# FINITE TEMPERATURE EFFECT ON STRINGS IN CURVED SPACE

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Using thermofield dynamics we study finite temperature effects on strings in curved space.

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In recent years there have been numerous studies in string theory [1]. In particular, thermodynamic properties of strings have been studied extensively [2–8]. However, in all these cases strings were considered in flat space-time (the reason for this is that it is not always easy to formulate string theory in curved space-time) and in this paper we shall study thermal effects on string in curved space. Here we shall follow the method of de Vega and Sanchez [9] to quantize strings in curved space and shall examine the possibility of stabilizing the tachyonic ground state through thermal effects.

To start with let us first consider the action for a string in an arbitrary $D$-dimensional manifold:

$$S = \frac{1}{2\pi\alpha}\int d\sigma d\tau \sqrt{g}\, g^{\mu\nu} G_{AB}(X)\partial_\mu X^A \partial_\nu X^B, \tag{1}$$

where $G_{AB}(X)$ and $g^{\mu\nu}$ denote respectively the space-time metric and the world sheet metric.

In the conformal gauge the string equations of motion are

$$\partial^2 X^A + \Gamma^A_{BC}(X)\partial X^B \partial X^C = 0, \tag{2}$$

where $\Gamma^A_{BC}(X)$ denotes the Christoffel connection.

The centre of mass motion is given by

$$\ddot{q}^A(\tau) + \Gamma^A_{BC}(q)\dot{q}^B(\tau) = 0. \tag{3}$$

Then one writes

$$X^A(\sigma, \tau) = q^A(\sigma, \tau) + \eta(\sigma, \tau) + \zeta(\sigma, \tau) + \ldots, \tag{4}$$

where $q^A(\sigma, \tau)$ is the exact solution of equation (3) and the other terms are perturbations around $q^A(\sigma, \tau)$. We note that for all types of curved spaces, the equations of motion do not admit solutions; hence we specialise to de Sitter space for which there exists exact solutions. The metric for the de Sitter space is given by

$$ds^2 = \left(\frac{R_0}{X^0}\right)^2 [(dX^0)^2-(dX^i)^2], \quad 1 \leqslant i \leqslant D-1, \tag{5}$$

where $R_0 = H^{-1} = 3/\Lambda$ and the curvature scalar is given by $R = HD(D-1)$. In this case we have

$$q^0(\tau) = \frac{gm}{r \sinh(gm\tau)}, \quad q^1(\tau) = r^1 - \frac{gm}{r} \coth(gm\tau),$$

$$q^i(\tau) = r^i, \quad 2 \leqslant i \leqslant D-1, \quad r^2 = \sum_{i=1}^{D-1} p^{i2}. \tag{6}$$

Using (6) the first order perturbations can be found and are given by [10]

$$\eta^0(\tau, \sigma) = \frac{1}{r \sinh^2(mg\tau)} [A_1(\sigma, \tau)+B(\sigma, \tau) \cosh^2(mg\tau)],$$

$$\eta^1(\sigma, \tau) = -\frac{1}{r \sinh^2(mg\tau)} [B(\sigma, \tau)+A_1(\sigma, \tau) \cosh^2(mg\tau)],$$

$$\eta^i(\sigma, \tau) = \frac{A_i(\sigma, \tau)}{P \sinh(mg\tau)}, \quad 2 \leqslant i \leqslant D-1, \tag{7}$$

where

$$A_\alpha(\sigma, \tau) = \sum_n [\gamma_n^\alpha e^{i(n\sigma-w_n\tau)} + \gamma_n^{\alpha+} e^{-i(n\sigma-w_n\tau)}],$$

$$B(\sigma, \tau) = \sum_n [\beta_n e^{in(\sigma-\tau)} + \beta_n^+ e^{-in(\sigma+\tau)}],$$

$$\beta_n = \beta_{-n}^+, \quad \beta_n^+ = \beta_{-n}, \quad w_n = \sqrt{n^2-m^2g}, \tag{8}$$

$\gamma$ and $\beta$ operators satisfy

$$[\gamma_n^\alpha, \gamma_l^{\beta+}] = \frac{m^2g^2}{4\pi w_n} \delta_{nl}\delta^{\alpha\beta}, \quad [\beta_n, \beta_l] = \frac{m^2g^2}{4\pi n} \delta_{n+l,0}. \tag{9}$$

In this case the mass spectrum is given by [10]

$$\frac{\pi m^2}{2} = (D-1)\varepsilon(m^2) + \sum_{n\neq 0} \Omega_n(m^2) \sum_{\alpha=1}^{D-1} a_n^{\alpha+} a_n^\alpha + O(g^2), \tag{10}$$

$$\Omega_n(m^2) = \sqrt{n^2 - m^2 g^2} + \frac{m^2 g^2}{2\sqrt{n^2 - m^2 g^2}}, \tag{11}$$

$$\varepsilon(m^2) = \sum_{n=1}^{\infty} \Omega_n(m^2), \tag{12}$$

$$a_n^\alpha = \frac{2\sqrt{\pi w_n}}{mg} \gamma_n^\alpha, \quad g = \frac{L_{Pl}}{R}. \tag{13}$$

Using $\zeta$-function regularization it can be easily shown the ground state is tachyonic and the coresponding mass given by

$$m_0^2 = -\frac{(D-1)}{12} \tag{14}$$

we shall now examine whether the ground state can be stabilized at finite temperature. In order to incorporate finite temperature we shall follow the thermofield dynamics approach [11]. In this approach the degrees of freedom is doubled with the introduction of tilde states. Thus, for example, there are operators $\tilde{a}_n^\alpha$ acting on the tilde states. These operators have the same commutation relations as the non-tilde ones and they commute with the non-tilde operators. Hence

$$[a_n^\alpha, \tilde{a}_m^\beta] = [a_n^\alpha, \tilde{a}_m^{\beta +}] = 0. \tag{15}$$

For any thermodynamic calculations it is necessary to determine the forms of the operators at finite temperature and these are given by

$$a_n^\alpha(\beta) = a_n^\alpha \cosh \theta_n^\alpha(\beta) + \tilde{a}_n^{\alpha +} \sinh \theta_n^\alpha(\beta), \tag{16}$$

$$\tilde{a}_n^\alpha(\beta) = \tilde{a}_n^\alpha \cosh \theta_n^\alpha(\beta) + a_n^{\alpha +} \sinh \theta_n^\alpha(\beta), \tag{17}$$

where

$$\cosh \theta_n^\alpha(\beta) = \frac{1}{(1-e^{-\beta n})}, \quad \sinh \theta_n^\alpha(\beta) = \frac{e^{-\beta n/2}}{(1-e^{-\beta n})}. \tag{18}$$

It is clear that the creation operators can be obtained by taking hermitian conjugates of (16) and (17). At this point we note that (16) and (17) together with their conjugates are invertible i.e., zero temperature operators can be expressed in terms of the finite temperature ones. For example we can write

$$a_n^\alpha = a_n^\alpha(\beta) \cosh \theta_n^\alpha(\beta) + \tilde{a}_n^{\alpha +}(\beta) \sinh \theta_n^\alpha(\beta). \tag{19}$$

Using these relations the mass spectrum at finite temperature is found to be

$$\frac{\pi m^2}{2} = (D-1)\varepsilon(m^2) + \sum_{n=1}^{\infty} \Omega_n(m^2) \sum_{\alpha=1}^{D-1} [a_n^{\alpha +}(\beta) a_n^\alpha(\beta) \cosh^2 \theta_n^\alpha(\beta)$$

$$+a_n^\alpha(\beta)\tilde{a}_n^{\alpha+}(\beta)\cosh\theta_n^\alpha(\beta)\sinh\theta_n^\alpha(\beta)$$

$$+\tilde{a}_n^{\alpha+}(\beta)a_n^\alpha(\beta)\sinh\theta_n^\alpha(\beta)\cosh\theta_n^\alpha(\beta)+\tilde{a}_n^\alpha(\beta)\tilde{a}_n^{\alpha+}(\beta)\sinh^2\theta_n^\alpha(\beta)]. \tag{20}$$

From (20) it follows that the mass of the thermal ground state is given by

$$\frac{\pi m_0^2}{2} = (D-1)\varepsilon(m_0^2)+(D-1)\sum_{n=1}^{\infty}\Omega_n(m_0^2)\frac{e^{-\beta n}}{(1-e^{-\beta n})}. \tag{21}$$

It is clear that the ground state mass (which is negative at zero temperature) is to become positive at finite temperature then it will vanish at a certain temperature, $T_c(=1/\beta_c)$ given by

$$\sum_{n=1}^{\infty}\frac{ne^{-\beta_c n}}{(1-e^{-\beta_c n})} = \tfrac{1}{12}. \tag{22}$$

Before we proceed to find $T_c$ from (22) it is necessary to show that the series in the L.H.S. of (22) is convergent. To this end we note the following inequality:

$$\sum_{n=1}^{\infty} ne^{-nx} = e^{-x}(1-e^{-x})^{-2} < S = \sum_{n=1}^{\infty}\frac{ne^{-x}}{1-e^{-nx}} < (1-e^{-x})\sum_{n=1}^{\infty} ne^{-nx}$$
$$= e^{-x}(1-e^{-x})^{-3}. \tag{23}$$

Hence the series on the L.H.S. of (22) is convergent. Evaluating this series we find

$$T_c \simeq 0.118\,\pi. \tag{24}$$

Therefore, for $T > T_c$ the ground state mass shifts to a positive temperature dependent mass while for $T < T_c$, the ground state mass remains negative. In other words, at temperature greater than the critical temperature $T_c$, the tachyonic ground state becomes stabilized. It is also interesting to note that since $e^{-\beta}(1-e^{-\beta})^{-N} \to 0$ as $\beta \to \infty$ for all $N > 0$, $S \to 0$ as $\beta \to \infty$ and from (21) we recover the zero temperature situation.

In this paper we have shown that for bosonic strings in curved space, namely de Sitter space, the tachyonic ground state becomes stabilized beyond a critical temperature $T_c$ given by (24). It may be noted here that the possibility of this sort of phenomena was conjectured by Leblanc [8] in case of string in flat space.

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