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THE PUZZLE OF VERY SOFT PHOTON PRODUCTION IN HADRONIC INTERACTIONS*

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We discuss the present status of experimental information on, and theoretical understanding of the production of very soft ($p_T < 30$ MeV/c or so) real and virtual photons in hadronic interactions. We expect that the data will undergo a time evolution towards the shape required by Low theorem and Landau-Pomeranchuk mechanism. Some recently suggested proposals for intermediate stage processes leading to enhancement of very soft photon production are qualitatively analyzed.

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1. Introduction

The production of very soft ($p_T < 30$ MeV/c or so) photons in hadronic reactions is expected to be given by bremsstrahlung radiation off incoming and outgoing hadrons and by the radiation from the intermediate stage of the collision. The bremsstrahlung is a well understood, essentially classical mechanism and the difference between data and bremsstrahlung off external legs can bring interesting information about the intermediate stage formed in collisions, in particular about its space-time dimensions.

The present experimental information is rather controversial. Data on very soft photon production in π^+p interactions at 10.5 GeV/c are consistent with bremsstrahlung from incoming and outgoing hadrons, whereas similar data obtained in K$^+$p interactions at 70 GeV/c and in p-Be and p-Al collisions at 450 GeV/c seem to be of the same shape as expected from bremsstrahlung off initial and final state hadrons, but the magnitude of very soft photon production is by a factor of 2–4 higher.

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This enhancement extending *uniformly* over the very soft region is believed to contradict very general statements: Low theorem and Landau-Pomeranchuk mechanism for suppressing very soft photon production from intermediate stages of the collision.

The purpose of the present paper is to review briefly the data and discuss the nature of the inconsistency of the available data with Low theorem and Landau-Pomeranchuk mechanism. In making so we shall have to distinguish two cases. In bubble chamber experiments the complete information on charged hadrons in the final state is available and the bremsstrahlung from incoming and outgoing hadrons can be calculated accurately. The discrepancy between very soft photon data and bremsstrahlung calculation is very well defined.

In electronic experiments only an incomplete information about the final state is available and the bremsstrahlung contribution has to be calculated by using approximate formulas. In this situation one has to calculate or estimate various possible corrections, mostly due to correlations in the final state, and only after that one can estimate the discrepancy between data and bremsstrahlung contributions.

Because of that the rest of the present paper has the following structure:

2. A brief review of data
3. Bremsstrahlung formula, Low theorem and Landau-Pomeranchuk mechanism
4. Exact and approximate formulas for bremsstrahlung in hadronic collisions
5. Corrections to approximate formulas due to correlations in the final state
   - 5a. Bose-Einstein correlations
   - 5b. $\pi^+\pi^-$ correlations from resonance decays
   - 5c. The increase of $\langle p_T \rangle$ with dN_{ch}/dy
   - 5d. Charge transfer dependence on dN_{ch}/dy
6. Models of intermediate stage contribution to very soft photon production
7. Concluding remarks.

2. *A brief review of data*

In a bubble chamber experiment Goshaw et al. [1] have studied very soft photon production in π^+p interactions at 10.5 GeV/c. The signal remaining after subtracting decay photons has been consistent with expectations based on bremsstrahlung from final state hadrons. The results are shown in Fig. 1. The total number of registred photons has been 840 ± 166 whereas bremsstrahlung calculation has lead to 671.

Bremsstrahlung calculations have been based on the exact formula

$$\omega \frac{d\sigma^{\gamma}}{d^3k} = \frac{\alpha}{4\pi^2} \sum_n \int d^3p_1 d^3p_2 \ldots d^3p_n \sum_{ij} \frac{-Q_i Q_j (p_i \cdot p_j)}{(p_i \cdot k)(p_j \cdot k)} \times \frac{d\sigma_n^{\text{hadr}}}{d^3p_1 \ldots d^3p_n}. \quad (1)$$

Here k denotes photon four-momentum $(\vec{k}, \omega)$, p_i are four-momenta of final and initial state hadrons, Q_i — charges in units of e and initial state hadrons have an additional minus sign in the sum over i, j. We shall come back to Eq. (1) later on.

Qualitatively different data has been obtained in another bubble chamber experiment.

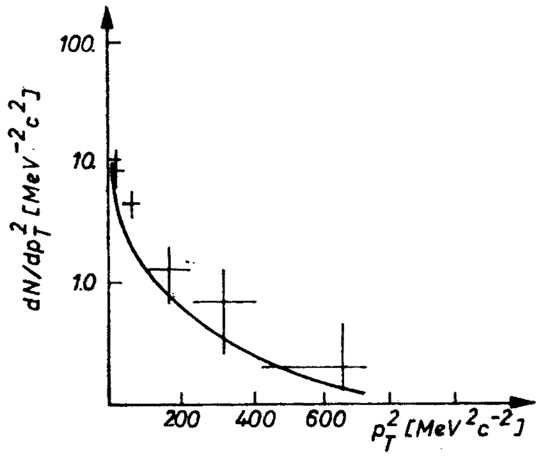

Fig. 1. p_T-dependence of direct very soft photons in π^+p collisions at 10.5 GeV/c [1]. Solid line denotes remsstrahlung calculations

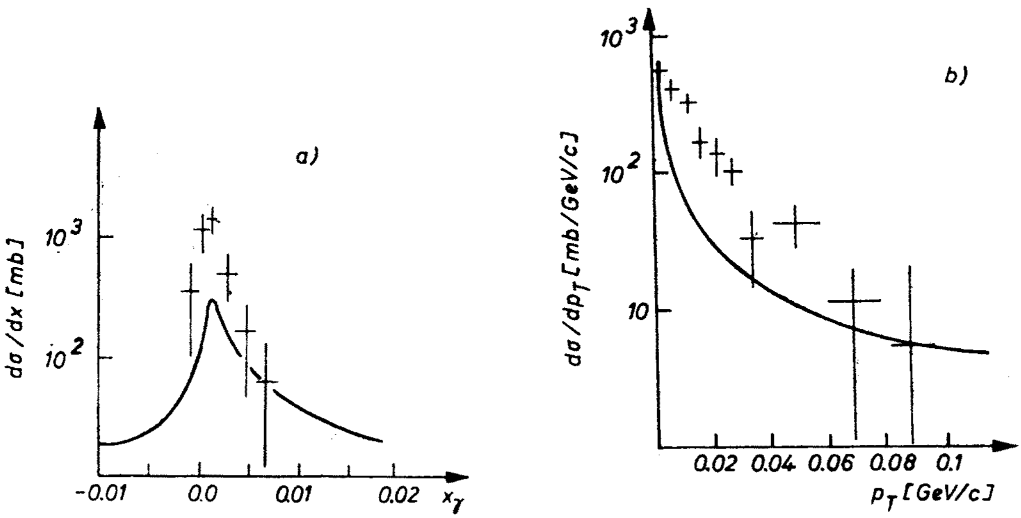

Fig. 2a. x-dependence of direct very soft photons as obtained in Ref. [2]. Solid line gives bremsstrahlung expectations; b. p_T-dependence of direct very soft photons as obtained by Chliapnikov et al. [2] (private communication by Y. Goldschmidt-Clermont to V. Hedberg, see Fig. 68 in [3])

Chliapnikov et al. [2] have studied direct very soft photon production in K^+p interactions at 70 GeV/c. After having subtracted photons coming from all known hadron decays they found signal similar in shape to bremsstrahlung but larger in size by a factor of about three. The results of Chliapnikov et al. are shown in Figs. 2a and 2b.

Most recent data on direct very soft photon production has been obtained in an electronic experiment in p-Be and p-Al interactions at 450 GeV/c by the NA-34 collaboration [4] at CERN. The result on p_T-dependence of direct very soft photons are presented in

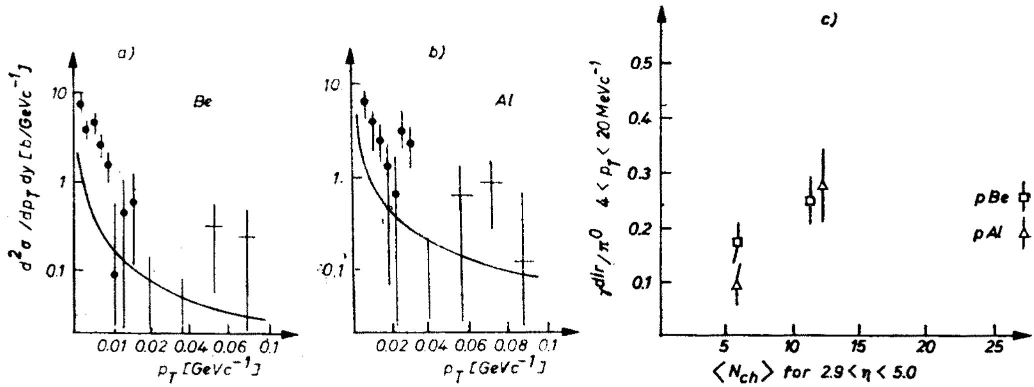

Fig. 3a. p_T-dependence of direct very soft photons at $y \sim 0$ in p-Be collisions at 450 GeV/c. Solid line denotes bremsstrahlung calculations based on the approximate formula Eq. (2); b. p_T-dependence of direct very soft photons at $y \sim 0$ in p-Al collisions at 450 GeV/c. Solid line — bremsstrahlung calculations based on the approximate formula Eq. (2); c. Multiplicity dependence of very soft photons at $y \sim 0$ with $4 \leqslant p_T \leqslant 20$ MeV/c. The symbol N_{ch} denotes multiplicity of charged hadrons with $2.9 < \eta < 5.0$ ($\eta \sim 3.5$ corresponding to c.m.s. rapidity $y \sim 0$)

Figs. 3a and 3b (Fig. 6.3 in Ref. [5]). The NA-34 group have also studied multiplicity dependence of direct very soft photons. It seems that [4, 5] production of these photons increases faster than linear, perhaps quadratically, with the rapidity density of hadrons at rapidities close to that of the photon. Results are shown in Fig. 3c (Ref. [4] and Fig. 5.4 in Ref. [5]).

It has to be stressed, however, that the data by NA-34 collaboration are preliminary and one has still to wait some time for final results. The statistics of the NA-34 data will be also significantly increased in the 1989 run.

In making comparison with bremsstrahlung calculations the NA-34 collaboration have used an approximate formula

$$\omega \frac{d\sigma^\gamma}{d^3k} = \frac{\alpha}{\pi^2 k_T^2}\left[\langle \Delta Q^2\rangle + 0.35\frac{dN_{ch}}{dy} + 0.05\left(\frac{dN_{ch}}{dy}\right)^2\right]\sigma^{hadr}. \qquad (2)$$

Here, k_T is the transverse momentum of photon at $y \sim 0$, $\langle \Delta Q^2\rangle$ is the average charge transfer squared, over $y \sim 0$ and dN_{ch}/dy is the rapidity density of charged hadrons at $y \sim 0$, e.g. at the same rapidity as the one of the photon. We shall come back to this formula later on.

Suppose now, for the moment, that all the data are correct. This, taken at face value, would indicate that at 10 GeV/c very soft photons are produced only by bremsstrahlung off initial and final state hadrons. At higher energies, starting at least with 70 GeV/c there appears some new mechanism, (presumably connected with what happens in the intermediate stage of the collision), which produces very soft photons of about the same shape as the bremsstrahlung but of a size by a factor of about 2–3 larger. In what follows we shall argue that this is very unlikely.

3. Bremsstrahlung formula, Low theorem and Landau-Pomeranchuk mechanism

Bremsstrahlung off incoming and outgoing hadrons is essentially a classical mechanism and it is not surprising that both classical and Feynman diagram-based arguments lead to the same result.

The classical formula for the number of photons emitted by a charged particle moving along trajectory $\vec{r}(t)$ is given as follows [6]

$$dN = \frac{\alpha}{4\pi^2}\left|\int d\vec{r}\times\vec{n}e^{i(\omega t-\vec{k}\cdot\vec{r})}\right|^2\frac{d^3k}{\omega}, \tag{3}$$

where ω, $\vec{k}$ is the photon energy and momentum and the rest of the notation is selfexplanatory. Note that Jackson [6] writes $\vec{n}\times(d\vec{r}\times\vec{n})$ instead of $d\vec{r}\times\vec{n}$ in Eq. (3) but so far as we are not interested in polarizations of photons, both formulae are equivalent.

In a special case when the velocity changes abruptly at $t = 0$ from $\vec{v}$ to $\vec{v}'$ the integral in Eq. (3) can easily be calculated (constributions from $t = \pm\infty$ are neglected because of $\omega \pm i\varepsilon$ conventions) and we obtain

$$dN = \frac{\alpha}{4\pi^2}\left|\frac{\vec{v}'\times\vec{n}}{i\omega(1-\vec{n}\cdot\vec{v}')} - \frac{\vec{v}\times\vec{n}}{i\omega(1-\vec{n}\cdot\vec{v})}\right|^2\frac{d^3k}{\omega}, \tag{4}$$

In fact, this is the whole entry ticket to bremsstrahlung calculations. The former term in Eq. (4) corresponds to an abrupt acceleration of a particle from rest to velocity $\vec{v}'$, the latter term corresponds to an abrupt stopping of a particle with velocity $\vec{v}$. Eq. (4) is easily generalized to the case of scattering of a charged particle on a scattering center, the result being

$$\omega\frac{d\sigma^\gamma}{d^3k} = \sigma^{\text{hard}}\frac{\alpha}{4\pi^2}\left|\frac{\vec{v}'\times\vec{n}}{i\omega(1-\vec{n}\cdot\vec{v}')} - \frac{\vec{v}\times\vec{n}}{i\omega(1-\vec{n}\cdot\vec{v})}\right|^2. \tag{5}$$

A little exercise shows also that Eq. (5) is equivalent to

$$\omega\frac{d\sigma^\gamma}{d^3k} = \sigma^{\text{hadr}}\frac{\alpha}{4\pi^2}(-a_\mu a^\mu), \tag{6a}$$

where

$$a_\mu = \frac{p'_\mu}{p'\cdot k} - \frac{p_\mu}{p\cdot k}, \tag{6b}$$

where p', p are four-momenta of the charged particle after and before the scattering and k is photon four-momentum.

For a multiparticle production the formula Eq. (6) is simply modified by adding to the amplitude a_μ the term $Q_i(p'_i)_\mu/(p'_i\cdot k)$ for any outgoing particle with charge Q_i (in units of the elementary charge e) and the term $-Q_i(p_i)_\mu/(p_i\cdot k)$ for any incoming particle. With this modification Eq. (6) reduces to Eq. (1) given above.

For the sake of completeness the standard derivation of Eq. (6) from Feynman diagrams is sketched in Appendix A.

Low theorem

In a simplified version Low theorem [7, 8] essentially says that at very low photon momentum ($k_\mu \to 0$) the amplitude for photon production is dominated by the bremsstrahlung radiation off incoming and outgoing charged particles.

To see the point let as consider a scattering of a charged particle on a neutral one. The scattering without radiation is shown in Fig. 4. The photon leg can be attached either to external (Figs. 5a, b) or to the internal charged leg (Fig. 5c). When the photon line

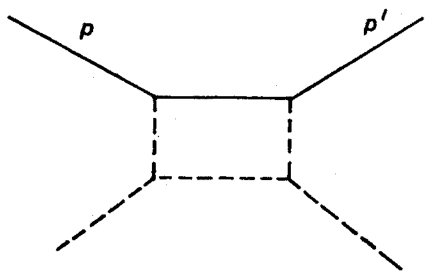

Fig. 4. Scattering of a charged particle (solid line) on a neutral one (dashed line)

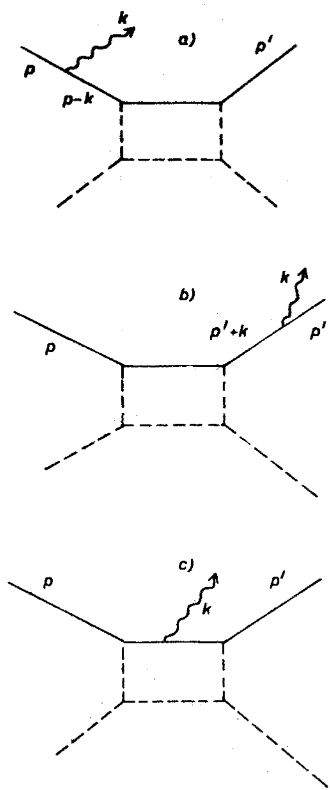

Fig. 5a. Photon line attached to the incoming charged particle; b. Photon leg attached to the outgoing charged particle; c. Photon leg attached to the internal charged particle line

is attached to the incoming line, there appears the charged particle propagator with the denominator $(p-k)^2-m^2 = p^2-m^2-2p\cdot k+k^2$. Since the incoming particle is on the mass shell $p^2-m^2 = 0$ and since the photon is real $k^2 = 0$. Because of that the denominator reduces to $-2p\cdot k$ and the amplitude corresponding to Fig. 5a becomes proportional to $1/p\cdot k$ developing a singularity at $k \to 0$. The same happens with the photon line attached to the external charged particle in Fig. 5b. The virtual charged particle in Fig. 5c is not on the mass shell and the amplitude corresponding to Fig. 5c is therefore not singular for $k \to 0$. For $k \to 0$ the whole amplitude is dominated by diagrams in Fig. 5a and 5b. The singularity is essentially connected with the infinite time of the propagation of a particle which is strictly on the mass shell. This is seen also from the classical expression in Eq. (3). The integral in Eq. (3) develops a singularity for $\omega \to 0$ (see Eq. (4)) only because of an infinite region of integration.

An interesting question, of course, is: at what photon energy ω the contribution of the amplitude in Fig. 5c can be comparable with those in Figs. 5a and 5b. The answer is intuitively obvious: the amplitude in Fig. 5c may be comparable to those in Fig. 5a and 5b for $\omega > 1/\Delta t$ where Δt is the time of the collision.

The statement can be qualitatively substantiated by what is usually referred to as the Landau-Pomeranchuk effect [9] or photon formation time (for a good exposition see e.g. Ref. [10]).

Landau-Pomeranchuk effect

The point is best seen in a case of a double scattering of a classical charged particle shown in Fig. 6. The corresponding amplitude entering Eq. (3) is defined as

$$A = \int d\vec{r} \times \vec{n} e^{i(\omega t - \vec{k}\cdot\vec{r})} \tag{7}$$

and a trivial calculation gives

$$\vec{A} = e^{i\omega(t_1 - \vec{n}\cdot\vec{r}_1)} \frac{\vec{v}_1 \times \vec{n}}{i\omega(1-\vec{n}\cdot\vec{v})} + [e^{i\omega(t_2-t_1)(1-\vec{n}\cdot\vec{v}_2)} - 1] \frac{\vec{v}_2 \times \vec{n}}{i\omega(1-\vec{n}\cdot\vec{v}_2)}$$
$$- e^{i\omega(t_2 - \vec{n}\cdot\vec{r}_2)} \frac{\vec{v}_3 \times \vec{n}}{i\omega(1-\vec{n}\cdot\vec{v}_3)} . \tag{8}$$

Up to phase factors the first and the third term in the r.h.s. correspond to what we find in Eq. (4). The second term represents a contribution due to radiation from the motion of charged particle between $(\vec{r}_1, t_1)$ and $(\vec{r}_2, t_2)$. It is the classical analogon of the photon line attached to the internal leg of a Feynman diagram in Fig. 5c.

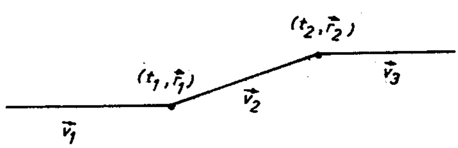

Fig. 6. Double scattering of a classical charged particle

The first and the third term in Eq. (8) have the singularity $1/\omega$ whereas the second term goes to a constant $(t_2-t_1)(\vec{v}_2\times\vec{n})$ for $\omega\to 0$. The second term is comparable with the other ones when the exponent in the square bracket is of the order of 1, that means for $\omega(t_2-t_1)(1-\vec{n}\cdot\vec{v}_2)\sim 1$, that means certainly for $\omega\Delta t\geqslant 1$, where $\Delta t = t_2-t_1$.

Note that the exponent in the second term can be also written as $(t_2-t_1)/t_f$ with $t_f = 1/\omega(1-\vec{n}\cdot\vec{v}_2)$ being the so called formation time of the photon [9, 10]. A little numerical exercise is very instructive.

The following table gives formation times Δt and corresponding frequencies ω_0. For $\omega<\omega_0$ the formation of photons is suppressed.

Δt	1 fm/c	5 fm/c	10 fm/c	20 fm/c	40 fm/c
ω_0	200 MeV	40 MeV	20 MeV	10 MeV	5 MeV

(9)

The Table shows, for instance that in order to produce very soft photons with $\omega\sim 10$ MeV from internal legs in an amount comparable to photons from external legs we need formation times of the order of 20 fm/c or larger.

4. Exact and approximate formulas for bremmstrahlung in hadronic collisions

The exact formula for bremsstrahlung emission in hadronic collision Eq. (1) has been derived in arguments below Eq. (6). In a non-relativistic notation the formula reads as follows

$$\omega\frac{d\sigma^\gamma}{d^3k} = \frac{\alpha}{4\pi^2}\frac{1}{\omega^2}\sum_n\int\frac{d\sigma^{\text{hadr}}}{d^3p_1\dots d^3p_n}d^3p_1\dots d^3p_n$$

$$\times\left|\sum_i Q_i\frac{\vec{v}_i\times\vec{n}}{1-\vec{v}_i\cdot\vec{n}}-\frac{\vec{v}_a\times\vec{n}}{1-\vec{v}_a\cdot\vec{n}}-\frac{\vec{v}_b\times\vec{n}}{1-\vec{v}_b\cdot\vec{n}}\right|^2. \tag{10}$$

Here a and b refer to particles in the initial state and subscript i refers to final state particles.

As pointed out by Cahn [11] and Rückl [12] the exact formula Eq. (12) can be approximated by a more simple expression. Suppose we are interested in production of photons at $y_{\text{c.m.}}\sim 0$. Most of final state particles are moving with velocities $|v_i|\sim 1$ either forward or backward. The photon is moving roughly perpendicularly to these particles so $\vec{v}_i\cdot\vec{n}\sim\vec{v}_a\cdot\vec{n}\sim\vec{v}_b\cdot\vec{n}\sim 0$. Denoting the total charge of forward (backward) moving particles as $Q'_a(Q'_b)$ the term within the symbol of the absolute value in Eq. (10) becomes

$$-\vec{v}_a\times\vec{n}(Q_a-Q'_a)-\vec{v}_b\times\vec{n}(Q_b-Q'_b)$$

since $\vec{v}_a = -\vec{v}_b$ and assuming $Q_a-Q'_a = -(Q_b-Q'_b)$ we have with $|\vec{v}_a|\sim 1$

$$\left(\omega\frac{d\sigma^\gamma}{d^3k}\right)_{\text{FB}}\sim\sigma^{\text{hard}}\frac{\alpha}{4\pi^2}\frac{1}{\omega^2}4\langle\Delta Q^2\rangle. \tag{11}$$

This contribution should include only particles moving with rapidity larger than some reasonable value, say $|y| > y_0 \sim 0.5$ and consequently the average charge transfer $\langle \Delta Q^2 \rangle$ should be understood also as a charge transfer over the interval $(-y_0, y_0)$.

Contribution of different hadrons within $(-y_0, y_0)$ is assumed to be incoherent. One hadron contributes

$$\left(\omega \frac{d\sigma^\gamma}{d^3k}\right)_1 = \frac{\alpha}{4\pi^2} \frac{1}{\omega^2} \sigma_{\text{hadr}} 4R,$$

where

$$4R = \frac{1}{\Delta y} \int\limits_{-y_0}^{y_0} dy \int\limits_0^\infty dp_T \int\limits_0^{2\pi} \frac{d\varphi}{2\pi} P(p_T) \left| \frac{\vec{v}(y, \vec{p}_T) \times \vec{n}}{1 - \vec{v}(y, \vec{p}_T) \cdot \vec{n}} \right|^2 \tag{12}$$

and the total bremsstrahlung becomes

$$\omega \frac{d\sigma^\gamma}{d^3k} = \sigma^{\text{hadr}} \frac{\alpha}{4\pi^2} \frac{1}{\omega^2} \left\{ 4\langle \Delta Q^2 \rangle + 4R \frac{dN_{\text{ch}}}{dy} \Delta y \right\}, \tag{13}$$

where $\Delta y = 2y_0$. In performing the averaging over p_T and φ we have used a phenomenologically motivated distribution

$$P(p_T, \varphi) = A^2 p_T e^{-Ap_T}, \tag{14}$$

with $A = 6\,[\text{GeV}/c]^{-1}$. Typical values of parameters are $\langle \Delta Q^2 \rangle \sim 1$ and $R \sim 0.44$ for $y_0 \sim 0.5$.

In Eq. (13) we have neglected all correlations between final state hadrons. The effects of correlations will be discussed in the next Section.

5. *Corrections to approximate formulas due to correlations in the final state*

Correlations between forward (backward) going particles are irrelevant since only the total forward (backward) charge contributes to $\langle \Delta Q^2 \rangle$. We shall be therefore interested only in correlations between particles within $-y_0 < y < y_0$ which will modify the second term in the r.h.s. of Eq. (13). The exact formula for bremsstrahlung radiation can be written as

$$\omega \frac{d\sigma^\gamma}{d^3k} = \frac{\alpha}{4\pi^2} \sum_n \int d^3p_1 \ldots d^3p_n \frac{d\sigma_n^{\text{hadr}}}{d^3p_1 \ldots d^3p_n} \left| \sum_{i=1}^n Q_i \vec{a}_i \right|^2, \tag{15a}$$

where

$$a_i = \frac{\vec{v}_i \times \vec{n}}{i\omega(1 - \vec{v}_i \cdot \vec{n})}. \tag{15b}$$

The quadratic term in Eq. (15a) in an explicite form gives

$$|\sum Q_i\vec{a}_i|^2 = \sum |\vec{a}_i|^2 + 2\sum_{i<j} Q_iQ_j \,\mathrm{Re}\,(\vec{a}_i\cdot\vec{a}_j^*). \tag{16}$$

If all correlations are neglected the second term vanishes, as can be easily checked. Eq. (16) shows also that only two-particle correlations count.

5.1. Bose-Einstein correlations

Consider a hadronic collision with N_+ positive and N_- negative pions within $-0.5 < y < 0.5$ in the final state. The choice of this interval is not quite arbitrary. Due to its denominator the bremsstrahlung amplitude $\vec{v}\times\vec{n}/[i\omega(1-\vec{n}\cdot\vec{v})]$ is concentrated in a cone and one can show that to photons at $y \sim 0$ only pions within the rapidity interval $-0.5 < y < 0.5$ will give a significant contribution. Bose-Einstein correlations between pions of the same sign can be described by a joint probability distribution

$$P_2(\vec{p}_1, \vec{p}_2) = P(\vec{p}_1)P(\vec{p}_2)C(\vec{p}_1, \vec{p}_2), \tag{17}$$

with $C(\vec{p}_1, \vec{p}_2)$ being given by a phenomenological expression

$$C(\vec{p}_1, \vec{p}_2) = c\left[1 + a\exp\left[-\frac{(\vec{p}_1-\vec{p}_2)^2}{2\sigma^2}\right]\right], \tag{18}$$

with $c \sim 1$, $a \sim 1$ and $\sigma \sim 100$ MeV/c.

The contribution of the second term in Eq. (18) to the bremsstrahlung is given via Eq. (16) and (15) as

$$\omega\frac{d\sigma^{\gamma}_{\mathrm{BE}}}{d^3k} = \frac{\alpha}{4\pi^2\omega^2}\sigma^{\mathrm{hadr}}\left(\tfrac{1}{2}N_+(N_+-1)+\tfrac{1}{2}N_-(N_--1)\right)I_{12}, \tag{19}$$

where

$$I_{12} = ca\int P(\vec{p}_1)P(\vec{p}_2)\exp\left(-\frac{(\vec{p}_1-\vec{p}_2)^2}{2\sigma^2}\right)\frac{\vec{v}_1\times\vec{n}}{1-\vec{v}_1\cdot\vec{n}}\cdot\frac{\vec{v}_2\times\vec{n}}{1-\vec{v}_2\cdot\vec{n}}\,d^3\vec{p}_1 d^3\vec{p}_2. \tag{20}$$

To estimate the r.h.s. in Eq. (19) we first put $N_+ = N_- = N_{\mathrm{ch}}/2$, $c \sim a \sim 1$ and evalute I_{12} in a simplified situation when both pions have $y = 0$ and constant velocity $v = \langle p_{\mathrm{T}}\rangle/\langle E\rangle$. In this situation we have

$$I_{12} = \frac{1}{2\pi}\frac{1}{2\pi}\int_0^{2\pi} d\phi_1\int_0^{2\pi} d\phi_2\,\frac{v^2\sin\phi_1\sin\phi_2}{(1-v\cos\phi_1)(1-v\cos\phi_2)}$$

$$\times\exp\left[-2(1-\cos(\phi_1-\phi_2))/\Delta^2\right], \tag{21}$$

where $\Delta = \sqrt{2}\sigma/p \sim \sqrt{2}\cdot 100\ \mathrm{MeV}/400\ \mathrm{MeV} \sim 0.35$.

A simple numerical integration then gives $I_{12} \sim 0.17$ which leads to

$$\omega \frac{d\sigma^{\gamma}_{BE}}{d^3k} = \frac{\alpha}{4\pi^2\omega^2}\sigma^{hadr}\,\tfrac{1}{4}\,N_{ch}(N_{ch}-2)0.17. \tag{22}$$

The comparison with Eq. (13) shows that

$$\frac{\text{bremsstrahlung enhancement due to BE}}{\text{bremsstrahlung without BE}} \sim 0.1\,\frac{N_{ch}}{4}. \tag{23}$$

The enhancement due to Bose-Einstein correlations depends quadratically on $N_{ch} = dN_{ch}/dy$ but it cannot explain the enhancement of bremsstrahlung by a factor of about three. For instance, for rather large rapidity density of $N_{ch} \sim 6$ the Bose-Einstein correlations will give an increase of less than 15%.

5.2. $\pi^+\pi^-$ correlations from resonance decays

We shall show here that back-to-back $\pi^+\pi^-$ correlations expected from ϱ^0 decay lead to the enhancement of bremsstrahlung relative to the case of uncorrelated $\pi^+\pi^-$. For a rough estimate consider a simplified situation. Suppose that we are observing photon at $y = 0$ in the direction perpendicular to the beam. The cross section for bremsstrahlung by π^+ and π^- both at $y = 0$ and produced with the same velocity $v = \langle p_T\rangle/(\langle p_T\rangle^2+m_\pi^2)^{1/2}$ is

$$\omega \frac{d\sigma^{\gamma}}{d^3k} = \sigma^{hadr}\frac{\alpha}{4\pi^2}\frac{1}{\omega^2}\left|\frac{\vec{v}_+\times\vec{n}}{1-\vec{v}_+\cdot\vec{n}} - \frac{\vec{v}_-\times\vec{n}}{1-\vec{v}_-\cdot\vec{n}}\right|^2. \tag{24}$$

If $\pi^+\pi^-$ are uncorrelated the cross-section should be averaged over directions of pions in the plane perpendicular to the beam axis. In this way we obtain

$$\omega \frac{d\sigma^{\gamma}}{d^3k} = \sigma^{hadr}\frac{\alpha}{4\pi^2}\frac{v^2}{\omega^2}I_{no\text{-}corr},$$

$$I_{no\text{-}corr} = \frac{1}{(2\pi)^2}\int d\phi d\phi'\left|\frac{\sin\phi}{1-v\cos\phi} - \frac{\sin\phi'}{1-v\cos\phi'}\right|^2. \tag{25}$$

Simple calculation gives $I_{no\text{-}corr} = (2/v^2)(\gamma-1)$. For the case of back-to-back correlation the cross-section is again given by Eq. (25) with $I_{no\text{-}corr}$ replaced by

$$I_{corr} = \frac{1}{2\pi}\int_0^{2\pi} d\phi\left[\frac{\sin\phi}{1-v\cos\phi} - \frac{\sin(\phi+\pi)}{1-v\cos(\phi+\pi)}\right]^2. \tag{26}$$

The calculation gives $I_{corr} = 2\gamma$. The ratio $R = I_{corr}/I_{no\text{-}corr} = (\gamma+1)/\gamma$ depends on a single parameter $\langle p_T\rangle$. This dependence is shown in Fig. 7. For realistic values of $\langle p_T\rangle \sim 0.4$ GeV the ratio is about 1.3. Note that our simplified model probably overestimates the effect and one can expect 10–20% of bremsstrahlung enhancement due to back-to-back correlations due to resonance decays. A quantitative estimate of the effect can be obtained only by Monte Carlo simulations of multiparticle production including resonance decays.

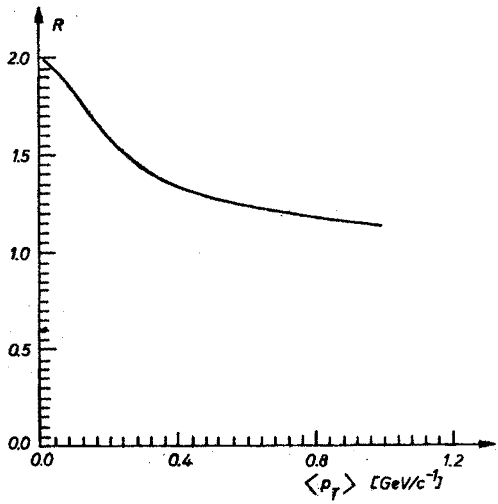

Fig. 7. The dependence of the ratio $R = I_{\mathrm{corr}}/I_{\mathrm{no\text{-}corr}}$ on the mean transverse momentum $\langle p_{\mathrm{T}} \rangle$

5.3. The increase of $\langle p_{\mathrm{T}} \rangle$ with dN_{ch}/dy

The amount of bremsstrahlung by a particle accelerated abruptly to velocity v increases with v. Because of that an increase of $\langle p_{\mathrm{T}} \rangle$ of final state hadrons leads to an increase in bremsstrahlung emission. The data [13] give evidence of the increase of $\langle p_{\mathrm{T}} \rangle$ with charged multiplicity. For the sake of simplicity we shall roughly estimate the effect by considering radiation emitted at $y = 0$ by a charged pion at $y = 0$. The bremsstrahlung emission is given as

$$\omega \frac{dN}{d^3k} = \frac{\alpha}{4\pi^2\omega^2} \left\langle \left| \frac{\vec{v} \times \vec{n}}{1 - \vec{v} \cdot \vec{n}} \right|^2 \right\rangle = \frac{\alpha}{4\pi^2\omega^2} I(\langle p_{\mathrm{T}} \rangle), \tag{27}$$

with

$$I(\langle p_{\mathrm{T}} \rangle) = \int_0^\infty A^2 p_{\mathrm{T}} e^{-Ap_{\mathrm{T}}} dp_{\mathrm{T}} \int_0^\pi \frac{d\phi}{2\pi} \frac{v^2(p_{\mathrm{T}}) \sin^2 \phi}{[1 - v(p_{\mathrm{T}}) \cos \phi]^2}, \tag{28}$$

where $A = 2/\langle p_{\mathrm{T}} \rangle$ and $v = p_{\mathrm{T}}/(p_{\mathrm{T}}^2 + m_\pi^2)^{1/2}$.

In Fig. 8 we present the dependence of $I(\langle p_{\mathrm{T}} \rangle)$ on $\langle p_{\mathrm{T}} \rangle$. The shape of $I(\langle p_{\mathrm{T}} \rangle)$ is practically linear. The cross-section for bremsstrahlung emission

$$\omega \frac{d\sigma^\gamma}{d^3k} = \frac{\alpha}{4\pi^2\omega^2} \sigma^{\mathrm{hadr}} \Delta y \frac{dN_{\mathrm{ch}}}{dy} I(\langle p_{\mathrm{T}} \rangle) \tag{29}$$

increases somewhat faster than linearly with dN_{ch}/dy but at 70 GeV/c and 450 GeV/c the increase is rather small, the effect being larger only at the highest ISR and Sp$\bar{p}$S energies.

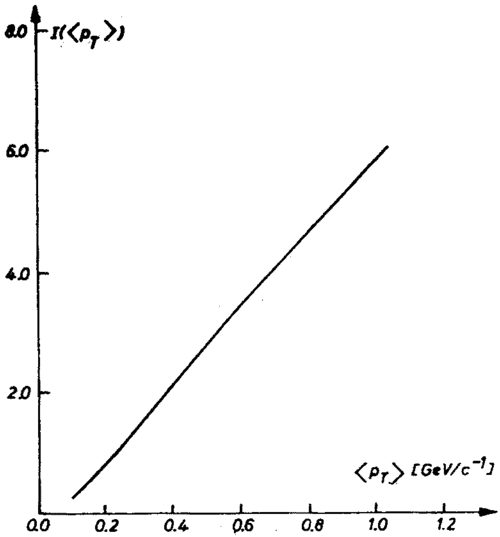

Fig. 8. Dependence of bremsstrahlung emission on $\langle p_T \rangle$ of radiating pions at $y = 0$

5.4. Charge transfer dependence on dN_{ch}/dy

The Cahn–Rückl formula Eq. (13) contains as an essential parameter $\langle \Delta Q^2 \rangle$. Charge transfer across $y_{c.m.} = 0$ has been studied in bubble chamber experiments and analyzed in cluster and parton models. The data show that at fixed s, $\langle \Delta Q^2 \rangle$ increases linearly [14] with total charged multiplicity. The data [14, 15, 16, 17] seem to suggest a linear increase of $\langle \Delta Q^2 \rangle$ with $(dN_{ch}/dy)_{y=0}$ and the same conclusion seems to follow also from neutral cluster [18, 19, 20] and parton models [21, 22]. It seems therefore that the term with $\langle \Delta Q^2 \rangle$ in Eq. (13) leads to linear dependence of bremsstrahlung emission on $(dN_{ch}/dy)_{y=0}$ but the issue deserves more detailed experimental and theoretical study.

The discussion of effects of correlations on bremsstrahlung emission can be summarized rather simply: the effects can increase the bremsstrahlung but not more than by about 50% and much faster than linear dependence (like quadratic), cannot be obtained in this way.

6. Models of intermediate stage contribution to very soft photon production

In this Section we shall consider a few models trying to obtain an increase of very soft photon production.

Shock-wave-like mechanism

The term shock-wave-like will be understood here in a broad sense as any mechanism in which the transverse velocity of final state particles is rapidly changed during the space-time evolution of the hadronic collision. As a typical situation we shall consider the following one. Suppose that when final state hadrons are created some collective mechanism gives to all of them an additional collective velocity v_c (the velocity of the shock-wave) in addition to their individual velocities. The collective mechanism stops at time t_1 and after that time hadrons have the velocity distribution observed in the final state. Time dependence of the velocity of a hadron becomes

$$v(t) = v_2 \quad \text{for} \quad t > t_1,$$

$$v(t) = v_1 = \frac{v_2+v_c}{1+v_2v_c} \quad \text{for} \quad t_1 > t > 0.$$

For simplicity we consider only hadron with $y = 0$.

For the number of emitted photons we obtain from Eq. (3)

$$dN = \frac{\alpha}{4\pi^2}|A_1+A_2|^2\frac{d^3k}{\omega}, \tag{31}$$

where

$$A_1 = \frac{\vec{v}_1\times\vec{n}}{i\omega(1-\vec{n}\cdot\vec{v}_1)}[e^{i\omega t_1(1-\vec{n}\cdot\vec{v}_1)}-1],$$

$$A_2 = -\frac{\vec{v}_2\times\vec{n}}{i\omega(1-\vec{n}\cdot\vec{v}_2)}[e^{i\omega t_1(1-\vec{n}\cdot\vec{v}_1)}], \tag{32}$$

when no shock-wave is present the amplitude corresponding to A_1 vanishes and in A_2 we have to put $t_1 = 0$.

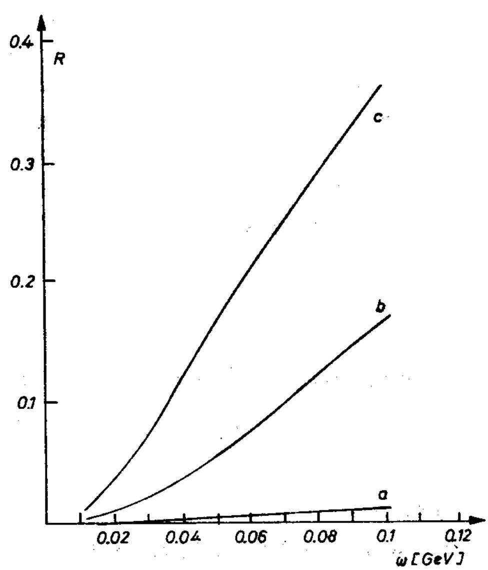

Fig. 9. The ratio of the additional bremsstrahlung during the shock-wave period to the bremsstrahlung emitted when no shock-wave is present. The three curves correspond to shock-wave duration of a — 1 fm/c, b — 5 fm/c and c — 10 fm/c. The ratio is plotted as a function of photon energy ω. Note the scale on the vertical axis

As before we average over p_T and ϕ by using $P(p_T, \phi)$ as given in Eq. (14). For definiteness we put $v_c = \sqrt{1/3}$.

In Fig. 9 we plot the ratio of the additional bremsstrahlung in the presence of a shock-wave to the bremsstrahlung emitted without the shock-wave.

The additional bremsstrahlung is rather small below $\omega = 20$ MeV/c even for shock-wave duration of 20 fm/c, which is completely unrealistic for hadron-hadron collision. The reason is obvious: it is the effect of the photon formation time discussed below Eq. (8).

Bremsstrahlung off a quark trying to escape from the intermediate system

The model is based on Refs. [23, 24]. It is assumed that during the hadronic, hadron-nucleus or nuclear collision an intermediate partonic system is formed. A quark is then trying to escape from the system but it is pulled back by chromoelectric flux tube. The situation is shown in Fig. 10. The bremsstrahlung emitted by the escaping quark depends

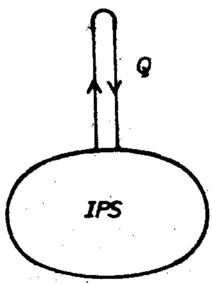

Fig. 10. Quark Q is escaping from the Intermediate Partonic System (IPS) and pulled back by the chromoelectric flux tube

on the time spent by the quark outside of the system and this, in turn, depends on the energy of the quark. Detailed calculations [25] show that the situation is very similar to the previous model. Even for a quark escaping with energy of 10 GeV, the emitted radiation is by a factor of ten smaller at photon energy of 20 MeV. Any realistic distribution of energies of escaping quarks leads to very soft photon spectrum of a shape substantially different from that of bremsstrahlung.

Van Hove's Cold Quark-Gluon Plasma

On the basis of studies of multiparticle production by a QCD showering mechanism [26, 27] Van Hove [28] has suggested that an intermediate parton system composed of Cold Quark-Gluon Plasma (CQGP) may be formed in hadronic collisions. The time-scale of the system is expected to be $\Delta t \sim 7$ fm/c. Characteristic rest-frame momenta of partons in a glob of CQGP may be as low as 20–30 MeV/c.

Van Hove's CQGP would emit very soft photons either by bremsstrahlung due to current fluctuations or in processes like $Q+\overline{Q} \to \gamma+g$ or $g+Q \to \gamma+Q$. In both cases one expects suppression of very soft photons below $\omega_0 \sim 20$ MeV. In the case of bremsstrahlung this is due to the finite formation time of photon in the case of partonic subprocesses the maximum of photon emission is expected to be around the average parton momentum of 20–30 MeV.

A very nice feature of Van Hove's scenario is a possibility to understand in a single picture various very soft phenomena, like intermittency fluctuations, peaks in inclusive hadron spectra at $p_T \to 0$ and peaks at low $|\vec{p}_1 - \vec{p}_2|$ in correlations of identical pions. The quantitative predictions for very soft photon production from Van Hove's model are not yet available.

Shuryak's dense pion gas [29]

A dense pion gas formed in heavy ion collisions and debatably also in K^+p, p-Be and p-Al collision would also radiate very soft photons either by current fluctuations or by $\pi^+\pi^-$ annihilations. It is however difficult to imagine that the spectrum would have the same shape as that of bremsstrahlung, the problems being similar to but still amplified in comparison with CQGP.

Charge oscillations in the intermediate partonic system

Bremsstrahlung production in a large system of oscillating charges is given by a straight-forward generalization of Eq. (3)

$$\omega \frac{dN^\gamma}{d^3k} = \frac{\alpha}{4\pi^2} \left| \int \vec{n} \times \frac{1}{e} \vec{j}(\vec{r}, t) e^{i(\omega t - \vec{k}\cdot\vec{r})} d^3\vec{r} dt \right|^2 . \quad (33)$$

The oscillations in a pre-equilibrium QGP has been studied by Białas and Czyż [30] and by Białas and Blaizot [31] and suggested as a possible source of bremsstrahlung enhancement by Emelyanov [32]. No detailed calculations are so far available and it is not at all clear how one can obtain very soft photon spectrum of a similar shape as the bremsstrahlung.

7. *Concluding remarks*

As seen from various attempts discussed above it is very difficult and probably impossible to obtain very soft photon spectrum of the bremsstrahlung shape from the intermediate stage mechanism (in a specific model this has been already pointed out by the Lund group [33]). The whole problem is clearly seen from the numbers in Eq. (9). In order to enhance very soft photon production at $\omega \sim 20$ MeV an intermediate stage extending in time over $\Delta t = 10$ fm/c is required and for $\omega \sim 10$ MeV one needs $\Delta t = 20$ fm/c what is certainly not realistic in hadronic or hadron-nucleus collisions.

An acceptable scenario would therefore require something of the form shown in Fig. 11. Very soft photon spectrum should be roughly consistent with bremsstrahlung expectations for $\omega < \omega_0$, with $\omega_0 > 20$ MeV and rising above these expectations for $\omega > \omega_0$. The value of ω_0 would thus give a valuable information about the size of the intermediate system.

It is also very difficult to obtain much faster than linear dependence of very soft photon production with dN_{ch}/dy. One would rather expect a linear dependence plus a small admixture of a quadratic one.

To sum up: there are essentially two possibilities for the evolution of the field in the

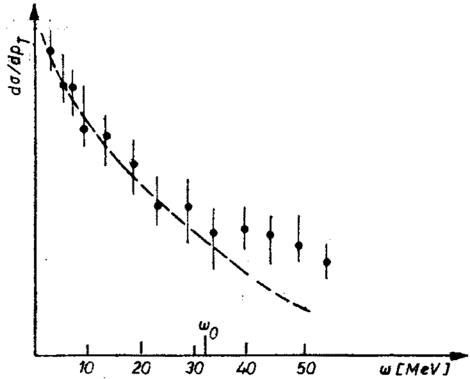

Fig. 11. Expected shape of very soft photon spectrum in hadronic collisions, dashed line — expectations from bremsstrahlung, ω_0 — photon energy at which significant deviations from bremsstrahlung calculations begin to be visible

near future. In the former one, the present indications given by the data (partly preliminary) will be confirmed and very soft photon production will become a real conundrum with data running against all expectations based on Low theorem and the concept of the formation time. Since these expectations are rooted rather deeply in quantum mechanics — in fact not very far from the principle of indeterminacy — this perspective is not very attractive. In the latter one more accurate data, to be obtained soon, will be consistent with bremsstrahlung expectations for $\omega \to 0$ and the deviations at larger ω will bring very interesting information.

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APPENDIX A

We shall sketch here, for the sake of completeness, the standard derivation [34] of the bremsstrahlung formula from Feynman diagrams and then present the covariant expression for bremsstrahlung in multiparticle production. The amplitude of Feynman diagram in Fig. 5a is (notation of Ref. [34])

$$M_a = e\sqrt{4\pi} \quad \bar{u}(p')M^{(0)} \frac{\hat{p}-\hat{k}+m}{(p-k)^2-m^2} \hat{\varepsilon}^* u(p).$$

Using $\hat{p}\hat{\varepsilon}^* = 2p\varepsilon^* - \hat{\varepsilon}^*\hat{p}$ and $\hat{p}u(p) = mu(p)$ and neglecting $\hat{k}$ in the numerator we obtain

$$M_a = -e\sqrt{4\pi}\, \bar{u}(p')M^{(0)} \frac{p\cdot\varepsilon^*}{(p\cdot k)} u(p).$$

The notation used is standard and $M^{(0)}$ is the scattering amplitude without the photon emission. Making the same for the diagram in Fig. 5b we find

$$M = M_a + M_b = \bar{u}(p')M^{(0)}u(p)\sqrt{4\pi}\, e\left(\frac{p'\cdot\varepsilon^*}{p'\cdot k} - \frac{p\cdot\varepsilon^*}{p\cdot k}\right) \tag{A1}$$

and

$$d\sigma^\gamma = d\sigma^{(0)} 4\pi\alpha \left|\frac{p'\cdot\varepsilon^*}{p'\cdot k} - \frac{p\cdot\varepsilon^*}{p\cdot k}\right|^2 \frac{d^3k}{(2\pi)^3 2\omega} \tag{A2}$$

summing over polarizations we find

$$d\sigma^\gamma = d\sigma^{(0)} 4\pi\alpha(-a^\mu a_\mu)\frac{d^3k}{(2\pi)^3 2\omega}, \tag{A3}$$

where

$$a_\mu = \frac{p'_\mu}{p'\cdot k} - \frac{p_\mu}{p\cdot k}.$$

Expressing (A2) in terms of velocities of the outgoing and ingoing particles, photon energy and direction we obtain

$$d\sigma^\gamma = d\sigma^{(0)} \frac{\alpha}{4\pi^2\omega}\left|\frac{\vec{v}'\times\vec{n}}{1-\vec{v}'\cdot\vec{n}} - \frac{\vec{v}\times\vec{n}}{1-\vec{v}\cdot\vec{n}}\right|^2 d\omega d\Omega \tag{A4}$$

which is equivalent to Eq. (5) used above. Bremsstrahlung emitted in a collision of two particles with charges e_1, e_2 and momenta p_1, p_2 leading to particles with charges Q_i and momenta q_i is still given by the expression (A3) only the amplitude a_μ has to be expressed as

$$a_\mu = \sum Q_i \frac{(q_i)_\mu}{q_i\cdot k} - e_1\frac{(p_1)_\mu}{p_1\cdot k} - e_2\frac{(p_2)_\mu}{p_2\cdot k}. \tag{A5}$$

This is equivalent to Eq. (6) in the text.

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