HIGHER ORDER CORRECTIONS TO JETS AND ONE PARTICLE INCLUSIVE PRODUCTION*

By P. Chiappetta**

LAPP, B.P. 110, 74941 ANNECY-le-Vieux Cedex, France

(Received October 12, 1989)

The present status of complete $O(\alpha_s^3)$ QCD corrections to jet production and one hadron inclusive production in hadronic collisions is reported.

PACS numbers: 12.38.-t

1. Introduction

Hadron jets from hadronic colliders are an important tool to test in a quantitative way the QCD improved parton model. The present situation, as we will see, does not allow to perform a quantitative comparison between QCD and experiment. Besides the uncertainty in the knowledge of integrated luminosity the main systematic experimental error is the lack of precision in the measurement of the jet energy in the calorimeter, leading to an overall uncertainty of $\pm 50\%$. On the theoretical side the data on inclusive jet production can be compared directly either to Born cross section (involving a two body $O(\alpha_s^2)$ parton parton scattering at subprocess level) or to sophisticated Monte Carlo programs including a part of higher order corrections. It should be stressed that the main theoretical uncertainty, i.e. the dependence of cross sections on scales M of evoluted parton densities (factorization scale) and of the strong coupling constant μ (renormalization scale) can be reduced by performing a complete $O(\alpha_s^3)$ calculation of subprocesses contributing to jet production:

$$H_1(K_1)+H_2(K_2) \to \text{jet}\,(P)+X \tag{1}$$

or inclusive production of a hadron:

$$H_1(K_1)+H_2(K_2) \to H_3(P)+X. \tag{2}$$

As an illustration if P_T is the transverse momentum of the jet, choosing $\mu = M = P_T/2$ or $\mu = M = 2P_T$, affects the Born cross section by a factor of 3–4 at present collider energies. Inclusive jet cross section is sensitive to the jet definition. We will focus on two

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* Invited lecture given at XXIX Cracow School of Theoretical Physics, Zakopane, Poland, June 2–12, 1989.

** Permanent address: Centre de Physique Théorique CNRS Luminy Case 907 F-13288, Marseille, Cedex 9.

algorithms. The first one, originating from e^+e^- collisions, is the Sterman-Weinberg scheme in which the jet is defined as an arbitrary set of hadrons within a cone of semi opening angle δ. For the second one, corresponding to UA1 algorithm, the jet is defined as the energy deposited inside a cone of radius R (where $R = \sqrt{\Delta\phi^2+\Delta\eta^2}$) in the pseudorapidity η-azimuthal angle ϕ space. Let us emphasize that the Born cross section has no explicit dependence on the jet definition, i.e. no term proportional to $\ln \delta$ or $\ln R$, and cannot be viewed as a jet cross section. The advantage of higher order calculations is that they explicitely depend on the jet definition. This evaluation of next to leading corrections started in the early 80's (when ISR energy was the highest one) for subprocesses involving quarks of different flavors [1]. The second motivation for studying now reactions (1) and (2) beyond leading order is that they could provide direct measurement of the gluon structure function. In DIS since there is no electroweak coupling of the gluon to the virtual photon, it is indirectly seen as a scaling violation of the structure function $F_2(x)$. Moreover DIS data are not sensitive to the gluon structure function in the large x region. Reactions from hadron hadron collisions — for which subprocesses involving gluons are dominant in certain kinematical regions — allow one direct extraction of gluon structure function. This has recently been done for direct photon experiments [2] ($H_1H_2 \to \gamma+X$). Within the optimized approach [3] to fix the scales μ and M good agreement with the precise BCDMS determination [4] leads to

$$xG(x) = A_g(1-x)^{n_g}, \tag{3}$$

where $n_g = 4.0\pm0.11$ $(+.8-.6)$ and $\Lambda = 231.5\pm17\pm50$ MeV. When a complete next to leading order calculation will be available, jet production from hadronic collisions will be well suited to perform a similar analysis and should improve our knowledge about the gluon structure function. This is why $O(\alpha_s^3)$ corrections to jet and one hadron inclusive production are important to test quantitatively QCD. Let us add that such a precise knowledge is crucial to establish any breakdown of the standard model like a firm limit on the compositeness scale of proton constituents.

The present status of the calculation is the following. Two groups are working on this subject. The first one [5] has performed a computation of $O(\alpha_s^3)$ corrections for all partonic subprocesses for one hadron production and inclusive jet production within a cone of semi aperture δ small. The second group [6] has performed an evaluation to $O(\alpha_s^3)$ of one jet inclusive cross section for a jet defined according to UA1 algorithm restricted to pure gluonic case.

After a presentation of the calculational method we will discuss numerical results for present $p\bar{p}$ colliders with peculiar attention about the reduced sensitivity to factorization and renormalization scales and about the dependence on jet size.

2. *Calculational method*

The method is based on the calculation [7] of full $O(\alpha_s^3)$ matrix elements for all $2 \to 2$ and $2 \to 3$ parton subprocesses in $n = 4-2\varepsilon$ dimensions. One starts from the expression of the matrix elements squared for real $2 \to 3$ parton subprocesses. Using algebraic manipulations with Macsyma and/or Reduce one performs the phase space integration of these real subprocesses. Since we are interested in one jet or one hadron inclusive production, i.e. we are considering one parton in the final state, we have to integrate over two four momenta. The basic idea is to express all integrals in terms of a few basic ones which are computed analytically. Once this has been done we are left with two types of infrared divergences i.e. terms like $1/\varepsilon^2$ and $1/\varepsilon$. The first ones are cancelled by adding virtual corrections. Let us take the example of inclusive gluon production from gluon gluon subprocesses. Two real subprocesses contribute: $gg \to ggg$ and $gg \to gq\bar{q}$. After adding virtual corrections to $gg \to gg$ and $gg \to q\bar{q}$ we are left with mass singularities we will absorb into the structure and fragmentation functions beyond leading order i.e. by adding terms like

$$\frac{\alpha_s(\mu^2)}{2\pi} H_{p_i p_i'}(x) d\sigma^{\text{Born}}(p_i p_j \to p_i p_k), \tag{4}$$

where

$$H_{p_i p_i'}(x) = P_{p_i p_i'}(x) \left\{ -\frac{1}{\varepsilon} - \ln 4\pi - \gamma_E + \ln \frac{M^2}{\mu^2} \right\} + f_{p_i p_i'}(x). \tag{5}$$

In Eq. (5) $P_{p_i p_i'}(x)$ is the Altarelli-Parisi Kernel whereas $f_{p_i p_i'}(x)$ is the finite $O(\alpha_s)$ correction for structure functions.

For $gg \to g+X$ we will add emission of one gluon from incoming legs, emission of a quark (or antiquark) from incoming legs followed by a $qg \to qg$ Born cross section, fragmentation of a gluon from an outgoing gluon, production of a $q\bar{q}$ pair from gg followed by fragmentation of the quark into a gluon. For one jet inclusive production the collinear divergences associated to final partons are automatically cancelled by adding contributions of one and two partons in the cone. As discussed in Ref. [5] the finite next to leading corrections to structure functions have only to be calculated for quarks [8]:

$$f_{qq}(x) = C_F \left\{ (1+x^2) \left[\frac{\ln(1-x)}{1-x} \right]_+ - \frac{3}{2} \frac{1}{(1-x)_+} - \frac{1+x^2}{1-x} \ln x + 3 + 2x - \left(\frac{9}{2} + \frac{\pi^2}{3} \right) \delta(1-x) \right\}. \tag{6}$$

As can been seen by looking at Eq. (6) they contain terms which become particularly large near the boundary of phase space ($x \to 0$ and $x \to 1$). In order to test the sensitivity to the factorization scheme we will consider two possibilities hereafter denoted as schemes CQ = 0 and CQ = 1. For choice CQ = 0 we will take $f_{p_i p_i'}(x) = 0$ except for quarks. For choice CQ = 1 we will incorporate in the f's the relevant kinematical factors obtained by multiplying $P_{p_i p_i'}(x)$ by $\ln\left(\frac{1-x}{x}\right)$ and for the finite terms $d_{p_i p_i'}(x)$ of fragmentation functions by multiplying $P_{p_i p_i'}(x)$ by $\ln((1-x)x^2)$. The complete expressions are obtained by imposing energy momentum sum rules. The cross section can finally be expressed in terms of convolution of partonic cross sections free of singularities with evoluted structure functions.

$$E\frac{d\sigma}{d^3P}=\frac{1}{\pi S}\sum_{ij}\int_{VW}^{V}\frac{dv}{1-v}\int_{VW/v}^{1}\frac{dw}{dw}F_{p_i}^{H_1}(x_1,M^2)F_{p_j}^{H_2}(x_2,M^2)$$

$$\times\left[\frac{1}{v}\frac{d\sigma^0_{ij\to\text{jet}}}{dv}(s,v)\delta(1-w)+\frac{\alpha_s(\mu^2)}{2\pi}K_{ij\to\text{jet}}(s,v,w;\mu^2,M^2,\delta)\right], \tag{7}$$

where $S=(K_1+K_2)^2$, $T=(K_1-P)^2$, $U=(K_2-P)^2$, $V=(1+T/S)$, $W=-U/(S+T)$, $X_1=VW/vw$, $X_2=(1-V)/(1-v)$ and $s=x_1x_2S$. The advantage of variables v and w is that we can express the partonic cross section in terms of distributions. The correction factor $K_{ij\to\text{jet}}$ can be written as:

$$K_{ij\to\text{jet}}(s,v,w;\mu^2,M^2,\delta^2)=\frac{\pi\alpha_s^2(\mu^2)}{2C_iC_js}$$

$$\times\left\{\left[c_1+\tilde{c}_1\ln\left(\frac{s}{M^2}\right)+\tilde{\tilde{c}}_1\ln\left(\frac{s}{E^2\delta^2}\right)+\hat{c}_1\ln\left(\frac{s}{\mu^2}\right)\right]\delta(1-w)\right.$$

$$+\left[c_2+\tilde{c}_2\ln\left(\frac{s}{M^2}\right)+\tilde{\tilde{c}}_2\ln\left(\frac{s}{E^2\delta^2}\right)\right]\frac{1}{(1-w)_+}$$

$$\left.+c_3\left[\frac{\ln(1-w)}{(1-w)}\right]_+\right\}+K'_{ij\to\text{jet}}(s,v,w,\delta), \tag{8}$$

where the coefficient C_i is equal to N for quarks and N^2-1 for gluons. As stressed in the introduction the calculation of group I is restricted to small δ values for which the kinematics is easier since the jet total momentum P is such that $P^2\approx 0$ and $E\approx|\vec{P}|$. The group II which computes the inclusive jet cross section for any value of δ splits the integral into a singular piece and a regular one. The singular piece is calculated analytically in order to ensure that the singularities are correctly cancelled whereas the finite left part is evaluated numerically using VEGAS multidimensional integration involving five variables. Our group has started to extend the jet calculation to large δ values using the exact analytical result for jet production within $\delta=0.1$ and performing the evaluation of remaining cone part by a Monte Carlo program. This computation has now been done for gluon-gluon subprocesses. Let us now focus on numerical results. Since the two groups have not yet performed a complete calculation at next to leading order, full quantitative comparison to collider data is not possible.

3. *Numerical results*

The questions we will address are the dependence on jet size, the reduced sensitivity to renormalization and factorization scales and to the factorization scheme. We will give results for jet cross section as a function of $\eta=2P_T/\sqrt{s}$ (where P_T is the transverse momentum of the jet) from $p\bar{p}$ collisions at $\sqrt{s} = 0.63$ and 1.8 TeV for $\theta_{cm} = 90°$. As shown in Fig. 1 the dependence of inclusive jet cross section on jet size R is logarithmic. We will now concentrate on scaled jet cross section $\Sigma = P_T^4 E \dfrac{d\sigma}{d^3P}$ for $\delta = 0.2$ and $N_F = 4$. We first plot in Figs 2a (resp. 2b) Σ for $\mu = M = 3P_T/4$ (resp. $\mu = M = 2P_T$) at $\sqrt{s} = 0.63$ TeV for CQ = 1. The comparison between the two figures shown that when higher order corrections are included the cross-section is quite stable, unlike the Born cross section which

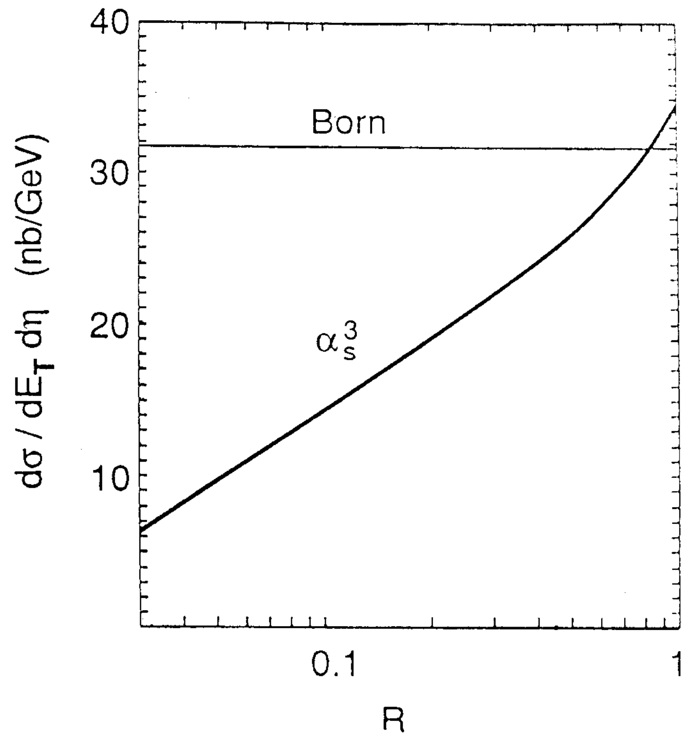

Fig. 1. Inclusive jet cross section $\dfrac{d\sigma}{dE_T d\eta}$ for $E_T = 50$ GeV, $\eta = 0$ and $\sqrt{s} = 1.8$ TeV at Born and $O(\alpha_s^3)$ level as a function of jet size R

is varying by a factor of 3. A similar behaviour is obtained for CQ = 0. Similar results hold at $\sqrt{s} = 1.8$ TeV. Let us mention that group II got a reduction of the sensitivity of the theoretical cross section due to the choice of mass scales by a factor of 2 to 3. Similar results have been obtained for one hadron inclusive production. The sensitivity to the factorization scheme affects mainly the gg subprocesses at large η but does not show up after sum of all contributions. This is shown more in detail in Fig. 3, where the K factor $= [O(\alpha_s^2)+O(\alpha_s^3)]/O(\alpha_s^2)$ — with α_s evoluted to 2 loops (1 loop) in the numerator (denominator) — is plotted for $M = \mu = 3P_T/4$, $2P_T$. The scheme corresponding to CQ = 1 looks therefore more stable perturbatively.

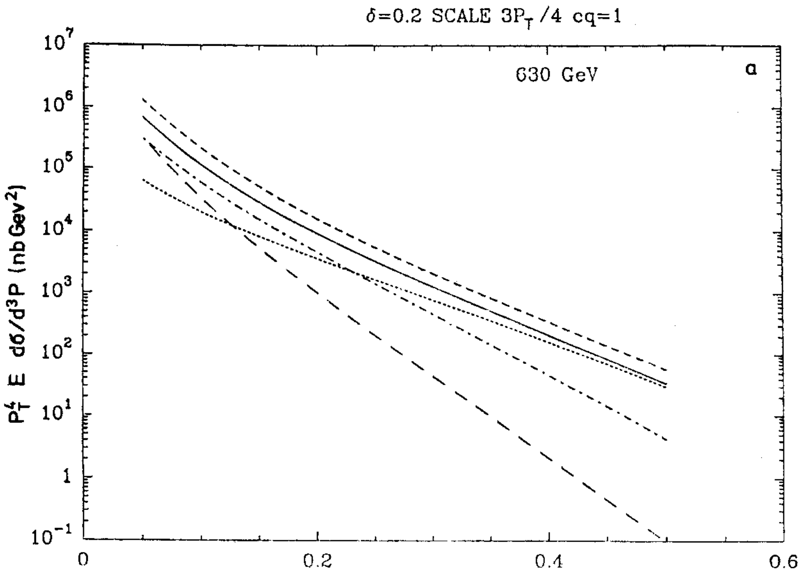

Fig. 2a. The scaled jet cross section $\Sigma = P_T^4 E \dfrac{d\sigma}{d^3P}$ as a function of $\eta = 2P_T/\sqrt{s}$ at $\sqrt{s} = 630$ GeV for $\delta = 0.2$, CQ = 1 and $\mu = M = 3P_T/4$. Dashed curve: Born cross section. Full curve: $O(\alpha_s^2)+O(\alpha_s^3)$ prediction for all subprocesses. Dot dashed curve: $O(\alpha_s^2)+O(\alpha_s^3)$ prediction for quark gluon subprocesses. Long dashed curve: $O(\alpha_s^2)+O(\alpha_s^3)$ prediction for gluon gluon subprocesses. Small dashed curve: $O(\alpha_s^2)+O(\alpha_s^3)$ prediction for all quark subprocesses

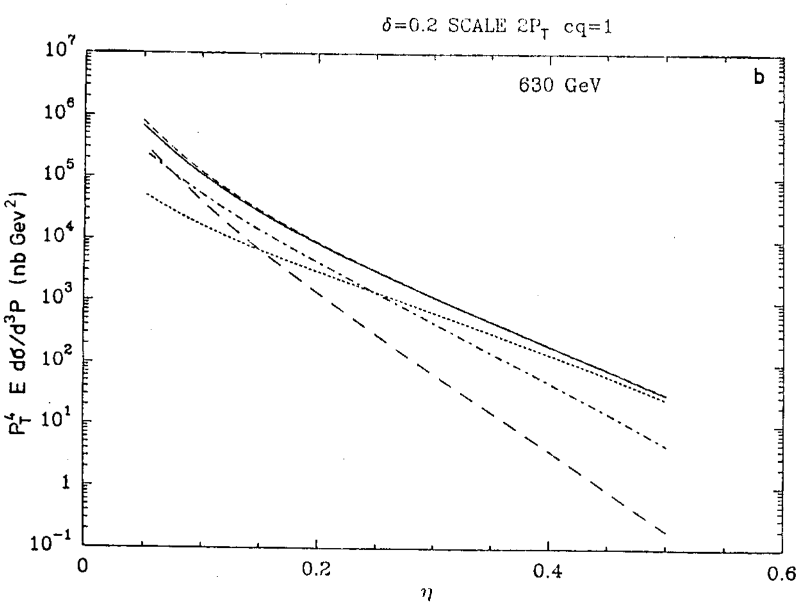

Fig. 2b. Same caption as for Fig. 2a for $\mu = M = 2P_T$

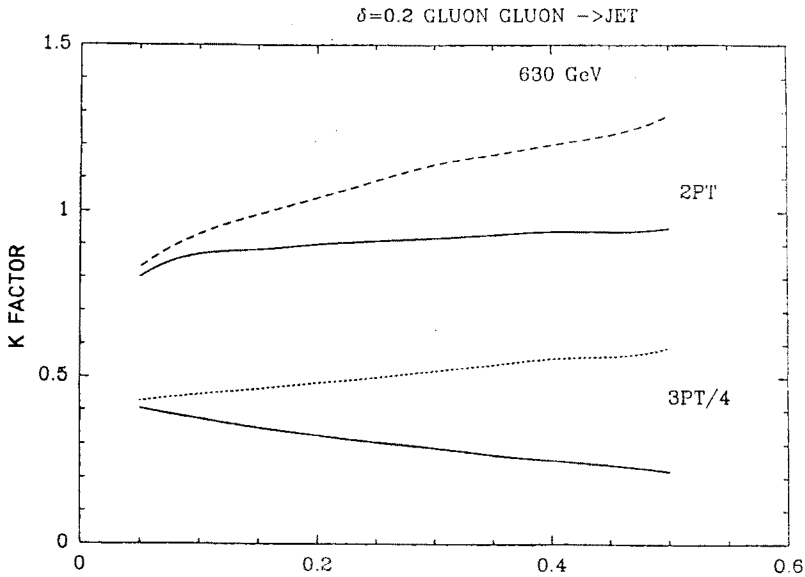

Fig. 3. The K factor for gluon gluon subprocesses at $\sqrt{S} = 630$ GeV for $\delta = 0.2$. Full curves: CQ = 1. Dashed curves: CQ = 0. Upper curves: $\mu = M = 2P_T$. Lower curves: $\mu = M = 3P_T/4$

4. *Conclusions*

We have presented the present status of full calculation of $O(\alpha_s^3)$ radiative corrections to high P_T inclusive jet cross section and one hadron inclusive production. Although the calculation is not yet complete we have shown that the theoretical uncertainty due to the arbitrary renormalization and factorization scales has been sizeably reduced, showing an excellent stability of $O(\alpha_s^2)+O(\alpha_s^3)$ cross section upon change of the scales in the range $\frac{3P_T}{4} \leqslant \mu, M \leqslant 2\,P_T$. The dependence on the factorization scheme is weak. Since analytical results for one hadron inclusive production and direct photon production at next to leading order have been performed a precise determination of γ/π^0 ratio is in progress.

It is a pleasure to thank the organizers of this XXIX Cracow School for such a successful and stimulating conference in a friendly atmosphere.

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