STRANGE MATTER BUBBLE FORMATION INSIDE NEUTRON MATTER*

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Initial stages of the phase transition of the neutron matter into strange matter at zero temperature and finite pressure are considered. Bubble formation is calculated numerically for pressures typical for a neutron star interior. The critical bubble size decreases rapidly with pressure, reaching minimum at $p \sim 20$ MeV/fm^3.

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1. Introduction

Recently much attention has been paid to the strange matter [1–3], which is dense matter built of u, d, and s quarks. In particular the question whether neutron stars could be in fact quark stars or have at least a strange matter core has been discussed by many authors [4]. Even if the strange matter is absolutely stable in bulk, we still have to know how the phase transition could start. The latter question is not trivial as we know that at zero pressure we need a very high order weak transition to produce a bubble of the strange matter which is stable against emitting a neutron [2].

In this paper we calculate the critical size of such strange matter bubbles inside the neutron matter under pressures typical for the interior of neutron stars. Within the Fermi-gas model [2–3] for the strange matter we find that the critical size rapidly decreases with pressure and, at least for the Friedman-Pandharipande neutron matter [5], is minimal for pressures of order $15 \div 30$ MeV/fm^3. Such pressures are much below 100 MeV/fm^3, a typical central pressure of the 1.4 $M_\odot$ neutron star.

In Section 2 we briefly present the equations of state we use for the strange and the neutron matter. We discuss there also the phase equilibrium in the bulk limit. In Section 3 we describe the strange matter bubble formation, and present, in particular, our results for the pressure dependence of the critical size. Since 4 contains summary and conclusions.

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2. *Equations of state and equilibrium in bulk*

In what follows we shall consider cold ($T = 0$) equations of state. First let us briefly summarize the strange matter equation of state [2–3].

Strange matter consists of u, d and s quarks. Only s quark is massive and we denote its mass by m. The flavour equilibrium is provided via weak interactions. The gas is strongly degenerated which results in more or less the same amount of each flavour. There is a little less heavy s quarks and we have to add some small number (order 10^{-4} per baryon) of electrons in order to maintain the electric charge neutrality. The equation of state is derived in terms of the MIT bag model [6]. We take the strong coupling constant α_s equal to 0 and thus we have a gas of free quarks inside a bag. The free energy reads

$$F = -V \sum_{i=\mathrm{u,d,s,e^-}} p_i + BV + \sum_{i=\mathrm{u,d,s,e^-}} \mu_i N_i, \tag{1}$$

where B is the bag constant and p_i and μ_i are partial pressures and chemical potentials of i-th species, respectively. Differentiating over V we get the condition for the pressure balance:

$$\sum_{i=\mathrm{u,d,s,e^-}} p_i = B + p, \tag{2}$$

where p denotes external pressure applied to the system. This equation combined with flavour equilibrium and charge neutrality can be solved for the chemical potentials of all species. Consequently we get all thermodynamic properties of the strange matter in terms of two parameters: B and m. For later use we present the formula for the Gibbs energy of strange matter

$$G_{\mathrm{sm}} = F + pV = \sum_i \mu_i N_i = \mu_{\mathrm{sm}} A, \tag{3}$$

where

$$\mu_{\mathrm{sm}} = \mu_{\mathrm{u}} + \mu_{\mathrm{d}} + \mu_{\mathrm{s}} \tag{4}$$

is the baryon chemical potential and

$$A = \tfrac{1}{3}(N_{\mathrm{u}} + N_{\mathrm{d}} + N_{\mathrm{s}}) \tag{5}$$

is the baryon number of the system.

For the neutron matter we use the v_{14}+TNI equation of state derived by Friedman and Pandharipande [5].

Now we can compare the neutron and strange matter phases at $T = 0$ and external pressure p. The phase which is realized in the bulk limit is the one with lower Gibbs energy per baryon (baryon chemical potential). We calculate the strange matter baryon chemical potentials μ_{sm} for three sets of B and m values:

$$B = 66.0\ \mathrm{MeV/fm^3}, \quad m = 150\ \mathrm{MeV}, \tag{6A}$$

$$B = 66.0\ \mathrm{MeV/fm^3}, \quad m = 180\ \mathrm{MeV}, \tag{6B}$$

$$B = 70.8\ \mathrm{MeV/fm^3}, \quad m = 150\ \mathrm{MeV}. \tag{6C}$$

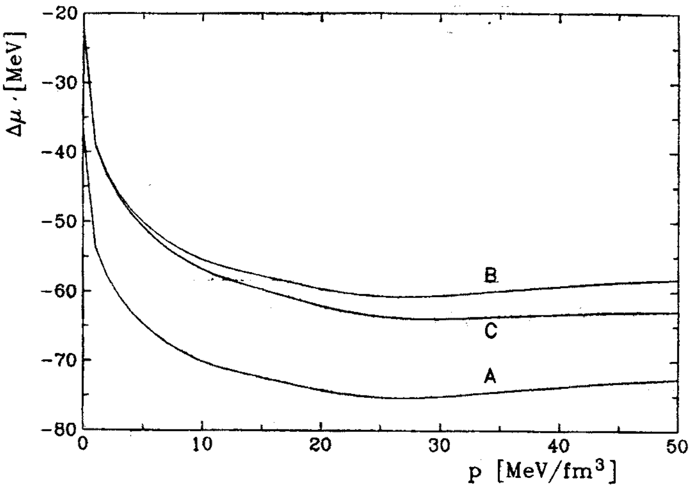

Fig. 1. Difference of the baryon chemical potentials of the strange and neutron matter for three sets (6) of B and m

At zero temperature and pressure μ_{sm} is equal to the energy per baryon and set (6A) gives $\mu_{sm} = 903$ MeV while both sets (6B) and (6C) give $\mu_{sm} = 918$ MeV. Thus all three sets give strange matter stable against decay into neutrons as well as into ^{56}Fe nuclei where energy per nucleon equals to 930 MeV.

In Fig. 1 we show the difference of baryon chemical potentials

$$\Delta\mu = \mu_{sm} - \mu_n \tag{7}$$

as a function of pressure p. The binding grows significantly for pressures up to about 25 MeV. The reason is that for relatively low pressures the neutron matter is much softer then the strange matter.

3. *Formation of bubbles*

Let us consider a system consisting of $N-A$ baryons in the neutron matter phase and a bubble of strange matter (quark bag) of baryon number A under external pressure p, as depicted in Fig. 2. Such a bubble must be produced by fluctuations and we have to calculate the Gibbs energy of the system in order to know if the bubble will grow or disappear.

The Gibbs energy reads

$$G(N, A, p) = G_{bag}(A, p) + G_n(N-A, p) + E_{surf}. \tag{8}$$

Both G_{bag} and G_n contain finite size effects and E_{surf} is the surface interaction energy. First we calculate G_{bag}. To this end we find the energy levels in the static spherical cavity approximation to the MIT bag model [6]. The standard solutions of the Dirac equation in a spherical well of radius R can be found in textbooks (see e.g. Ref. [8]). Up to normaliz-

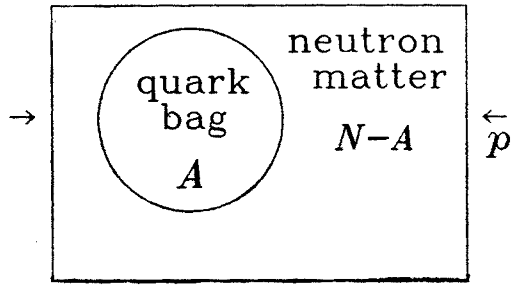

Fig. 2. A strange matter bubble of baryon number A inside neutron matter under external pressure p

ing factor they read

$$\psi_{pjl\sigma}(\vec{r}) = \begin{pmatrix} \sqrt{\omega+m}\, j_l(pr)\Omega_{jl\sigma}(\theta, \varphi) \\ -\sqrt{\omega-m}\, j_{l'}(pr)\Omega_{jl'\sigma}(\theta, \varphi) \end{pmatrix}, \tag{9}$$

where p, ω, m are quark momentum, energy and mass; j, σ its angular momentum and z-projection and $\Omega_{jl\sigma}(\theta, \varphi)$ are spherical spinors. l and l' are best given in terms of the "parity parameter" ε:

$$\varepsilon = \pm 1, \quad l = j + \frac{\varepsilon}{2}, \quad l' = j - \frac{\varepsilon}{2}. \tag{10}$$

Under spatial reflection $\psi_{pjl\sigma}$ gets multiplied by $i(-1)^l$.

Imposing the linear boundary condition

$$-i\vec{\gamma}\hat{r}\psi = \psi \quad \text{at} \quad r = R \tag{11}$$

we arrive at the following equation for energy modes

$$j_n(x) = \frac{mR}{x} + \varepsilon\sqrt{1 + \left(\frac{mR}{x}\right)^2}\, j_{n+1}(x), \tag{12}$$

where $x = pR$ and $n = j - \frac{1}{2}$.

For each quantum number n and ε there is an infinite series $x_{\varepsilon ni}$, $i = 1, 2, \ldots$ of radial excitations with energies

$$\omega_L = \sqrt{m^2 + x_L^2/R^2}, \tag{13}$$

where by L we denote the set εni.

The total energy of the bag with radius R containing $3A$ quarks reads

$$E(R) = BV - \frac{Z_0}{R} + \sum_L N_L^0 \omega_L^0 + \sum_L N_L \omega_L, \tag{14}$$

with

$$\sum_L N_L^0 + \sum_L N_L = 3A.$$

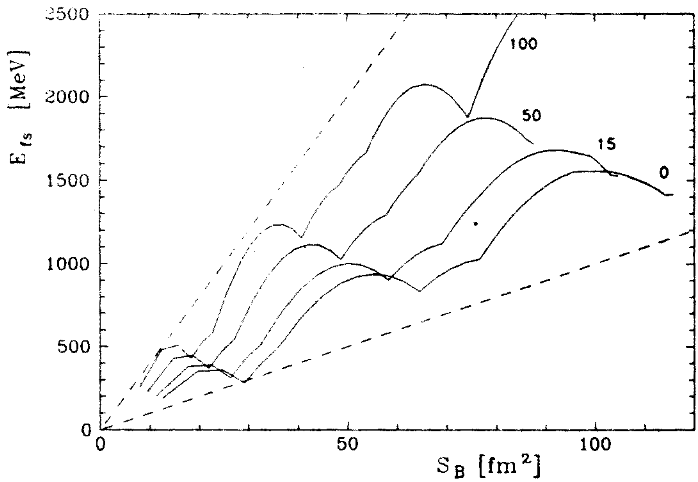

Fig. 3. Finite size energy E_{fs} versus bag surface area S_B for the set (6A) of B and m. The numbers at the curves denote pressure in MeV/fm³. Dashed lines correspond to $E_{fs} = \sigma S_B$ with $\sigma = 10$ and 40 MeV/fm², respectively

Here V is the bag volume, Z_0 is zero mode energy [6] and N_L^0, ω_L^0 and N_L, ω_L denote the number and energy of massless (u, d) and massive (s) quarks at L-th level, respectively. At finite external pressure p we need in fact the Gibbs energy

$$G_{\text{bag}}(A, p) = E(R) + pV. \tag{15}$$

To find the actual bag Gibbs energy we have to minimize Eq. (15) with respect to R. In our numerical calculation we minimize it also with respect to strangeness S (= − number of s quarks) for each A.

G_{bag} obtained in this way contains finite size energy. We calculate it as

$$E_{fs} = G_{\text{bag}}(A, p) - A\mu_{sm}(p). \tag{16}$$

In Fig. 3 we show E_{fs} versus bag surface $S_B = 4\pi R^2$ for $m = 150$ MeV, $B = 66$ MeV/fm³ and $p = 0$, 15, 50, 100 MeV/fm³. Two dashed lines correspond to $E_{fs} = \sigma S_B$ for $\sigma = 10$, 40 MeV/fm². Thus we see that if we parametrized E_{fs} by some surface tension σ its value would be surely higher than 10 MeV/fm².

Let us proceed now with the last two terms in Eq. (8). The finite size effects in G_n come from nuclear interactions. As the quark bag is colour neutral E_{surf} should be also of nuclear type. Hence we expect the finite size effects in G_n and E_{surf} to be not greater than in usual nuclear matter. The latter correspond to the surface tension below 1 MeV/fm² [7]. Thus they are much smaller than E_{fs} coming from the strange matter and we simply neglect them in the following. We take $E_{\text{surf}} = 0$ and

$$G_n(N, p) = \mu_n N. \tag{17}$$

Eq. (8) becomes

$$G(N, A, p) = G_{\text{bag}}(A, p) + \mu_{\text{n}}(p)(N - A). \tag{18}$$

Now we are ready to look at the strange matter bubble formation. We compare the system containing a bubble with baryon number A with the one containing no bubbles. The Gibbs energy difference reads

$$\Delta G(A, p) = G(N, A, p) - G(N, 0, p)$$
$$= G_{\text{bag}}(A, p) - \mu_{\text{n}}(p)A. \tag{19}$$

$\Delta G(A, p)$ equals to the minimal work system has to perform in order to produce a bubble of size A. If ΔG is negative then the system with the bubble is favoured over the one without it. We define the critical bubble size, A_{crit}, as the value of A above which ΔG is negative. This definition is rather conservative as, usually, by the critical size one denotes the size of the smallest bubble which has higher probability to grow then to disappear. In other words we expect the actual critical size to be smaller than A_{crit}. On the other hand we are aware of the simplicity of the model we use and do not want to rely on the detailed structure of the bumps in the A-dependence of ΔG.

Fig. 4 shows the results of numerical calculation of ΔG for $m = 150$ MeV, $B = 66$ MeV/fm^3. We plot there ΔG versus A for external pressure $p = 0, 1, 2, 5, 15, 30, 50, 100$ MeV/fm^3. For $p = 0$ the critical bubble has $A_{\text{crit}} = 60$ and the strangeness $-S = 24$. Such a fluctuation requires 24-th order weak transition and is therefore very unlikely to appear. For relatively low pressures (up to 15 MeV/fm^3) ΔG decreases rapidly with p which results in much easier bubble formation than for $p = 0$. For pressures above about 20 MeV/fm^3 ΔG grows again. This behaviour is analogous to the bulk case (see Fig. 1), and can possibly indicate that the Friedman-Pandharipande neutron matter is too

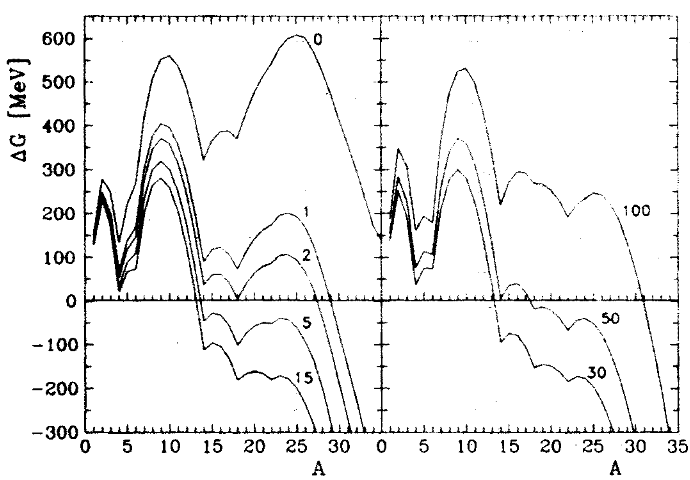

Fig. 4. Gibbs energy difference, ΔG, of the strange and neutron matter for the set (6A) of B and m. The numbers over the curves denote pressure in MeV/fm^3

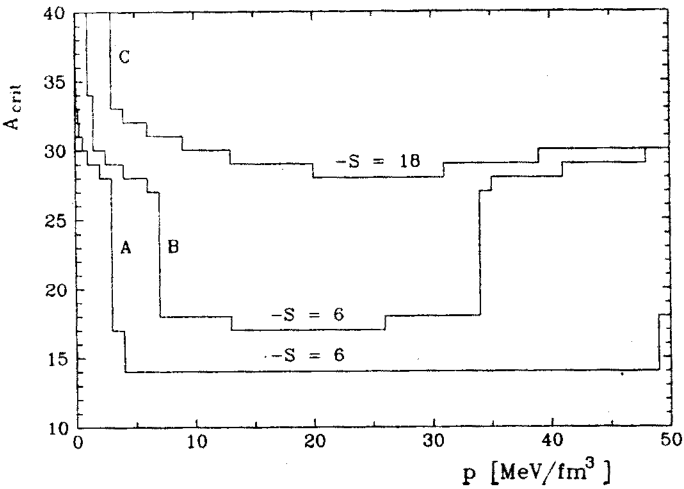

Fig. 5. Critical size A_{crit} for all three sets (6) of B and m. The strangeness of smallest critical bubbles is given at the curves

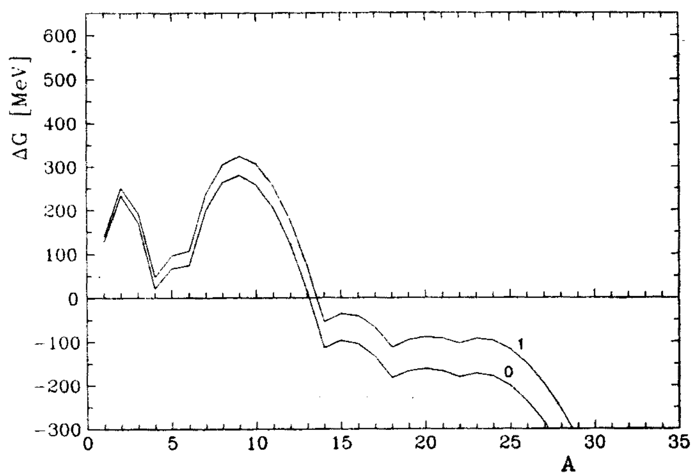

Fig. 6. Additional surface energy effect on ΔG for the set (6A) of B and m. The numbers over the curves denote surface tension in $\mathrm{MeV/fm^2}$

soft at high pressure. The bumpy structure of ΔG suggests that the bubble formation goes through a sequence of metastable states. These states correspond to the closed shells of bag orbitals.

In Fig. 5 we show A_{crit} as the function of p for three sets (6) of B and m. In all three cases the smallest critical bubbles are obtained at some finite and relatively low pressure. The strangeness of these bubbles is given at the curves. For $B = 66\ \mathrm{MeV/fm^3}$ (sets (6A) and (6B)) it is $-S = 6$ which makes the bubble formation much more likely than without external pressure.

Finally, we test how much our results could be changed by taking into account the surface effects of neutron matter. To this end we add σS_B to the right hand side of Eq. (19). The curves in Fig. 6 show ΔG for B and m of set (6A), $p = 15$ MeV/fm^3 and $\sigma = 0$ and 1 MeV/fm^2. We see that the effect is practically negligible and the critical size remains the same.

4. *Summary and conclusions*

We have investigated the strange matter bubble formation inside the neutron matter under finite pressure. The strange matter equation of state depends on the values of the bag constant B and the strange quark mass m. All three sets of B and m used in our calculation give strange matter stable in the bulk limit at external pressure $p = 0$.

Using Friedman-Pandharipande [5] equation of state of neutron matter we have found that the bubbles are most easily produced at pressures in the range of $15 \div 30$ MeV/fm^3. The central pressure of a typical neutron star is much higher. For example, a 1.4 $M_\odot$ neutron star has the central pressure of 150 MeV/fm^3 and the radius of about 10 km. The pressure drops below 30 MeV/fm^3 at distances above 7 km from the center. Thus we see that the phase transition should start in quite distant layers rather than close to the center of the neutron star.

The actual kinetics of the phase transition requires much more detailed analysis of fluctuations which could produce a bubble of, say, $A = 14$ and $-S = 6$. We expect that it should go by formation of metastable bubbles corresponding to the minima of ΔG (see Fig. 4) and quite possibly by two- or three-body collisions of these states with each other or strange baryons (Σ^-, Λ^0, ...) present in the neutron matter.

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REFERENCES

[1] E. Witten, *Phys. Rev.* **D30**, 272 (1984).
[2] E. Farhi, R. L. Jaffe, *Phys. Rev.* **D30**, 2379 (1984).
[3] T. Chmaj, W. Słomiński, *Phys. Rev.* **D40**, 165 (1989).
[4] P. Haensel, J. L. Zdunik, R. Schaeffer, *Astron. Ap.* **160**, 121 (1986); A. V. Olinto, *Phys. Lett.* **192B**, 71 (1987); C. Alcock, E. Farhi, A. V. Olinto, *Astrophys. J.* **310**, 261 (1986).
[5] B. Friedman, V. R. Pandharipande, *Nucl. Phys.* **A361**, 502 (1981).
[6] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, V. F. Weisskopf, *Phys. Rev.* **D9**, 3471 (1974); A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, *Phys. Rev.* **D10**, 2599 (1974); T. De Grand, R. L. Jaffe, K. Johnson, J. Kiskis, *Phys. Rev.* **D12**, 2060 (1975).
[7] G. Baym, H. A. Bethe, C. J. Pethick, *Nucl. Phys.* **A175**, 225 (1971); D. G. Ravenhall, C. D. Bennet, C. J. Pethick, *Phys. Rev. Lett.* **28**, 978 (1972).
[8] A. Messiah, *Quantum Mechanics*, vol. II, North Holland, Amsterdam 1966.