Vol. B21 (1990) ACTA PHYSICA POLONICA No 3

INSTANTONS IN QCD AND SOLITON MODELS OF HADRONS

By E. A. Dudas

Iron and Steel Group Enterprises, Călăraşi, Romania*

(*Received February 6, 1989; revised version received May 29, 1989*)

Using an analogy to a well known soliton model of hadrons the existence of the quark sea appearing in deep inelastic lepton-hadron scattering is justified. Then using the existence of $\langle\bar{\psi}\psi\rangle$ condensate in QCD we insert in the QCD functional integral a Lorentz scalar field which describes the quark-anti-quark pairs, and produces a dynamical mass for quarks.

PACS numbers: 12.38.Lg

1. *Introduction*

It is widely accepted today that quantum chromodynamics (QCD) is the quantum theory of strong interactions, due to its remarkable successes in describing high energy interactions and the classification of hadrons [1]. Unfortunately, very little has been done toward an understanding of the hadronic structure: the QCD coupling constant increases with distance and we are yet unable to handle quantum theories with a coupling constant greater than one. That is why the soliton models of hadrons, of phenomenological nature, are so extensively used in fitting the experimental data, in understanding the confinement of quarks [2, 3]. The discovery of the powerful topological methods has raised the bid for theoretical attempts toward hadronic structure and confinement [4, 5].

In this paper we try to describe through an effective SU(3) flavour octet pseudoscalar field the quark-antiquark pairs from the quark sea observed in the deep inelastic scattering of leptons on hadrons, a field very close to a soliton one. The same field describes the meson cloud surrounding a hadron at low energies. In fact, it can be argued [6] that at high energies this cloud enters the hadron and reveals the quark-antiquark structure, by the quark sea. This SU(3) flavour octet field is produced by a process of condensation in the analogy of superconductivity and superfluidity [7].

A dynamical mass for quarks appears which is the same as that obtained by operator product expansion of the quark propagator (nonperturbative part). Confinement possibilities are studied considering quark interactions only with this field, extending the Wilson criterion.

* Address: Iron and Steel Group Enterprises, CTC Department, Str. Prel. Bucureşti nr 162, Călăraşi 8500, Romania.

2. *A soliton model for hadrons*

The major problem of QCD today is to explain colour confinement, the experimental fact that there are no "coloured" objects in nature. The most believable indications related are the lattice calculations [8], but a rigorous proof widely accepted does not exist. In practice, confinement is considered as an input for the study of the related properties of hadrons.

We will consider a phenomenological model for th QCD vacuum in which it is characterized by a colour dielectric constant $k < 1$. It will have antiscreening properties; the interaction between two coloured charges becomes stronger in vacuum. The introduction of a colour charge in this medium will produce a hole, a sphere. The energy of the system is stable for a fixed radius and is proportional to $\frac{1}{k}$. If $k = 0$, the energy becomes infinite. So coloured objects with finite energy cannot exist. We can argue that the energy of configurations consisting of a quark-antiquark pair with opposite colours or of three quarks with complementary colours is finite, but an infinite work is needed for breaking these configurations into constituents; the model predicts colour confinement.

If we immerse a hadron into QCD vacuum we will obtain a hole of radius R filled with valence quarks and outside the vacuum with $k \simeq 0$. Within the hole $k = 1$. The transition region with $k = k(x)$ will produce a contribution to the system energy. We introduce a phenomenological field $\vec{\phi}$ which carries this energy, and fulfils boundary conditions

$$\vec{\phi} = \begin{cases} 0, & k = 1, \\ \vec{\phi}_{\text{vac}}, & k = 0. \end{cases}$$

$\vec{\phi} : (\phi^a)_{a=1,8}$ is a Lorentz pseudoscalar and a SU(3) flavour octet field. It is a colour singlet.

In the following by "a, b, c" we will denote flavour indices and by "l, m, n" the colour indices.

The quantum system will be described by

$$\mathscr{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \tfrac{1}{4}\, kF^l_{\mu\nu}F^{l\mu\nu}$$
$$+\tfrac{1}{2}(\partial_\mu\vec{\phi})(\partial^\mu\vec{\phi}) - U(\vec{\phi}) - f\bar{\psi}\gamma_5 \frac{\lambda_a}{2}\phi_a\psi, \qquad (1)$$

where ψ is the quark field, A^l_μ the gluon gauge field, $(\lambda^a)_{a=1,8}$ are Gell-Mann matrices satisfying SU(3) algebra

$$\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right] = if^{abc}\frac{\lambda^c}{2}$$

and $D_\mu = \partial_\mu - ig\frac{\lambda^l}{2}A^l_\mu$, g being the colour coupling constant.

The potential U has the property

$$U(\vec{\phi} = 0) = U(\vec{\phi} = \vec{\phi}_{\text{vac}}) = 0.$$

The lagrangian (1) reveals a Yukawa-type coupling, with the effect that

$$m \to m+f\gamma_5 \frac{\lambda a}{2}\phi_a \equiv m+f\gamma_5 \frac{\vec{\lambda}}{2}\vec{\phi}.$$

If we choose $f\vec{\phi}_{\text{vac}}\frac{\vec{\lambda}}{2} \to \infty$, m will tend to infinity and quarks automatically stay inside the region where $\vec{\phi} \simeq 0$, i.e. inside the hole.

The radius R of the hole is the scale of confinement. In the first approximation $\vec{\phi}$ is a classical field related to collective long range effects in QCD: the short wavelength components do not exist in this approximation. In computing the lowest energy of the system, $\vec{\phi}$ is independent of time. Neglecting the quark masses and gluon exchange, treating $\vec{\phi}$ as a classical field $\vec{\phi}_0$ we obtain [9, 10] the energy of the system

$$E = N\varepsilon + \int d^3\vec{x}[\tfrac{1}{2}(\vec{\nabla}\phi^a)(\vec{\nabla}\phi^a)+U(\phi^a)], \tag{2}$$

where N is the number of quarks and ε is the lowest quark energy.

The equation for ψ is

$$\left(-i\vec{\alpha}\vec{\nabla}+\beta f\gamma_5\frac{\vec{\lambda}}{2}\vec{\phi}_0\right)\psi = i\frac{\partial\psi}{\partial t}, \tag{3}$$

where $\psi = (\psi_f^c)$, c being the colour and f the flavour. Using the finite volume normalization, we write

$$\psi_f^c(t,\vec{x}) = \sum_n [(a_f^c)_n\chi_n(\vec{x})e^{-i\varepsilon_n t}+(b_f^c)_n^+\bar{\chi}_n(\vec{x})e^{i\varepsilon_n t}].$$

From (3) we obtain

$$\left(-i\vec{\alpha}\vec{\nabla}+\beta f\gamma_5\frac{\vec{\lambda}}{2}\vec{\phi}_0\right)\begin{cases}\chi_n\\ \bar{\chi}_n\end{cases} = \varepsilon_n\begin{cases}\chi_n\\ \bar{\chi}_n\end{cases}$$

so

$$\varepsilon_n = \int d^3\vec{x}\chi^+\left(-i\vec{\alpha}\vec{\nabla}+\beta f\gamma_5\frac{\vec{\lambda}}{2}\vec{\phi}_0\right)\chi, \qquad \varepsilon = \text{inf.}\ \varepsilon_n. \tag{4}$$

The functional differentiation $\dfrac{\delta E}{\delta\phi_0^a} = 0$ gives us the equation satisfied by $\vec{\phi}_0$:

$$-\vec{\nabla}^2\phi_0^a+\frac{\partial U}{\partial\phi_0^a} = -fN\chi^+\beta\gamma_5\frac{\lambda^a}{2}\chi, \tag{5}$$

where χ is the quark wave function corresponding to ε and β is the γ_0 Dirac matrix.

Outside the hadron, (5) has a soliton-type solution if $m_{\vec{\phi}_0}$ is high and $U = \dfrac{m_{\phi_0}^2}{2\phi_{\text{vac}}^2}\vec{\phi}^2(\vec{\phi}-\vec{\phi}_{\text{vac}})^2$

$$\phi_0^a \simeq \tfrac{1}{2}\left[1+\text{th}\frac{m_{\phi_0}}{2}(r-R)\right]\phi_{\text{vac}}^a. \tag{6}$$

We consider a model where (6) is very approximate, $m_{\vec{\phi}_0}$ is not too high. Equivalently, the transition region inside-outside is not just a wall but is of order of the hadron radius itself (the scale of the transition region is $\sim m^{-1}$). In this case, as we will see, we are able to consider the quantum excitations of $\vec{\phi}$ as quark-antiquark pairs inside the hadron and the classical field $\vec{\phi}_0$ as the meson cloud surrounding the hadron. The correct classical field for us is

$$\vec{\phi}_0'^a = \frac{e^{-m_{\phi_0} r}}{r} \phi_0^a. \tag{7}$$

Recall that (6) is the one-dimensional Kink soliton of the ϕ^4 theory.

For simplicity, we have supposed that the ϕ^a field mass matrix M_ϕ appearing in $U(\vec{\phi})$ is of the form $M_{\phi_0} = m \times 1$, where 1 is the 8×8 unit matrix.

In the following Section we will again neglect gluon exchange and the quark masses.

3. *Soliton quantization*

At high energies, taking $\vec{\phi}$ as a classical field is unsatisfactory, because of short range effects. The quantum field $\vec{\phi}'$ fluctuates around its classical value $\vec{\phi}_0'$:

$$\phi'^a = \phi_0'^a + \hbar \phi_1'^a + \ldots$$

We put

$$\Phi'^a(x) = \phi_0'^a(\vec{x}) + \eta^a(t, \vec{x}) \quad \text{and} \quad V[\vec{\Phi}] = \int d^3\vec{x} [\tfrac{1}{2} (\vec{\nabla}\phi'^a)(\vec{\nabla}\phi'^a) + U(\phi'^a)].$$

To quantize the $\vec{\phi}'$ field we apply the WKB method

$$V[\vec{\phi}'] = V[\vec{\phi}_0'] + \tfrac{1}{2} \int d^3\vec{x} d^3\vec{y} \left. \frac{\delta^2 V}{\delta\vec{\phi}'(x)\delta\vec{\phi}'(y)} \right|_{\vec{\phi}' = \vec{\phi}_0'}$$

$$\times [\vec{\phi}'(x) - \vec{\phi}_0'(\vec{x})] [\vec{\phi}'(y) - \vec{\phi}_0'(\vec{y})] + \ldots$$

$$= V[\vec{\phi}_0'] + \tfrac{1}{2} \int d^3\vec{x} \eta^a(x) \left(-\vec{\nabla}\delta_{ab}^2 + \frac{\partial^2 U}{\partial\phi_a \partial\phi_b} \right)_{\phi' = \phi_0'} \eta^b(y) + \ldots . \tag{8}$$

We consider in (8) the value $\vec{\phi}' = \vec{\phi}_0$ as an approximate stationary point supposing that the factor $\dfrac{e^{-mr}}{r}$ in (7) oscillates slowly.

We solve the Schrödinger problem

$$\left[-\vec{\nabla}^2 \delta_{ab} + \frac{\partial^2 U}{\partial\phi_0^a \partial\phi_0^b} \right] \eta_i^a(\vec{x}) = \omega_{ib}^2 \eta_i^b(\vec{x}) \tag{9}$$

and expand

$$\eta^a(x) = \sum C_i^a(t)\eta_i^a(\vec{x}) \tag{9'}$$

without summation over indexes "a" and "b" in (9) and (9'). $\eta_i^a(\vec{x})$ are normalized according to

$$\int d^3\vec{x}\eta_i^a(\vec{x})\eta_j^b(\vec{x}) = \delta_{ij}\delta^{ab}.$$

We introduce the creation and destruction operators for the quanta of soliton sector of $\vec{\phi}$ field by

$$d_i^a = \frac{1}{\sqrt{2\omega_{ia}}}(\dot{C}_i^a + i\omega_{ia}C_i^a),$$

$$d_i^{+a} = \frac{1}{\sqrt{2\omega_{ia}}}(\dot{C}_i^a - i\omega_{ia}C_i^a).$$

Then

$$\phi'^a(x) = \frac{e^{-m_{\phi_0}r}}{r}\phi_0^a(x) + i\sum_i (d_i^{+a}e^{i\omega_{ia}t} - d_i^a e^{-i\omega_{ia}t})\eta_i^a(\vec{x}), \tag{10}$$

$\eta_i^a(\vec{x})$ are the wave functions of the soliton field quanta.

Then the hamiltonian of the system is

$$H = \sum_{n,f,c} [\varepsilon(a_f^c)_n^+(a_f^c)_n + (b_f^c)_n^+(b_f^c)_n] + \sum_{i,a}\omega_{ia}(d_i^+)^a d_i^a$$
$$+ \sum_{i,a}\sqrt{\frac{2}{\omega_{ia}}}\int d^3\vec{x} f\psi^+(x)\gamma_5\beta\frac{\lambda^a}{2}(d_i^{+a}e^{i\omega_{ia}t} - d_i^a e^{-i\omega_{ia}t})$$
$$\times \eta_i^a(\vec{x})\psi(x) + \int d^3\vec{x}\,\tfrac{1}{2}(\vec{\nabla}\phi_0'^a)(\vec{\nabla}\phi_0'^a) + U(\phi_0'^a). \tag{11}$$

The energy of a state composed of $N = \sum_n N_n$ quarks and $M = \sum_{i,a} n_{ia}$ quanta of soliton is

$$E = H|N, M\rangle = \sum_n N_n\varepsilon_n + \sum_{i,a} n_{ia}\omega_{ia} + V[\phi_0'] + \langle N, M|H_{\rm int}|N, M\rangle. \tag{12}$$

4. *Quark-antiquark pairs from deep inelastic scattering resonances*

The frequency spectrum ω_{ia} is determined by (9). With the change of function

$$\eta_i^a = \frac{1}{r}\chi_i^a$$

the equation is exactly solvable [3]. It has two discrete levels followed by a continuum. The discrete levels are (we suppose again that the eight quanta of $\vec{\phi}'$ field have the same mass)

$$\omega_{0a}^2 = 0, \quad \chi_0^a(z) = \frac{1}{\mathrm{ch}^2 z},$$

$$\omega_{1a}^2 = \tfrac{3}{2}\, m_{\phi_0}^2, \quad \chi_1^a(z) = \frac{\mathrm{sh}\, z}{\mathrm{ch}^2 z}, \quad \text{where } z = \frac{m_{\phi_0}}{2}(r-R). \tag{13}$$

The continuum levels, labelled $q \in \boldsymbol{R}$ are

$$\omega_{qa}^2 = m_{\phi_0}^2(\tfrac{1}{2} - q^2 + 2), \quad \chi_q^a(z) = e^{iqz}(3\mathrm{th}^2 z - 1 - q^2 - 3iq\, \mathrm{th}\, z).$$

The energy of a state is

$$E = \sum_n N_n \varepsilon_n + V[\vec{\phi}_0'] + \sum_a \sqrt{\tfrac{3}{2}}\, m_{\phi_0} n_{1a}$$

$$+ \sum_{q_i, a} m_{\phi_0} n_{q_i a}(\tfrac{1}{2} q_i^2 + 2)^{1/2} + \langle N, M | H_{\mathrm{int}} | N, M \rangle. \tag{14}$$

The $\omega = 0$ modes are spurious appearing because of the lagrangian invariance at continuous transformations and they have no associated quanta.

The ω_{1a} modes are not spurious and we interpret them as describing the excited states of hadrons, i.e., the hadron resonances.

The continuum modes ω_{qa} can be interpreted as quark-anti-quark pairs from the quark sea, which appears in addition to valence quarks at high energies inside the hadron.

The quantum field η^a is spatially confined because of (13). The soliton quanta are localized essentially inside and near the surface of the hadron. Considering R as an input, we will redefine the hadron radius $R(q)$ by the condition $\chi_q^a(r = 0) = 0$, which implies

$$3\mathrm{th}^2 \frac{m_{\phi_0}}{2} R - 1 - q^2 + 3iq\, \mathrm{th} \frac{m_{\phi_0}}{2} R = 0.$$

For example, for $q = 0$ we obtain $R \simeq \dfrac{3.73}{m_{\phi_0}}$.

As a rough estimate using $m_\Delta - m_N \sim 300$ MeV we obtain $m_{\phi_0} \sim 200$ MeV. Of course, we are not able to reproduce the correct masses of hadrons because our formulae are SU(6) invariant (we have neglected the gluon exchange and the quark mass parameters in $\mathscr{L}$).

5. Bosonization in QCD instantons and the quark condensate

The gauge theory of QCD has nonperturbative features related to topological configurations of finite action [11] which modify the structure of the QCD vacuum [12]. We believe that these configurations may provide a theoretical ground for the phenomenological field ϕ_a introduced earlier. More precisely it can be argued that $\langle \theta | : \bar{\psi}\psi : | \theta \rangle$ vanishes for a coupling constant less than a value of the order one and is nonvanishing for a greater coupling constant [13]. From renormalization group arguments inside the hadron the coupling constant is weak. Then we obtain a condensate outside the hadron like the ones in the preceding section. At high energies, the corresponding quanta penetrate into the hadron and generate the quark sea. We will justify the SU(3) octet Lorentz pseudoscalar field as being a composed field leading to a bosonization process like in superconductivity [7].

Let us consider the vacuum-to-vacuum amplitude in euclidean space

$$\langle 0|0\rangle = \int [d\bar{\psi} d\psi d\vec{A}_\mu d\vec{C}^\dagger d\vec{C}] e^{-\int d^4x[\mathscr{L}^{\text{gauge}}(\vec{A}_\mu)+\mathscr{L}^{\text{fermion}}(\vec{A}_\mu,\psi)+\mathscr{L}^{\text{fix}}(\vec{A}_\mu)+\mathscr{L}^{\text{ghost}}(\vec{A}_\mu,C)]}, \quad (15)$$

where $\mathscr{L}^{\text{fix}}$ fixes the gauge and $\mathscr{L}^{\text{ghost}}$ is the corresponding Fadeev–Popov ghost term with $\vec{C}$ the ghost field.

Applying the steepest-descent method, we may write

$$\mathscr{L} = \mathscr{L}(A^{\text{cl}}) + \vec{A}^{\text{qu}}_\mu M_1^{\mu\nu} \vec{A}^{\text{qu}}_\nu + \bar{\psi} M_2 \psi + \vec{C}^+ M_3 \vec{C} + \text{higher orders in the quantum fields,} \quad (16)$$

where we have used

$$A^l_\mu = (A^l_\mu)^{\text{cl}} + (A^l_\mu)^{\text{qu}} \equiv (A^l_\mu)^0 + (A^l_\mu)^{\text{qu}}$$

and M_1, M_2, M_3 are some matrices.
$(A^l_\mu)^0$ belongs to an equivalence class described by the Pontryagin index $\nu \in Z$.

It is easily seen that due to topological configurations

$$\frac{\delta^4 S}{\delta\psi(x_1)\delta\bar{\psi}(x_2)\delta\psi(x_3)\delta\bar{\psi}(x_4)} \neq 0. \quad (17)$$

The classical equations of motion for the system are

$$(\gamma_\mu D^0_\mu - m)\psi_0(x) = 0, \qquad D^0_\mu (F^l_{\mu\nu})^0 = \bar{\psi}_0 \frac{\lambda^l}{2} \gamma_\nu \psi_0, \quad (18)$$

where

$$D^0_\mu = \partial_\mu - ig\frac{\lambda^l}{2}(A^l_\mu)^0 \quad \text{and} \quad F^l_{\mu\nu} = \partial_\mu A^l_\nu - \partial_\nu A^l_\mu + g f^{lmn} A^m_\mu A^n_\nu.$$

But

$$\frac{\delta^2 S}{\delta\psi(x_1)\delta\bar{\psi}(x_2)} = (\gamma^\mu D^\mu - m)\delta^4(x_1 - x_2)$$
$$= (\gamma^\mu D^\mu_{qu} - m - g\gamma_\mu \frac{\vec{\lambda}}{2}\vec{A}^0_\mu)\delta^4(x_1 - x_2).$$

Due to nontriviality of $\vec{A}_\mu$ and using (18b) we obtain $\vec{A}^0_\mu = f(\bar{\psi}_0, \psi_0)$. Consequently, we obtain (17).

We arrive at a nonlocal $(\bar{\psi}\psi)^2$-type interaction, in agreement with the more rigorous proof of 't Hooft [14, 15].

Let us write

$$\mathscr{L} = \mathscr{L}(A^0)+\vec{A}_\mu^{\text{qu}}M_1^{\mu\nu}\vec{A}_\nu^{\text{qu}}+\bar{\psi}M_2\psi+\vec{C}^+M_3\vec{C}+\int d^4y\bar{\psi}\psi M_4(x,y)\bar{\psi}\psi. \tag{19}$$

We introduce in $\langle 0|0\rangle$ a Lorentz scalar field which compensates the last term written in (19):

$$\sqrt{\det M_5}\int [d\phi^a]e^{-\int d^4x d^4y \phi^a(x)M_5{}^{ab}(x,y)\phi^b(y)} = 1. \tag{20}$$

This field must have (imposed by us) the property

$$\langle\phi^a\rangle = \frac{1}{\Lambda^2}\langle\theta|:\bar{\psi}\gamma_5\frac{\lambda^a}{2}\psi:|\theta\rangle,$$

where $|\theta\rangle$ is the QCD vacuum and Λ a parameter with dimension of mass.

We then apply the perturbation theory for

$$\phi_a^* = \phi_a-\frac{1}{\Lambda^2}\bar{\psi}\gamma_5\frac{\lambda_a}{2}\psi, \quad \langle\phi_a^*\rangle = 0. \tag{21}$$

The value of M_5 is correlated with M_4 which does not contain time derivates (neither spatial ones), so ϕ_a will not be an independent degree of freedom for the quantum system: it is a composed field.

Let us now suppose that $M_5^{ab}(x,y) = \delta^4(x-y)M_5^{ab}$, a rather poorly justified approximation, but a very simple one to work with.

Inserting (20) and (21) into $\langle 0|0\rangle$ and omitting the asterisk, we obtain:

$$\langle 0|0\rangle = \int [d\psi d\bar{\psi} d\vec{A}_\mu d\vec{C}^+ d\vec{C} d\phi^a]\exp\left\{-S-\int d^4x\left[\phi^a M_5^{ab}\phi^b-\frac{1}{\Lambda^2}\bar{\psi}\gamma_5\right.\right.$$
$$\left.\left.\times\frac{\lambda_a}{2}\psi M_5^{ab}\phi^b-\frac{1}{\Lambda^2}\phi^a M_5^{ab}\bar{\psi}\gamma_5\frac{\lambda^b}{2}\psi\right]\right\}.$$

Taking into account that $\dim|M_5| = \text{mass}^2$ we can in our approximation write

$$M_5^{ab} = -\tfrac{1}{2}\mu^2\delta^{ab}, \tag{22}$$

where $\mu = m_{\phi_0}$ introduced earlier.

Returning to Minkowski space we have

$$\mathscr{L} = \bar{\psi}(i\gamma^\mu D_\mu-m)\psi-\tfrac{1}{4}F_{\mu\nu}^l F_l^{\mu\nu}+\mathscr{L}^{\text{fix}}(A_\mu)+\mathscr{L}^{\text{ghost}}-\frac{\mu^2}{2}\phi^a\phi^a-f\bar{\psi}\gamma_5\frac{\lambda^a}{2}\phi^a\psi, \tag{23}$$

where $f = \dfrac{\mu^2}{\Lambda^2}$.

Neglecting the gauge and ghost fields, we obtain Euler-Lagrange equations

$$\phi^a = -\frac{1}{\Lambda^2}\bar{\psi}\gamma_5\frac{\lambda^a}{2}\psi, \quad \left(i\gamma^\mu\partial_\mu - m - f\gamma_5\frac{\lambda^a}{2}\phi^a\right)\psi \neq 0. \tag{24}$$

It is clear from (24) that ϕ^a is a composed field and that it coincides with the phenomenological field introduced in (1).

In [13] it is argued that

$$\langle\bar{\psi}\psi\rangle = \begin{cases} 0, & g < \frac{8}{9} \\ \neq 0, & g > \frac{8}{9} \end{cases}$$

so using (24) we recognize the property of the previous phenomenological field

$$\phi_0^a = \langle\phi^a\rangle = \begin{cases} 0, & g < \frac{8}{9} \\ \neq 0, & g > \frac{8}{9} \end{cases}.$$

We associate the running coupling constant of value $g = \frac{8}{9}$ with the size of the hadron. As we see from

$$\mathscr{L}^{(0)} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m - f\gamma_5\frac{\lambda^a}{2}\phi^a\right)\psi - \tfrac{1}{2}\mu^2\phi^a\phi^a$$

even if we take $m = 0$ (chiral symmetry) the quark acquires a dynamical mass of the form

$$m_{\text{dyn}} \sim f\phi_0^a \sim \langle\theta|\bar{\psi}\gamma_5\frac{\lambda^a}{2}\psi|\theta\rangle. \tag{25}$$

6. *Chiral symmetry breaking through $\langle\bar{\psi}\psi\rangle$ condensate*

The nonvanishing of $\langle\bar{\psi}\psi\rangle$ is known to produce a dynamical breaking of the chiral symmetry in QCD. But the quanta of this field are the quanta of ϕ^a. We study the lifetime of these quanta by elementary current algebra methods.

Considering the chiral symmetry $SU(3)_L \times SU(3)_R$ we obtain the following charges:

$$Q_a = \int d^3\vec{x}\,\psi^+(x)\frac{\lambda_a}{2}\psi(x), \quad Q_{5a} = \int d^3\vec{x}\,\psi^+(x)\gamma_5\frac{\lambda_a}{2}\psi(x)$$

which are time independent if the quark mass matrix is zero in $\mathscr{L}_{QCD}$. The charges are generated by the SU(3) vector and axial vector currents:

$$J_\mu^a = \bar{\psi}\gamma_\mu\frac{\lambda^a}{2}\psi, \quad J_\mu^{5a} = \bar{\psi}\gamma_5\gamma_\mu\frac{\lambda^a}{2}\psi.$$

The chiral symmetry is spontaneously broken in the sense

$$Q_a|\theta\rangle = 0, \quad Q_{5a}|\theta\rangle \neq 0.$$

Using the anticommutation relations for quark fields we may obtain

$$\left[Q_{5a}, \bar{\psi}\gamma_5 \frac{\lambda_b}{2}\psi\right] = if_{abc}\bar{\psi}\frac{\lambda_c}{2}\psi, \tag{26}$$

$$\langle\theta|\left[Q_{5a}, \bar{\psi}\gamma_5 \frac{\lambda_b}{2}\psi\right]|\theta\rangle$$

$$= \sum_n \delta^3(\vec{p}_\theta - \vec{p}_n)\Big[e^{-i(E_\theta - E_n)t}\langle\theta|J_0^{5a}|n\rangle\,\langle n|\bar{\psi}\gamma_5\frac{\lambda^b}{2}\psi|\theta\rangle$$

$$-e^{i(E_\theta - E_n)t}\langle\theta|\bar{\psi}\gamma_5\frac{\lambda^b}{2}\psi|n\rangle\cdot\langle n|J_0^{5a}|\theta\rangle\Big] = i\langle\theta|\bar{\psi}\frac{\lambda_c}{2}\psi|\theta\rangle f_{abc}. \tag{27}$$

So there will be excitations $|n\rangle$ of vacuum (but not massless) such that

$$E_n = E_\theta \quad \text{and} \quad \langle\theta|J_0^{5a}|n\rangle \neq 0.$$

The matrix element of the axial current is proportional to the disintegration constant of the quanta, which are the quark-antiquark pairs from the quark sea.

It is interesting to note that J_0^{5a} are very close in structure to our ϕ^a field.

If we put $v^a = \langle\theta|\bar{\psi}\frac{\lambda_a}{2}\psi|\theta\rangle$, from (27) we obtain a formula in which

$$\langle\theta|J_0^{5a}|n\rangle \sim \sqrt{v^a}$$

in contrast to the usual π mesons, when $\langle 0|J_0^{5a}|\pi\rangle \sim v^a$.

It seems that the quark-antiquark pair has a smaller disintegration constant than the pion.

The scale of chiral symmetry breaking is given by [16] $\langle\bar{\psi}\psi\rangle = -(250\ \text{MeV})^3$.

7. *Dynamical masses for quarks*

The quark condensate produces a dynamical quark mass through a mechanism analogous to the Higgs mechanism in the electroweak theory. We remark also that (25) is close to the nonperturbative effective quark mass obtained by developing the quark propagator in the Wilson operator product expansion (for zero mass)

$$\int d^4x e^{iqx}\langle\theta|T\psi(x)\bar{\psi}(0)\,|\theta\rangle$$

$$= \frac{1}{\not{q}} + C_2(q)\,\langle\theta|:\bar{\psi}\psi:|\theta\rangle + C_3(q)\,\langle\theta|:F^l_{\mu\nu}F^{l\mu\nu}:|\theta\rangle + \ldots,$$

where $C_i(q)$ are Wilson coefficients computable through renormalization group methods.

In the deep euclidean region we obtain [17]

$$m(Q) \simeq m_0(M)\left[\frac{g^2(Q)}{g(M)}\right]^{\frac{12}{27}} + \frac{4g^2(Q)}{Q^2}\langle\bar{\psi}\psi\rangle\left[\frac{g^2(Q)}{g^2(M)}\right]^{-\frac{12}{27}} + C_3(q)\langle F^l_{\mu\nu}F^{l\mu\nu}\rangle + \dots$$

M is the point at which we renormalize, $m_0(M)$ is the perturbative renormalization mass and $Q^2 = -q^2$.

In the electroweak theory the vacuum expectation value of the Higgs field leads to spontaneous symmetry breaking $SU(2)_L \times U(1) \to U(1)$. The Yukawa interaction fermions--Higgs field leads to a mass term for fermions which can be interpreted as inertia due to particle motion in the bosonic Higgs condensate, in the analogy to the electron mass shift in solid state physics. In electroweak theory we have a space-time independent condensate and consequently a constant mass. In this paper we have a space-dependent condensate, like in ferromagnetism, for instance. The transition region inside-outside the hadron is like the Bloch walls. In [18, 19] the gluon condensate $F^l_{\mu\nu}F^{l\mu\nu}$ is used to provide confinement. But its quanta can be interpreted like gluon pairs, something like glue-balls, which must appear in deep inelastic scattering also. In contrast to our soliton field it will be confined inside the hadron.

8. Conclusions

We can describe, using a SU(3) pseudovector field, the meson field surrounding a hadron and the quark-antiquark pairs at high energies inside the hadron, but our formulae are somewhat phenomenological, because the field ϕ'^a itself is phenomenological.

It is interesting to study quark confinement in a model where quarks interact only with the field ϕ^a. In this respect, we use the Wilson criterion [20] in a modified form. The energy of a quark-antiquark pair separated by a distance R is obtained applying the formula

$$e^{-E(R)T} = \frac{\int [d\phi^a] e^{-S_{\phi^a} + iqf\gamma_5 \frac{\lambda^a}{2}\oint_\Gamma ds\phi^a}}{\int [d\phi^a] e^{-S_{\phi^a}}},$$

where $S_{\phi}a = -\dfrac{\mu^2}{2}\displaystyle\int d^4x\phi^a\phi^a$, q is the quark charge and ds is the world line element on the loop Γ generated by the pair. We obtain

$$e^{-E(R)T} = e^{\frac{q^2f^2}{2\mu^2}\lambda^a\lambda^a\oint_\Gamma\oint_\Gamma dsds'}.$$

But $\oint_\Gamma\oint_\Gamma dsds' \sim RT$, so we obtain $E(R) \sim R$. Quarks will be confined in this model (of course, if we accept this extension of the Wilson criterion).

I'm very indebted to dr. M. Visinescu for his help and encouragements. I would also like to thank Mr. G. Mârzac, the general director of Iron and Steel Group Enterprises Calarasi for kindly permitting me to carry out this work.

REFERENCES

[1] F. J. Yndurain, *Quantum chromodynamics*, Springer-Verlag, Berlin 1983.
[2] W. Bardeen et al., *Phys. Rev.* **D11**, 1094 (1975).
[3] R. Rajaraman, *Solitons and instantons in quantum field theory*, North-Holland, Amsterdam 1982.
[4] C. Callan, R. Dashen, D. Gross, *Phys. Rev.* **D17**, 2717 (1978).
[5] C. Callan, R. Dashen, D. Gross, *Phys. Rev.* **D19**, 1826 (1979).
[6] G. E. Brown, *Nucl. Phys.* **A478**, 79C (1988).
[7] V. N. Popov, *Functional integrals in quantum field theory and statistical physics*, Reidel, New York 1983.
[8] M. Bander, *Phys. Rep.* **75**, 205 (1981).
[9] T. D. Lee, *Particle physics and introduction to field theory*, Harwood, New York 1981.
[10] R. Friedberg, T. D. Lee, *Phys. Rev.* **D15**, 1694 (1977).
[11] A. Belavin et al., *Phys. Lett.* **59B**, 85 (1975).
[12] E. Shuryak, *Phys. Rep.* **115**, 485 (1984).
[13] J. Finger, D. Horn, J. Mandula, *Phys. Rev.* **D20**, 3253 (1979).
[14] G. 't Hooft, *Phys. Rev. Lett.* **37**, 8 (1976).
[15] G. 't. Hooft, *Phys. Rev.* **D14**, 3432 (1976).
[16] A. B. Migdal, *Nucl. Phys.* **A478**, 95c (1988).
[17] H. Georgi, H. Politzer, *Phys. Rev.* **D14**, 1829 (1976).
[18] M. Baker, J. Ball, F. Zachariasen, *Phys. Rev.* **D37**, 1036 (1988).
[19] M. Baker, J. Ball, F. Zachariasen, *Phys. Rev. Lett.* **61**, 521 (1988).
[20] K. Wilson, *Phys. Rev.* **D10**, 2445 (1974).