it coincides with the asymptotic freedom formula (14). $\mathrm{QCD_h}$ describes the glueball masses in the region expected today [18].

We have shown that for the light quarks the action turns into the chiral Lagrangian in the low-energy limit which is almost independent of the form of the potential [22]. The latter explains the fact of existence of a lot of models of the low-energy hadron physics.

So, the rising potential leads to constituent quark and gluon masses and to the chiral Lagrangian, i.e. to hadronization, but not to confinement [6].

5. *Discussion*

There is a lot of papers, devoted to the gluon propagator behaviour. They contain contradictory results. For example, the $1/k^4$ [23] behaviour is avoided in the axial gauge studies [24]. Moreover, G. B. West has proved that the analyticity properties make it impossible for the $g^{\mu\nu}$ term to be more singular than $1/k^2$ in the infrared region [25]. His conclussion is the following: *this behaviour is obviously not sufficient to infer an area law for the Wilson loop and it is not possible to prove confinement from the IR behaviour of the gluon propagator in the axial gauge* [25].

N. Brown and M. R. Pennington [26] have used a covariant gauge (the Landau gauge) to study the behaviour of the gluon propagator. It was found, the gluon propagator was strongly enhanced like $1/k^4$ at low momenta. The price to get this result is a *plus prescription* not determined by the theory but put in *by hand. The justification here lies in the physically meaningful result* (area law) and *in elimination of infrared divergences* [26].

In this paper we have shown that there exists the unique gauge invariant method of describing gluon and quark propagators in hadrons. With respect to the total momentum of a hadron the gluon interaction is separated in the static (i.e. Coulomb-like) potential and in the quasi-particle transversal excitations (i.e. transversal gluons). We remove the *static* infrared divergences in the *QCD* perturbation theory by the modification of the Coulomb potential in accordance with the experimental data on J/ψ.

This modification of the *non*physical (*non*particle) part of the theory leads to the constituent mass of the transversal gluon. The gluon propagator has the usual analytic properties and moreover, for large transversal momenta it turns into the parton propagator of the standard *QCD* perturbation theory.

In contrast with West's result [27], we have found that the rising potential does not lead to confinement which in our opinion has topological character [6].

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