ONE-PHOTON EXCHANGE QUASIPOTENTIALS OF TWO-BODY SYSTEMS

By E. A. Dey, V. N. Kapshaj and G. Yu. Tyumenkov

Department of Physics, F. Skorina University, Gomel*

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In the covariant single-time approach to the quantum field theory the expressions of the one-photon exchange quasipotentials (the kernels of three-dimensional integral equations for the relativistic wave functions) of two-body systems are obtained. The systems of particles with spins (1/2, 1/2), (1/2, 0) and (0, 0) are considered. In the calculations the double-time Green's functions are used. It is shown that the obtained quasipotentials coincide with the corresponding Feynman amplitudes on the energy-shell.

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The covariant single-time approach to the quantum field theory [1–3] was successfully applied to investigation of the static characteristics of bound states (mass-spectra, magnetic moments of hadrons, etc.) and to the description of elastic and deep-inelastic scattering of compound particles (see, for example, [4, 5]). The basic object of this approach is the covariantly determined single-time wave function of a bound state [6] which has certain advantages over the Bethe–Salpeter wave function [7, 8]. The single-time wave function has the probabilistic interpretation and it is found as a solution of a three-dimensional integral equation [2, 3]. These advantages are consequences of the elimination of relative time which has no physical meaning. The role of the principle of causality in the problem of single-time reduction in the quantum field theory has been shown in Refs. [9, 10].

The kernel of a three-dimensional integral equation, the quasipotential, depends on the total energy of the system as a parameter. The procedure of its construction based on the Green's function has been formulated in Ref. [2]. It has been shown in [11] that the quasipotential can be constructed by using the retarded part of Green's function. However, as a rule, the determination of the quasipotential by an other methos [12] based on the physical scattering amplitude was used. The complementary determinations are required here because the amplitude is known only on the energy-shell, while the equation for the wave function is used outside the energy-shell. The quasipotential of a system of

* Address: F. Scorina University, Sovietskaya 104, Gomel 246699, USSR.

two scalar particles interacting by the one-spinless-boson exchange was found in [13]. The authors of the paper [13] applied the covariant double-time Green function for this purpose. Subsequently analogous questions were considered in Refs. [14–16].

In the present paper the one-photon exchange quasipotentials for two-body systems are found by the Green's function method. A system of two spinor particles, a system of two scalar particles and a system of spinor and scalar particles are considered. The calculations are performed in the α-gauge.

The covariant single-time wave function of a two-body system is taken in the following form [2, 6]

$$(2\pi)^4\delta(P-K)\Psi_K(p|\lambda) = \int \exp(ip_1x_1+ip_2x_2)\delta(\lambda x_1-\lambda x_2) \times \langle 0|\Psi_1(x_1)\Psi_2(x_2)\,|P_n\rangle d^4x_1d^4x_2. \tag{1}$$

In this formula $\Psi_1(x_1)$, $\Psi_2(x_2)$ are the Heisenberg operators of interacting fields, $|P_n\rangle$ is a vector from the complete set describing the bound state with the 4-momentum P_n and mass M_n, and besides $P_{0n}^2 = \boldsymbol{P}_n^2+M_n^2$, n are all other quantum numbers. The time-like unit vector $\lambda_\mu(\lambda^2 = \lambda_0^2-\boldsymbol{\lambda}^2 = 1)$ characterizes the system in which the times of particles 1 and 2 are equated, as a rule, $\lambda_\mu = P_\mu/\sqrt{P^2}$. Here the total and relative momenta are introduced:

$$P = p_1+p_2; \quad p = \eta_2p_1-\eta_1p_2; \quad P_n^2 = M_n^2, \tag{2}$$

and

$$\eta_1 = \frac{M_n^2+m_1^2-m_2^2}{2M_n^2}; \quad \eta_2 = \frac{M_n^2-m_1^2+m_2^2}{2M_n^2},$$

where m_1 and m_2 are the masses of particles 1 and 2.

The Lorentz transformation laws for the spinor operators and for the state vectors allows us to establish the following property of the wave function [6]:

$$\Psi_P(p|\lambda) = S_1(L_\lambda)S_2(L_\lambda)\Psi_{\mathring{P}_n}(\mathring{p}|\mathring{\lambda}).$$

Here $S_i(L_\lambda)$, $i = 1,2$ are the boost matrices which for the scalar and spinor fields are of the forms, respectively,

$$S(L_\lambda) = 1; \quad S(L_\lambda) = \sqrt{\frac{\lambda_0+1}{2}}\left[\frac{(\boldsymbol{\lambda}\times\boldsymbol{\alpha})}{\lambda_0+1}+1\right],$$

where $\boldsymbol{\alpha}$ are the Dirac matrices. The vectors with zero on their top result from the Lorentz transformation L_λ^{-1}:

$$\mathring{\lambda} = L_\lambda^{-1}\lambda = (1, \boldsymbol{0}); \quad \mathring{p} = L_\lambda^{-1}p.$$

The covariant double-time Green's function of particles 1 and 2 has, by analogy with [2], the form

$$(2\pi)^4\delta(P-K)\tilde{G}(P;\, p, k|\lambda) = \int \exp(ip_1x_1+ip_2x_2-ik_1y_1-ik_2y_2)$$

$$\times \langle 0|T\Psi_1(x_1)\Psi_2(x_2)\bar{\Psi}_2(y_2)\bar{\Psi}_1(y_1)\,|0\rangle$$

$$\times\,\delta(\lambda x_1-\lambda x_2)\delta(\lambda y_1-\lambda y_2)d^4x_1d^4x_2d^4y_1d^4y_2. \tag{3}$$

The momenta K and k are expressed through k_1 and k_2, in analogy with (2). The Lorentz transformation law for (3) is

$$\tilde{G}(P;\, p,\, k|\lambda) = S_1(L_\lambda)S_2(L_\lambda)\tilde{G}(\mathring{P};\, \mathring{p},\, \mathring{k}|\mathring{\lambda})S_1^{-1}(L_\lambda)S_2^{-1}(L_\lambda).$$

For the function $\tilde{G}$ one can obtain the spectral representation [2, 6]:

$$\tilde{G}(\mathring{P};\, \mathring{p},\, \mathring{k}|\mathring{\lambda}) \equiv \tilde{G}(\mathring{P};\, \mathring{\boldsymbol{p}},\, \mathring{\boldsymbol{k}}) = \tilde{G}_{\text{ret}}(\mathring{P}:\, \mathring{\boldsymbol{p}},\, \mathring{\boldsymbol{k}})+\tilde{G}_{\text{adv}}(\mathring{P};\, \mathring{\boldsymbol{p}},\, \mathring{\boldsymbol{k}}), \tag{4}$$

$$\tilde{G}_{\text{ret}}(\mathring{P};\, \mathring{\boldsymbol{p}},\, \mathring{\boldsymbol{k}}) = \sum_n \frac{\Psi_{\mathring{P}}(\mathring{\boldsymbol{p}})\bar{\Psi}_{\mathring{P}}(\mathring{\boldsymbol{k}})}{\mathring{P}_0-\sqrt{\mathring{\boldsymbol{P}}^2+M_n^2}+i0} + \dots . \tag{5}$$

The wave function $\Psi_{\mathring{P}}(\mathring{\boldsymbol{p}})$ in formula (5) has, in accordance with (1), the following form:

$$\Psi_{\mathring{P}}(\mathring{\boldsymbol{p}}) \equiv \Psi_{\mathring{P}}(\mathring{p}|\mathring{\lambda}) = \int \langle 0|\Psi_1(\eta_2\mathring{\boldsymbol{x}})\Psi_2(-\eta_1\mathring{\boldsymbol{x}})|\mathring{P}\rangle \exp{(i\mathring{\boldsymbol{p}}\mathring{\boldsymbol{x}})}d\mathring{\boldsymbol{x}}. \tag{6}$$

To obtain the quasipotential equation for spin particles, the wave function (6) and the Green function (4) must be projected on the positive-frequency subspace [6] with the help of spinors $u(\mathring{\boldsymbol{p}}_{1,2})$, $u(\mathring{\boldsymbol{k}}_{1,2})$, where in accordance with (2)

$$\boldsymbol{p}_1 = \eta_1\mathring{\boldsymbol{P}}+\mathring{\boldsymbol{p}}; \quad \mathring{\boldsymbol{p}}_2 = \eta_2\mathring{\boldsymbol{P}}-\mathring{\boldsymbol{p}};$$

$$\mathring{\boldsymbol{k}}_1 = \eta_1\mathring{\boldsymbol{K}}+\mathring{\boldsymbol{k}}; \quad \mathring{\boldsymbol{k}}_2 = \eta_2\mathring{\boldsymbol{K}}-\mathring{\boldsymbol{k}}.$$

The spinor are normalized by the condition $\overset{+}{u}{}^\sigma(p)\, u^r(p) = \delta^{\sigma r}$. For the system of spinor and scalar particles the projection is realized in the form:

$$X^\sigma_{\mathring{P}}(\mathring{\boldsymbol{p}}) = \overset{+}{u}{}^\sigma(\mathring{\boldsymbol{p}}_1)\Psi_{\mathring{P}}(\mathring{\boldsymbol{p}}), \tag{7}$$

$$\tilde{G}^\sigma_r(\mathring{P};\, \mathring{\boldsymbol{p}},\, \mathring{\boldsymbol{k}}) = \overset{+}{u}{}^\sigma(\mathring{\boldsymbol{p}}_1)\tilde{G}(\mathring{P};\, \mathring{\boldsymbol{p}},\, \mathring{\boldsymbol{k}})u^r(\mathring{\boldsymbol{k}}_1).$$

For a system of two spinor particles the projection is defined as

$$X^{\sigma_1\sigma_2}_{\mathring{P}}(\mathring{\boldsymbol{p}}) = \overset{+}{u}{}^{\sigma_1}(\mathring{\boldsymbol{p}}_1)\overset{+}{u}{}^{\sigma_2}(\mathring{\boldsymbol{p}}_2)\Psi_{\mathring{P}}(\mathring{\boldsymbol{p}}),$$

$$\tilde{G}^{\sigma_1\sigma_2}_{r_1r_2}(\mathring{P};\, \mathring{\boldsymbol{p}},\, \mathring{\boldsymbol{k}}) = \overset{+}{u}{}^{\sigma_1}(\mathring{\boldsymbol{p}}_1)\overset{+}{u}{}^{\sigma_2}(\mathring{\boldsymbol{p}}_2)\tilde{G}(\mathring{P};\, \mathring{\boldsymbol{p}},\, \mathring{\boldsymbol{k}})u^{r_1}(\mathring{\boldsymbol{k}}_1)u^{r_2}(\mathring{\boldsymbol{k}}_2). \tag{8}$$

For a system of two scalar particles there is no projection operation. However, for the notation being general we put $X_{\mathring{P}}(\mathring{\boldsymbol{p}}) \equiv \Psi_{\mathring{P}}(\mathring{\boldsymbol{p}})$.

The quasipotential of a two-body system can be expressed in terms of the total (causal) Green's function and its retarded part as follows:

$$V = \hat{G}_{(0)}^{-1} - \hat{G}^{-1}; \quad V_{ret} = \hat{G}_{(0)ret}^{-1} - \hat{G}_{ret}^{-1}; \quad \hat{G} = i\tilde{G}/(2\pi)^3;$$

and in the second order of perturbation theory

$$V = \hat{G}_{(0)}^{-1}\hat{G}_{(2)}\hat{G}_{(0)}^{-1}; \quad V_{ret} = \hat{G}_{(0)ret}^{-1}\hat{G}_{(2)ret}\hat{G}_{(0)ret}^{-1}. \tag{9}$$

The functions $X_{\mathring{p}}$, introduced in formulae (7) and (8) satisfy the equation

$$[\hat{G}_{(0)}^{-1} - V] * X_{\mathring{p}} = 0; \quad [\hat{G}_{(0)ret}^{-1} - V_{ret}] * X_{\mathring{p}} = 0. \tag{10}$$

The symbol (*) means the integration over the three-dimensional momentum.

Explicit expressions of free double-time Green's functions can be easily obtained. So in each case we consider, we have the relation (here and later on zero over the vectors P, $\boldsymbol{p}$, $\boldsymbol{k}$ is omitted)

$$\hat{G}_{(0)}(P; \boldsymbol{p}, \boldsymbol{k}) = i(2\pi)^3 \hat{G}_{(0)}(P; \boldsymbol{p})\delta(\boldsymbol{p}-\boldsymbol{k}).$$

For a system of particles with spins (1/2,0) and (1/2, 1/2) we obtain, respectively,

$$\hat{G}_{(0)r}^{\sigma}(P; \boldsymbol{p}) = \frac{\delta_r^\sigma}{2\omega_2^p} R_p; \quad \hat{G}_{(0)r_1r_2}^{\sigma_1\sigma_2}(P; \boldsymbol{p}) = R_p \delta_{r_1}^{\sigma_1}\delta_{r_2}^{\sigma_2}, \tag{11}$$

and for a system of spinless particles

$$\hat{G}_{(0)}(P; \boldsymbol{p}) = \frac{A_p - R_p}{4\omega_1^p\omega_2^p}; \quad \hat{G}_{(0)ret}(P; \boldsymbol{p}) = -\frac{R_p}{4\omega_1^p\omega_2^p}. \tag{12}$$

In formulae (11) and (12) the following notation

$$\omega_j^p = \sqrt{\boldsymbol{p}_j^2 + m_j^2}, \quad j = 1, 2;$$

$$R_p = (P_0 - \omega_1^p - \omega_2^p + i0)^{-1}; \quad A_p = (P_0 + \omega_1^p + \omega_2^p - i0)^{-1}$$

is used.

It is to be noted that the projected functions (11) coincide with the retarded functions for the same systems. It follows from the explicit expression of R_p.

Now let's find the explicit form of quasipotentials for considered systems. The double-time Green's functions $\tilde{G}$, their inverse functions and, consequently, the quasipotential will be found by perturbation theory. The translational invariance allows us to write down the momentum representation of the 4-time Green's function in the form

$$G(P, p; K, k) = (2\pi)^4\delta(P-K)G(P; p, k).$$

The double-time function $\tilde{G}(P; \boldsymbol{p}, \boldsymbol{k})$ is expressed through the function $G(P; p, k)$ in the following way [2, 6]

$$\tilde{G}(P; \boldsymbol{p}, \boldsymbol{k}) = \frac{1}{(2\pi)^2}\int dp_0 dk_0 G(P; p, k). \tag{13}$$

The Lagrangians of interaction of spinor and scalar fields with the electromagnetic field are, respectively, of the forms

$$\mathscr{L}_1(x) = g : \bar{\Psi}(x)\gamma_\mu \Psi(x) : A^\mu(x),$$

$$\mathscr{L}_1(x) = ig : \Psi^+(x)\partial_\mu \Psi(x) - \Psi(x)\partial_\mu \Psi^+(x) : A^\mu(x) + g^2 : \Psi^+(x)\Psi(x)A_\mu(x)A^\mu(x) :.$$

In the second order of perturbation theory for the 4-time Green's functions of considered systems we have

$$G_{(2)}(P; p, k) = D_1(\eta_1 P + p)\Gamma_1^\mu D(\eta_1 P + k)D_{\mu\nu}(p-k) \times D_2(\eta_2 P - p)\Gamma_2^\nu D_2(\eta_2 P - k), \tag{14}$$

where $D_j(p)$ are the propagators of spinor and scalar fields

$$D_j(p) = \frac{\hat{p} + m_j}{p^2 - m_j^2 + i0}; \quad D_j(p) = \frac{1}{p^2 - m_j^2 + i0}, \tag{15}$$

and Γ_j^μ are the vertex factors

$$\Gamma_{1,2}^\mu = \gamma^\mu; \quad \Gamma_1^\mu = (2\eta_1 P + p + k)^\mu; \quad \Gamma_2^\nu = (2\eta_2 P - p - k)^\nu. \tag{16}$$

We have chosen the photon propagator in the α-gauge:

$$D_{\mu\nu}(q) = \frac{1}{q^2 + i0}\left[g_{\mu\nu} + (\alpha - 1)\frac{q_\mu q_\nu}{q^2 + i0}\right]. \tag{17}$$

Consider the case of two spinor particles. Using formulae (14)–(17) and the technique of contour integration, we find the following expression for the double-time Green's function from (13):

$$\hat{G}^{\sigma_1\sigma_2}_{(2)r_1r_2}(P; \boldsymbol{p}, \boldsymbol{k}) = -(2\pi)^{-3} g_1 g_2 R_p R_k \bar{u}^{\sigma_1}(\boldsymbol{p}_1)\gamma^\mu u^{r_1}(\boldsymbol{k}_1) \times \bar{u}^{\sigma_2}(\boldsymbol{p}_2)\gamma^\nu u^{r_2}(\boldsymbol{k}_2)\,[g_{\mu\nu} C(P; \boldsymbol{p}, \boldsymbol{k}) + (\alpha - 1)g_{\mu 0}g_{\nu 0} B(P; \boldsymbol{p}, \boldsymbol{k})],$$

where

$$C(P; \boldsymbol{p}, \boldsymbol{k}) = (2W)^{-1}(R_{12} + R_{21}),$$

$$B(P; \boldsymbol{p}, \boldsymbol{k}) = R_p^{-1} R_k^{-1}(4W^3)^{-1}[W(R_{12}^2 + R_{21}^2) - R_{12} - R_{21} + R_p + R_k],$$

g_1 and g_2 are the charges of particles 1 and 2. Here a further notation is introduced:

$$W = |\boldsymbol{p} - \boldsymbol{k}|, \quad \Omega_{ij} = \omega_i^p + \omega_j^k + W,$$

$$R_{ij} = (P_0 - \Omega_{ij} + i0)^{-1}.$$

By the definition (9), we find the quasipotential

$$V^{\sigma_1\sigma_2}_{(2)r_1r_2}(P; \mathbf{p}, \mathbf{k}) = R_p^{-1}R_k^{-1}\hat{G}^{\sigma_1\sigma_2}_{(2)r_1r_2}(P; \mathbf{p}, \mathbf{k}), \tag{18}$$

and now we can write the quasipotential equation (10) for a bound state of two spinor particles explicitly:

$$R_p^{-1}X_P^{\sigma_1\sigma_2}(\mathbf{p}) = -(2\pi)^{-3}g_1g_2\int d\mathbf{k}\bar{u}^{\sigma_1}(\mathbf{p}_1)\gamma^\mu u^{r_1}(\mathbf{k}_1)\times\bar{u}^{\sigma_2}(\mathbf{p}_2)\gamma^\nu u^{r_2}(\mathbf{k}_2)$$
$$\times[g_{\mu\nu}C(P; \mathbf{p}, \mathbf{k})+(\alpha-1)g_{\mu 0}g_{\nu 0}B(P; \mathbf{p}, \mathbf{k})]X_P^{r_1r_2}(\mathbf{k}). \tag{18a}$$

The analogous procedure allows us to get the expression of double-time Green's function for system of spinor and scalar particles in the second order of perturbation theory:

$$\hat{G}^\sigma_{(2)r}(P; \mathbf{p}, \mathbf{k}) = (2\pi)^{-3}g_1g_2(8W\omega_2^p\omega_2^k)^{-1}\bar{u}^\sigma(\mathbf{p}_1)\gamma^\mu u^r(\mathbf{k}_1)$$
$$\times[C_\mu(P; \mathbf{p}, \mathbf{k})+(\alpha-1)B_\mu(P; \mathbf{p}, \mathbf{k})].$$

Here the following notation is used:

$$C_\mu(P; \mathbf{p}, \mathbf{k}) = R_kR_{12}[R_pf_\mu^{(+)}(\mathbf{p}, \mathbf{k})+\Omega_{22}^{-1}f_\mu^{(-)}(-\mathbf{p}, \mathbf{k})]+(\mathbf{p}\leftrightarrow\mathbf{k}),$$

$$B_\mu(P; \mathbf{p}, \mathbf{k}) = R_kR_{12}(2W)^{-1}\{R_pf_\nu^{(+)}(\mathbf{p}, \mathbf{k})\,[\tilde{q}^\nu\tilde{q}_\mu R_{12}$$
$$+(2W)^{-1}(\tilde{q}^\nu\tilde{q}'_\mu+\tilde{q}'^\nu\tilde{q}_\mu)]+\Omega_{22}^{-1}f_\nu^{(-)}(-\mathbf{p}, \mathbf{k})$$
$$\times[\tilde{q}^\nu\tilde{q}_\mu(R_{12}-\Omega_{22}^{-1})+(2W)^{-1}(\tilde{q}^\nu\tilde{q}'_\mu+\tilde{q}'^\nu\tilde{q}_\mu)]\}+(\mathbf{p}\leftrightarrow\mathbf{k}),$$

$$f^{(\pm)\mu}(\mathbf{p}, \mathbf{k}) = (-\omega_2^p\mp\omega_2^k, \mathbf{p}\pm\mathbf{k}),$$

$$\tilde{q}^\mu = (W, \mathbf{k}-\mathbf{p}), \quad \tilde{q}'^\mu = (W, \mathbf{p}-\mathbf{k}).$$

The expression in parentheses is the previous term with the shown substitution $(\mathbf{p}\leftrightarrow\mathbf{k})$. In this case the quasipotential is expressed through $\hat{G}^\sigma_{(2)r}$ by the relation

$$V^\sigma_{(2)r}(P; \mathbf{p}, \mathbf{k}) = 4\omega_2^p\omega_2^kR_p^{-1}R_k^{-1}\hat{G}^\sigma_{(2)r}(P; \mathbf{p}, \mathbf{k}). \tag{19}$$

The equation for the relativistie wave function of a system of scalar and spinor particles has the form

$$2\omega_2^p(P_0-\omega_1^p-\omega_2^p)X_P^\sigma(\mathbf{p}) = (2\pi)^{-3}g_1g_2\int d\mathbf{k}(2W)^{-1}\bar{u}^\sigma(\mathbf{p}_1)\gamma^\mu u^r)\mathbf{k}_1)$$
$$\times R_p^{-1}R_k^{-1}[C_\mu(P; \mathbf{p}, \mathbf{k})+(\alpha-1)B_\mu(P; \mathbf{p}, \mathbf{k})]X_P^r(\mathbf{k}). \tag{19a}$$

For a system of two scalar particles interacting via the one-photon exchange, in accordance with (13) and (14), we obtain the explicit form of functions $\hat{G}_{(2)\text{ret}}$ and $\hat{G}_{(2)\text{adv}}$ which determine the function $\hat{G}_{(2)}$ by the formula (4)

$$\hat{G}_{(2)\text{ret}}(P; \mathbf{p}, \mathbf{k}) \equiv \hat{G}_{(2)\text{ret}}(P_0, \mathbf{P}; \mathbf{p}, \mathbf{k})$$
$$= g_1g_2(2\pi)^{-3}(32W\omega_1^p\omega_2^p\omega_1^k\omega_2^k)^{-1}[C_1(P; \mathbf{p}, \mathbf{k})+(\alpha-1)B_1(P; \mathbf{p}, \mathbf{k})],$$
$$\hat{G}_{(2)\text{adv}}(P; \mathbf{p}, \mathbf{k}) \equiv \hat{G}_{(2)\text{adv}}(P_0, \mathbf{P}; \mathbf{p}, \mathbf{k}) = \hat{G}_{(2)\text{ret}}(-P_0, \mathbf{P}; \mathbf{p}, \mathbf{k}),$$

where

$$C_1(P_0, \boldsymbol{P}; \boldsymbol{p}, \boldsymbol{k}) = [(\omega_1^p - \omega_1^k)(\omega_2^p - \omega_2^k) - (\boldsymbol{p}_1 + \boldsymbol{k}_1)(\boldsymbol{p}_2 + \boldsymbol{k}_2)]$$

$$\times (\omega_1^p + \omega_2^p + \omega_1^k + \omega_2^k)^{-1}(\Omega_{11}^{-1} + \Omega_{22}^{-1})R_p$$

$$-(\boldsymbol{p}_1 + \boldsymbol{k}_1)(\boldsymbol{p}_2 + \boldsymbol{k}_2)(\Omega_{11}^{-1} - R_k)(\Omega_{22}^{-1} - R_p)R_{12}$$

$$-[(\omega_1^p - \omega_1^k)\Omega_{11}^{-1} - (\omega_1^p + \omega_1^k)R_k][(\omega_2^p - \omega_2^k)\Omega_{22}^{-1}$$

$$+(\omega_2^p + \omega_2^k)R_p]R_{12} + (\boldsymbol{p} \leftrightarrow \boldsymbol{k}),$$

$$B_1(P_0, \boldsymbol{P}; \boldsymbol{p}, \boldsymbol{k}) = 2\omega_1^k W^{-2}(\omega_2^k R_p - \omega_2^p R_{12} + \omega_2^p W R_{12}^2) + (\boldsymbol{p} \leftrightarrow \boldsymbol{k}).$$

Now it is not difficult to get the quasipotentials from (9):

$$V_{(2)}(P; \boldsymbol{p}, \boldsymbol{k}) = g_1 g_2 (2\pi)^{-3}[8W(\omega_1^p + \omega_2^p)(\omega_1^k + \omega_2^k)R_p R_k A_p A_k]^{-1}$$

$$\times \{C_1(P_0, \boldsymbol{P}; \boldsymbol{P}, \boldsymbol{k}) + C_1(-P_0, \boldsymbol{P}; \boldsymbol{p}, \boldsymbol{k}) + (\alpha - 1)[B_1(P_0, \boldsymbol{P}; \boldsymbol{p}, \boldsymbol{k})$$

$$+ B_1(-P_0, \boldsymbol{P}; \boldsymbol{p}, \boldsymbol{k})]\}, \tag{20}$$

$$V_{(2)\mathrm{ret}}(P; \boldsymbol{p}, \boldsymbol{k}) = g_1 g_2 (2\pi)^{-3}(2WR_p R_k)^{-1}$$

$$\times [C_1(P_0, \boldsymbol{P}; \boldsymbol{p}, \boldsymbol{k}) + (\alpha - 1)B_1(P_0, \boldsymbol{P}; \boldsymbol{p}, \boldsymbol{k})], \tag{21}$$

and the corresponding equations from (10), (12), (20) and (21)

$$4\omega_1^p \omega_2^p [(\omega_1^p + \omega_2^p)^2 - P_0^2] X_P(\boldsymbol{p}) = g_1 g_2 (2\pi)^{-3} \int d\boldsymbol{k}$$

$$\times [4W(\omega_1^k + \omega_2^k)R_p R_k A_p A_k]^{-1}\{C_1(P_0, \boldsymbol{P}; \boldsymbol{p}, \boldsymbol{k}) + C_1(-P_0, \boldsymbol{P}; \boldsymbol{p}, \boldsymbol{k})$$

$$+ (\alpha - 1)[B_1(P_0, \boldsymbol{P}; \boldsymbol{p}, \boldsymbol{k}) + B_1(-P_0, \boldsymbol{P}; \boldsymbol{p}, \boldsymbol{k})]\} X_P(\boldsymbol{k}), \tag{20a}$$

$$4\omega_1^p \omega_2^p (\omega_1^p + \omega_2^p - P_0) X_P(\boldsymbol{p}) = g_1 g_2 (2\pi)^{-3} \int d\boldsymbol{k}(2WR_p R_k)^{-1}$$

$$\times [C_1(P_0, \boldsymbol{P}; \boldsymbol{p}, \boldsymbol{k}) + (\alpha - 1)B_1(P_0, \boldsymbol{P}; \boldsymbol{p}, \boldsymbol{k})] X_P(\boldsymbol{k}). \tag{21a}$$

It is necessary to note that equations (18a)–(21a) belong to the spectral problems, and what is more, the spectral parameter P_0 (the energy of a two-body system) is present in the left-hand sides of the equations and in the kernels too. At present the methods of analytic and numerical solutions [17] of such equations are being actively elaborated.

The wave functions $X_P^{r_1 r_2}(\boldsymbol{p})$, $X_P^r(\boldsymbol{p})$ and $X_P(\boldsymbol{p})$ (the solutions of the equations obtained in this paper) can be used for solving many problems of relativistic two-body bound systems. In the general case, they are normalized by the condition [6]

$$\frac{1}{(2\pi)^6} \int d\boldsymbol{p} d\boldsymbol{k} X_P^*(\boldsymbol{p}) \frac{\partial}{\partial \sqrt{P^2}} [\tilde{G}(P; \boldsymbol{p}, \boldsymbol{k})]_{\sqrt{P^2} = M_n} \times X_P(\boldsymbol{k}) = 2Mn.$$

On the energy-shell when $P_0 = \omega_1^p + \omega_2^p = \omega_1^k + \omega_2^k$ the quasipotentials (18) — (21) coincide with the corresponding Feynman amplitudes:

$$V_{(2)r_1r_2}^{\sigma_1\sigma_2}(P; \boldsymbol{p}, \boldsymbol{k})|_{P_0=\omega_1 p,k+\omega_2 p,k} = -g_1g_2(2\pi)^{-3}\bar{u}^{\sigma_1}(\boldsymbol{p}_1)\gamma^\mu u^{r_1}(\boldsymbol{k}_1)$$

$$\times \bar{u}^{\sigma_2}(\boldsymbol{p}_2)\gamma_\mu u^{r_2}(\boldsymbol{k}_2)\left[(\omega_2^p-\omega_2^k)^2-(\boldsymbol{p}-\boldsymbol{k})^2+i0\right]^{-1}$$

$$= -g_1g_2(2\pi)^{-3}\bar{u}^{\sigma_1}(\boldsymbol{p}_1)\gamma^\mu u^{r_1}(\boldsymbol{k}_1)\times\bar{u}^{\sigma_2}(\boldsymbol{p}_2)\gamma_\mu u^{r_2}(\boldsymbol{k}_2)\frac{1}{q^2+i0};$$

$$V_{(2)r}^{\sigma}(P; \boldsymbol{p}, \boldsymbol{k})|_{P_0=\omega_1 p,k+\omega_2 p,k} = -g_1g_2(2\pi)^{-3}\bar{u}^{\sigma}(\boldsymbol{p}_1)\gamma_\mu u^{r}(\boldsymbol{k}_1)\frac{(p_2+k_2)^\mu}{q^2+i0};$$

$$V_{(2)}(P; \boldsymbol{p}, \boldsymbol{k})|_{P_0=\omega_1 p,k+\omega_2 p,k} = -g_1g_2(2\pi)^{-3}(p_1+k_1)^\mu(p_2+k_2)_\mu\frac{1}{q^2+i0}.$$

Here the 4-momenta p_1, p_2, k_1, k_2 belong to the mass-shell and $q = k_2-p_2$ is the transfer momentum. Therefore, these quasipotentials can be used like the off-energy-shell amplitudes in the three-body problem of quantum field theory.

The generalization of these methods to the nonabelian gauge theories will be published separately.

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