CHARGES, MONOPOLES AND UNIT SYSTEMS

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Using an explicit notation for units, it is argued that electromagnetic fields measured by means of a charge and the corresponding fields measured by a monopole, are different physical entities. This result is related to the persistent failure of the experimental quest for monopoles.

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New papers reporting the failure of monopole search have recently been published [1]. These papers join a long list of articles describing experiments with the same negative result. References to earlier works can be found in the literature [2–7]. Today there are just two reports of a single monopole event which have not been completely refuted [8, 9]. These articles have already been criticized by other researchers who claim that the result is hard to be reconciled with their own conclusions [10–13] or that its foundation is not well established [14]. It is interesting to note that the author of one of the positive reports of a monopole detection [8] has already admitted that a spurious cause for the event cannot be ruled out [15]. Considering these results, people working on the subject conclude that "what the field of monopoles would really need would be some real monopoles" [3], that "the magnetic monopole continues to be strangely elusive" [4] and that "the field has reverted to those willing to do long and difficult work, with improved null result as the most likely outcome" [5].

This unsatisfactory situation calls for a reexamination of the theory used as a foundation of contemporary quest for monopoles. The present work is devoted to a new point of this theory. It discusses some aspects of the underlying assumption saying that electromagnetic fields measured by means of a charge are the same physical entities as fields measured by means of a monopole. Hereafter, this assumption is called the identity assumption. The following analysis adheres to the tensorial formulation of electromagnetic quantities. Introducing an explicit notation for units, it shows that the identity assumption does not hold.

In the subsequent analysis, Greek indices range from 0 to 3 and Latin ones range from

1 to 3. Unless stated explicitly otherwise, units where the speed of light $c = 1$ are employed. In these units, the metric tensor is

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{1}$$

In the following discussion, the validity of the identity assumption is postulated and problems stemming from it are derived. Let us examine an experiment where classical electromagnetic fields are measured by means of a charge. The results can be written in the following tensorial form

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}. \tag{2}$$

This experiment is repeated and the same fields are measured by means of a monopole. The tensorial form of the new quantities is

$$F^{*\mu\nu} = \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z & -E_y \\ B_y & -E_z & 0 & E_x \\ B_z & E_y & -E_x & 0 \end{pmatrix}. \tag{3}$$

Comparing the results of the two measurements, one finds that the identity assumption states, for example, that $F^{10} = F^{*23}$, etc. The plain meaning of this statement says that *noncovariant relations* exist between the tensors $F^{\mu\nu}$ and $F^{*\mu\nu}$, since *different* components of these two tensors are considered *identical.* Quantum mechanical arguments deriving related problems of monopole theory have been published in the literature a long time ago [16–18]. Evidently, noncovariant expressions are unacceptable as elements of an electromagnetic theory.

An attempt to solve this dilemma uses the following tensorial relations

$$F^{*\mu\nu} = \tfrac{1}{2}\,\varepsilon^{\mu\nu\alpha\beta} g_{\alpha\gamma} g_{\beta\delta} F^{\gamma\delta}, \tag{4}$$

where $\varepsilon^{\alpha\beta\gamma\delta}$ is the completely antisymmetric unit tensor of the fourth rank. The subsequent discussion proves that the tensorial relations (4) do not settle the problem.

A basic requirement which should be satisfied by any physical equation says that it must be correct for any set of units. Let us examine the previous results in a system of units where the speed of light $c \neq 1$. In special relativistic notation, this transformation does not affect the metric tensor because, in this theory, it is accustomed to take $x^0 = ct$. In other words, in the new system of units, $x'^{\mu} = x^{\mu}$ and no coordinate transformation entails the introduction of the new units. A different insight into the situation is achieved if the tensorial formalism of general relativity is used.

In this formalism, coordinate transformations are not restricted to the Lorentz ones. Under this extended group of transformations, any tensor transforms according to the well known relations. For example, $T'^{\mu}{}_{\nu} = T^{\alpha}{}_{\beta} \dfrac{\partial x'^{\mu}}{\partial x^{\alpha}} \dfrac{\partial x^{\beta}}{\partial x'^{\nu}}$. In particular, the new units where $c \neq 1$ relate the new coordinates to the old ones according to the following relations: $x'^{0} = x^{0}/c$, $x'^{i} = x^{i}$. At this point one can directly utilize the well established tensorial formalism used in general relativity. The new metric is

$$g'_{\mu\nu} = \begin{pmatrix} c^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{5}$$

In the new system of units, the antisymmetric unit tensor of the fourth rank takes the following form [19, 20]

$$\eta^{\alpha\beta\gamma\delta} = \varepsilon^{\alpha\beta\gamma\delta}/c. \tag{6}$$

Assume that a measurement of electromagnetic fields by means of a charge yields, in the new system of units, the results written symbolically in (2). In this system the tensorial relations (4) are

$$F'^{*\mu\nu} = \tfrac{1}{2}\, \eta^{\mu\nu\alpha\beta} g'_{\alpha\gamma} g'_{\beta\delta} F'^{\gamma\delta}. \tag{7}$$

Substituting (2), (5) and (6) into (7) and relying on the postulated validity of the identity assumption, one finds the quantities measured by means of a monopole

$$F'^{*\mu\nu} = \frac{1}{c} \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & c^2E_z & -c^2E_y \\ B_y & -c^2E_z & 0 & c^2E_x \\ B_z & c^2E_y & -c^2E_x & 0 \end{pmatrix}. \tag{8}$$

As pointed out previously, in the new system of units, $c \neq 1$. Hence, the quantities (2), obtained from a charge-dependent experiment, are different from the corresponding monopole-dependent values of (8). A basic property of any classical physical entity is that, under identical circumstances, it always yields the same results. The satisfaction of this requirement must be independent of the method of measurement and of the system of units as well. In particular, if the identity assumption holds then the two ways of measuring classical electromagnetic fields, either by charges or by monopoles, must yield the same numerical values. Evidently, a comparison of (2) with (8) proves that this requirement is violated by the identity assumption.

The tensorial relations (4) are introduced for the purpose of rescuing the identity assumption from a state of covariance violation. The foregoing discussion proves that, although covariance is apparently restored, other basic physical requirements, like the validity of physical relations in all systems of units, do not hold and (4) cannot be considered a satisfactory solution of the dilemma. Therefore, it is concluded that the identity assumption is unphysical.

It is interesting to note that the results of the present article can also be deduced by means of different arguments. Indeed, it has already been shown that examinations of the metric of a curved space-time [7] and that of a noninertial frame of reference [21] yield analogous conclusions. However, the present discussion looks more fundamental. Indeed, the former derivations rely on general relativity, namely on a specific physical theory. On the other hand, the present deduction uses a fundamental requirement which says that a physical equation should be correct in every system of units.

The failure of the identity assumption raises another problem. If electromagnetic fields measured by charges and those measured by monopoles are different physical entities then the related assumption saying that these fields have the same dynamical properties is not self-evident. (Henceforth, this latter assumption is called the dynamical assumption.) In particular, it is not clear why electromagnetic fields associated with monopoles must be endowed with the property of accelerating charges according to the Lorentz law of force.

The dynamical assumption is a cardinal element upon which current experiments designed for monopole detection are based. The complete failure of these experiments provides another indication questioning the validity of the dynamical assumption [22].

This point is pertinent to another problematic aspect of presently accepted monopole theories, namely, the inability to construct a regular canonical formalism of a classical charge-monopole system. Indeed, it is shown that if these two problematic issues, namely the identity assumption and the dynamical one, are not imposed upon the system then an alternative classical regular canonical theory can be constructed [23]. A basic point of this approach is the evidence that monopoles have not yet been detected. Therefore, there is no experimental determination of their equations of motion. In this case, it looks promising to start not from specific equations of motion but from well established principles of physics. As postulated, the new theory retains the regular canonical structure of ordinary classical electrodynamics. Its equations of motion are characterized by the following property: fields associated with charges do not accelerate monopoles and fields of monopoles do not accelerate charges; charges and monopoles interact indirectly by means of fields of real photons. A prediction of this theory says that current attempts of monopole detection, which are eventually based upon the acceleration of a charge in fields of a monopole, are bound to fail [6, 7]. Actually, the persistent negative results of the quest for monopoles are compatible with this prediction.

REFERENCES

[1] T. E. Ebisu, T. Watanabe, *J. Phys.* **G11**, 883 (1985); S. Bermon, P. Chaudhari, C. C. Chi, C. D. Tesche, C. C. Tsuei, *Phys. Rev. Lett.* **55**, 1850 (1985); T. Kajita et al., *J. Phys. Soc. Jpn* **54**, 4065 (1985); T. Hara et al., *Phys. Rev. Lett.* **56**, 553 (1986); P. B. Price, M. H. Salamon, *Phys. Rev. Lett.* **56**, 1226 (1986); M. W. Cromar, A. F. Clark, F. R. Fickett, *Phys. Rev. Lett.* **56**, 2561 (1986); J. Incandela, H. Frisch, S. Somalwar, M. Kuchnir, H. R. Gustafson, *Phys. Rev.* **D34**, 2637 (1986); T. Gentile et al., *Phys. Rev.* **D35**, 1081 (1987); S. Nakamura et al., *Phys. Lett.* **183B**, 395 (1987); T. Tsukamoto et al., *Europhysics Lett.* **3**, 39 (1987); G. E. Masek et al., *Phys. Rev.* **D35**, 2758 (1987); M. T. Shefko et al., *Phys. Rev.* **D35**, 2917 (1987); J. Bartlet et al., *Phys. Rev.* **B36**, 1990 (1987); B. Barish, G. Liu, C. Lane, *Phys. Rev.* **D36**, 2641 (1987); P. B. Price, R. Guoxiao,

K. Kinoshita, *Phys. Rev. Lett.* **59**, 2523 (1987); T. Ebisu, T. Watanabe, *Phys. Rev.* **D36**, 3359 (1987).
[2] Particle Data Group, *Phys. Lett.* **111B**, 111 (1982).
[3] G. Giacomelli, *Riv. Nuovo Cimento* **7**, 1 (1984). (No. 12).
[4] J. Preskill, *Ann. Rev. Nuc. Part. Science* **34**, 461 (1984).
[5] D. H. Groom, *Phys. Rep.* **140**, 323 (1986).
[6] E. Comay, *Lett. Nuovo Cimento* **43**, 150 (1985).
[7] E. Comay, *Helv. Phys. Acta* **58**, 1009 (1985).
[8] B. Cabrera, *Phys. Rev. Lett.* **48**, 1378 (1982).
[9] A. D. Caplin, M. Hardiman, M. Koratzinos, J. C. Schouten, *Nature* **321**, 402 (1986).
[10] D. E. Groom, E. C. Loh, N. H. Nelson, D. M. Ritson, *Phys. Rev. Lett.* **50**, 573 (1983).
[11] G. Tarle, S. P. Ahlen, T. M. Liss, *Phys. Rev. Lett.* **52**, 90 (1984); *Phys. Rev.* **D30**, 884 (1984).
[12] F. Kajino et al., *Phys. Rev. Lett.* **52**, 1373 (1984); *J. Phys.* **G10**, 447 (1984).
[13] J. Incandela et al., *Phys. Rev. Lett.* **53**, 2067 (1984).
[14] C. N. Guy, *Nature* **325**, 463 (1987).
[15] B. Cabrera, R. Gardner, R. King, *Phys. Rev.* **D31**, 2199 (1985).
[16] D. Zwanziger, *Phys. Rev.* **B137**, 647 (1965).
[17] S. Weinberg, *Phys. Rev.* **B138**, 988 (1965).
[18] C. R. Hagen, *Phys. Rev.* **B140**, 804 (1965).
[19] L. D. Landau, E. M. Lifshitz, *The Classical Theory of Fields*, Pergamon, Oxford 1975, p. 232.
[20] J. L. Synge, *Relativity: the General Theory*, North-Holland, Amsterdam 1966, pp. 18, 356.
[21] E. Comay, *Phys. Lett.* **B187**, 111 (1987).
[22] It is interesting to note that Dirac, who introduced monopoles into quantum mechanics, became skeptical of their existence (see: Dirac's letter to A. Salam in *Monopoles in Quantum Field Theory*, N. S. Craigie, P. Goddard and W. Nahm Editors, World Scientific, Singapore 1982, p. iii; L. Halpern, *Found. Phys.* **15**, 257 (1985)).
[23] E. Comay, *Nuovo Cimento* **80B**, 159 (1984); *Nuovo Cimento* **89B**, 224 (1985).