MATRIX REALIZATION OF STRING ALGEBRA AXIOMS AND CONDITIONS OF INVARIANCE

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The matrix representations of Witten's and B-algebras of the field string theory in finite dimensional space of the ghost states are suggested for the case of Virasoro algebra truncated to its SU(1,1) subalgebra. In this case all algebraic operations of Witten's and B-algebras are realized in explicit form as some matrix operations in the graded complex vector space. The structure of string action coincides with the universal non-linear cubic matrix form of action for the gauge field theories. These representations lead to matrix conditions of theory invariance which can be used for finding of the explicit form of corresponding operators of the string algebras.

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1. *Introduction*

String field theory [1–4] as a gauge theory is known to be based on the application of string algebras which are specified on the string functionals $\phi(x^\mu(\sigma), c(\sigma), \bar{c}(\sigma))$, depending on the string coordinates $x^\mu(\sigma)$, the ghosts $c(\sigma)$ and the antighosts $\bar{c}(\sigma)$. The string fields form the Z_2-graded vector space, each element of which has a degree equal $(-1)^{|\phi|} = 1$ or $(-1)^{|\phi|} = -1$, where $|\phi|$ is the parity of the element ϕ determined by the ghost number of the corresponding functional ϕ. In this space the operations of multiplication $(\phi_1 * \phi_2)$ or $(\phi_1 \circ \phi_2)$ and of exterior differentiation $Q\phi$, and either the corresponding operation of integration $\int \phi$ or scalar product (ϕ_1, ϕ_2) are also specified.

In the first case $(*, Q, \int)$ we come to the system of Witten's axioms

$$\phi_1 * (\phi_2 * \phi_3) = (\phi_1 * \phi_2) * \phi_3, \tag{1}$$

$$Q(\phi_1 * \phi_2) = Q\phi_1 * \phi_2 + (-1)^{|\phi_1|}\phi_1 * Q\phi_2, \tag{2}$$

$$Q^2 = 0, \tag{3}$$

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$$\int Q\phi = 0, \tag{4}$$

$$\int \phi_1 * \phi_2 = (-1)^{|\phi_1||\phi_2|} \int \phi_2 * \phi_1, \tag{5}$$

where

$$\text{degree}\,(\phi_1 * \phi_2) = (-1)^{|\phi_1|+|\phi_2|}, \quad \text{degree}\, Q\phi = (-1)^{|\phi|+1}. \tag{6}$$

The action for the string field is determined from the condition of invariance under gauge transformation

$$\delta\varphi = \frac{1}{g} Q\varepsilon + \phi * \varepsilon - \varepsilon * \phi, \tag{7}$$

where ε is a functional in the space Z_2, $|\varepsilon| = 1$, and takes the form

$$S_W = \int \{\phi * Q\phi + \tfrac{2}{3} g\phi * \phi * \phi\}. \tag{8}$$

In the second case $(\circ, Q, (\ldots, \ldots))$ we take the system of axioms of *B*-algebra [2, 3] in the form

$$\phi_1 \circ \phi_2 = (-1)^{|\phi_1||\phi_2|+1} \phi_2 \circ \phi_1, \tag{9}$$

$$(\phi_1, \phi_2 \circ \phi_3) = (-1)^{|\phi_1|(|\phi_2|+|\phi_3|)} (\phi_3, \phi_1 \circ \phi_2), \tag{10}$$

$$(\phi_1, \phi_2) = (-1)^{|\phi_2||\phi_2|} (\phi_2, \phi_1), \tag{11}$$

$$Q(\phi_1 \circ \phi_2) = Q\phi_1 \circ \phi_2 + (-1)^{|\phi_1|} \phi_1 \circ Q\phi_2, \tag{12}$$

$$(\phi_1, Q\phi_2) = (-1)^{|\phi_1|+1} (Q\phi_1, \phi_2). \tag{13}$$

The action in the framework of axioms (9)–(13) is given by

$$S_B = (\phi, Q\phi) + \tfrac{1}{3} g(\phi, \phi \circ \phi), \tag{14}$$

and the gauge transformation is

$$\delta\phi = Q\varepsilon + g\phi \circ \varepsilon. \tag{15}$$

In both cases the operator Q is BRST-charge operator of corresponding Virasoro algebra.

Note that the structure of expressions for action (8), (14) coincides with the universal non-linear cubic matrix differential form action for the gauge field theories [5]

$$S = \int dx \{\psi\alpha\psi + \tfrac{2}{3} g\psi M\psi\psi\}, \tag{16}$$

where

$$\alpha = \alpha^\mu \partial_\mu + \alpha^0,$$

in which all the theory properties are determined by five square matrices α^μ, α^0 and cubic matrix M, and the gauge transformation takes the form

$$\delta\psi = Q^\mu \partial_\mu \varepsilon + gN\psi\varepsilon, \tag{17}$$

where $\psi = \{\psi_A(x)\}$, $\varepsilon = \{\varepsilon_a(x)\}$, $A = 1, 2, \ldots, r$, $a = 1, 2, \ldots, q$ are the vectors, $Q^\mu = (Q^\mu_{Aa})$, $N = (N^a_{AB})$ are the matrices with constant elements.

In this case the conditions of action invariance (16) under the transformations (17) can be given by the following system of matrix equations [6]

$$(\alpha^0+\tilde{\alpha}^0)Q^\mu = 0, \quad \tilde{\alpha}^\mu Q^\nu = 0, \quad \alpha^{(\mu}Q^{\nu)} = 0,$$

$$(\alpha^0+\tilde{\alpha}^0)N+\tilde{N}(\alpha^0+\tilde{\alpha}^0) = 0, \quad (\alpha^\mu+\tilde{\alpha}^\mu)N = 0, \tag{18}$$

$$\alpha^\mu N+\tilde{N}\alpha^\mu+(\sum_\pi M)Q^\mu = 0, \tag{19}$$

$$\sum_\pi \{(\sum_\pi M)N\} = 0, \tag{20}$$

where

$$\tilde{\alpha}^\mu_{AB} = \alpha^\mu_{BA}, \quad \tilde{\alpha}^0_{AB} = \alpha^0_{BA}, \quad \tilde{N}^a_{AB} = N^a_{BA},$$

$$\sum_\pi (M_{ABC}) = 1/3!(M_{ABC}+M_{BCA}+M_{CAB}+M_{ACB}+M_{BAC}+M_{CBA}).$$

It should be noted the specification of gauge group representations in the space of the field ψ for source fields will be sufficient to obtain the explicit forms of matrices α^μ, α^0, M, and, hence, to determine the action (16) from the matrix conditions of invariance (18)–(20).

2. *Matrix realization of Witten's algebra*

There arises a problem of realization of operators of Witten's and B-algebras as some matrix operations satisfying axioms (1)–(5), (9)–(13) and of representation of the conditions of action invariance (8), (14) in the form of matrix equations. Note also, that in Witten's papers the string field is interpreted as a sort of "infinite matrix". At the same time, as the string algebras have a nontrivial mathematical structure, let us consider now the possibility of matrix representation of the above algebras imposing a number of simplified assumptions in terms of finite dimensional realization. Such model realizations were proposed in [7, 8] and turned out to be useful to elucidate different properties of string algebras. In [7] as a Q operator was taken the BRST-operator of the so-called "truncated Virasoro algebra"

$$[\varrho_n, \varrho_m] = (n-m)\varrho_{n+m}, \quad n, m = 0, +, -, \tag{21}$$

thus corresponding to the subalgebra generators SU(1, 1) of the Virasoro algebra. In this case the ghost and antighost operators satisfy the following anticommutation relations

$$\{\bar{c}_n, c_m\} = \delta_{n+m,0}, \{c_n, c_m\} = \{\bar{c}_n, \bar{c}_m\} = 0, \tag{22}$$

and the ground state $|0\rangle$ is defined by the conditions

$$\bar{c}_0|0\rangle = \bar{c}_+|0\rangle = c_+|0\rangle = 0 \tag{23}$$

Thus there are only eight ghost states, which correspond to the ghost numbers $N_g = \pm 3/2$, $\pm 1/2$, that is

$$\bar{c}_-|0\rangle, \quad N_g = -3/2,$$
$$|0\rangle, \quad c_0\bar{c}_-|0\rangle, \quad c_-\bar{c}_-|0\rangle, \quad N_g = -1/2,$$
$$c_0|0\rangle, \quad c_-|0\rangle, \quad c_0c_-\bar{c}_-|0\rangle, \quad N_g = 1/2,$$
$$c_0c_-|0\rangle, \quad N_g = 3/2. \tag{24}$$

According to (24) we introduce the string fields as the zero-, one-, two- and three-forms of the following structure

$$|\varepsilon\rangle_0, \quad |A_0, A_+, A_-\rangle_1, \quad |F_0, F_+, F_-\rangle_2, \quad |D\rangle_3, \tag{25}$$

where ε, A_n, F_n, D are complex 2×2 matrices and the index $p = N_g+3/2 = 0, 1, 2, 3$ corresponds to the form degree. The operators $*$, Q, $\int$ are introduced in [7] to satisfy Witten's axioms (1)–(5) as follows:

$$|\varepsilon_1\rangle_0 * |\varepsilon_2\rangle_0 = |\varepsilon_1\varepsilon_2\rangle_0, \quad |\varepsilon\rangle_0 * |A_0, A_+, A_-\rangle_1 = |\varepsilon A_0, \varepsilon A_+, \varepsilon A_-\rangle_1, \tag{26}$$

$$|A\rangle_1 * |B\rangle_1 = |A_+B_- + A_-B_+, A_+B_0 + A_0B_+, A_-B_0 + A_0B_-\rangle_2, \tag{27}$$

$$|A\rangle_1 * |F\rangle_2 = |A_0F_0 + A_+F_- + A_-F_+\rangle_3, \tag{28}$$

$$Q|\varepsilon\rangle_0 = |[\varrho_0, \varepsilon], [\varrho_+, \varepsilon], [\varrho_-, \varepsilon]\rangle_1, \tag{29}$$

$$Q|A\rangle_1 = |\{\varrho_+, A_-\} + \{\varrho_-, A_+\}, \{\varrho_0, A_+\} + \{\varrho_+, A_0\}, \{\varrho_0, A_-\} + \{\varrho_-, A_0\}\rangle_2, \tag{30}$$

$$Q|F\rangle_2 = |[\varrho_0, F_0] + [\varrho_+, F_-] + [\varrho_-, F_+]\rangle_3, \quad Q|D\rangle_3 = 0, \tag{31}$$

$$\int |\rangle_p = 0, \quad p = 0, 1, 2, \quad \int |D\rangle_3 = \mathrm{Tr}\, D. \tag{32}$$

Here ϱ_n are complex 2×2 matrices of the following form

$$\varrho_0 = \tfrac{1}{2}\sigma_3, \quad \varrho_+ = \tfrac{1}{2}(i\sigma_1 - \sigma_2), \quad \varrho_- = \tfrac{1}{2}(i\sigma_1 + \sigma_2), \tag{33}$$

σ_1, σ_2, σ_3 are Pauli matrices.

We show that for algebra (21) the system of axioms of Witten's algebra as well as of B-algebra may be realized on the basis of matrix multiplication in some finite dimensional vector space. In this case the action of the model is represented as in (16) and the conditions of action invariance under transformation have matrix form and are the consequence of string algebra axioms.

Note that the operators $*$, Q transform forms (25) with different degrees one into another,

$$|\rangle_{p_1} * |\rangle_{p_2} = |\rangle_{p_1+p_2}, \quad p_1+p_2 \leqslant 3, \quad |\rangle_{p_1} * |\rangle_{p_2} = 0, \quad p_1+p_2 > 3, \tag{34}$$

$$Q|\rangle_p = |\rangle_{p+1}, \quad p = 0, 1, 2, \quad Q|\rangle_{p_3} = 0. \tag{35}$$

Let us introduce the linear vector space in which states (24) with all different ghost numbers are the basic vectors and the matrices ε, A_n, F_n, D determine the vector coordinates in this basis. The operators $*$ and Q will have representation in this space in the form of matrices. The explicit form of the matrices satisfying axioms (1)–(5) may be found from relations (26)–(32).

Then introduce a vector in the space of all different ghost states (24)

$$\psi = \{\psi^p\} = \{\psi^0, \psi^1, \psi^2, \psi^3\} = \{\psi_A\}, \tag{36}$$

where ψ^p are the forms corresponding to the ghost numbers $-3/2$, $-1/2$, $1/2$, $3/2$ and are given by

$$\psi^0 = \{\psi^0_{\alpha\beta}\} = \{\varepsilon_{\alpha\beta}\}, \qquad \psi^1 = \{\psi^1_{n\alpha\beta}\} = \{A_{n\alpha\beta}\},$$

$$\psi^2 = \{\psi^2_{n\alpha\beta}\} = \{F_{n\alpha\beta}\}, \qquad \psi^3 = \{\psi^3_{\alpha\beta}\} = \{D_{\alpha\beta}\}. \tag{37}$$

Here $A = (0\alpha\beta,\ 1n\alpha\beta,\ 2n\alpha\beta,\ 3\alpha\beta)$, $n = 0, +, -$, $\alpha, \beta = 1, 2$. The components ε, A_n, F_n, D are arbitrary constant complex 2×2 matrices.

Then, the operation $\psi * \varphi$ may be introduced as the multiplication of cubic matrix by two arbitrary vectors

$$\psi * \varphi = \Lambda\psi\varphi = (\Lambda^C_{AB}\psi_B\varphi_C). \tag{38}$$

The next condition on matrix Λ follows from the associativity of multiplication (1)

$$\Lambda^E_{AB}\Lambda^C_{ED} = \Lambda^D_{AE}\Lambda^C_{EB}, \tag{39}$$

and property (6) results in limitation

$$\Lambda^C_{AB} = 0, \qquad (-1)^{|A|} \neq (-1)^{|B|+|C|}. \tag{40}$$

The cubic matrix $\Lambda = (\Lambda^C_{AB})$ is determined by the expression

$$\Lambda = (\Lambda_p) = (\Lambda_{0\alpha\beta}, \Lambda_{1n\alpha\beta}, \Lambda_{2n\alpha\beta}, \Lambda_{3\alpha\beta}), \tag{41}$$

and the matrices Λ_p are of the following structure

$$\Lambda_0 = \begin{pmatrix} \Lambda^0_{00} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Lambda_1 = \begin{pmatrix} 0 & \Lambda^0_{11} & 0 & 0 \\ \Lambda^1_{10} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\Lambda_2 = \begin{pmatrix} 0 & 0 & \Lambda^0_{22} & 0 \\ 0 & \Lambda^1_{21} & 0 & 0 \\ \Lambda^2_{20} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Lambda_3 = \begin{pmatrix} 0 & 0 & 0 & \Lambda^0_{33} \\ 0 & 0 & \Lambda^1_{32} & 0 \\ 0 & \Lambda^2_{31} & 0 & 0 \\ \Lambda^3_{30} & 0 & 0 & 0 \end{pmatrix}. \tag{42}$$

The matrix elements Λ^r_{pq} may be expressed in explicit form in terms of Kronecker's symbols as

$$\Lambda^0_{00} = \Lambda^0_{33} = \Lambda^3_{30} = \Omega, \quad \Omega = (\Omega^{\gamma\delta}_{\alpha\beta,\varepsilon\sigma}) = (\delta_{\alpha\gamma}\delta_{\delta\varepsilon}\delta_{\beta\sigma}),$$
$$\Lambda^0_{11} = \Lambda^0_{22}, \quad \Lambda^1_{10} = \Lambda^2_{20}, \quad \Lambda^1_{32} = \Lambda^2_{31},$$
$$\Lambda^0_{11} = (\Lambda^0_{1k,1n}) = (\delta_{kn}\Omega), \quad \Lambda^{1n}_{1k,0} = \delta_{kn}\Omega,$$
$$\Lambda^1_{21} = (\Lambda^{1n}_{20,1m}, \quad \Lambda^{1n}_{2+,1m}, \quad \Lambda^{1n}_{2-,1m})$$
$$= ((\delta_{n,+}\delta_{m,-}+\delta_{n,-}\delta_{m,+})\Omega, (\delta_{n,+}\delta_{m,+}+\delta_{n,+}\delta_{m,-})\Omega, (\delta_{n,-}\delta_{m,0}+\delta_{n,0}\delta_{m,-})\Omega),$$
$$\Lambda^1_{32} = (\Lambda^{1k}_{3,2n}) = ((\delta_{k,0}\delta_{n,0}+\delta_{k,+}\delta_{n,-}+\delta_{k,-}\delta_{n,+})\Omega). \tag{43}$$

The operator Q may be represented as a square matrix in the space ψ. The nilpotency of Q means that $Q_{AB}Q_{BC} = 0$, and from (6) we obtain $Q_{AB} = 0$ for $(-1)^{|A|} \neq (-1)^{|B|+1}$. Property (2) in the matrix form reads

$$Q_{AC}\Lambda^B_{CD} = \Lambda^D_{AC}Q_{CB}+(-1)^{|B|}\Lambda^B_{AC}Q_{CD}. \tag{44}$$

To determine explicit form of the operator Q for (21) we represent it in the form of square matrix in the space ψ as:

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ x_1 & 0 & 0 & 0 \\ 0 & x_2 & 0 & 0 \\ 0 & 0 & x_3 & 0 \end{pmatrix}, \tag{45}$$

where the matrices x_1, x_2, x_3 are of the following structures

$$x_1 = \begin{pmatrix} \varrho_0^- & 0 & 0 \\ 0 & \varrho_+^- & 0 \\ 0 & 0 & \varrho_-^- \end{pmatrix}, \quad x_2 = \begin{pmatrix} 0 & \varrho_-^+ & \varrho_+^+ \\ \varrho_+^+ & \varrho_0^+ & 0 \\ \varrho_-^+ & 0 & \varrho_0^+ \end{pmatrix},$$
$$x_3 = (\varrho_0^-, \varrho_+^-, \varrho_-^-). \tag{46}$$

The matrices $\varrho_k^\pm$ are given by

$$\varrho_k^\pm = \varrho_k\lambda^\pm, \quad \varrho_k^\pm F_n = [\varrho_k, F_n]_\pm, \tag{47}$$

where the cubic matrices take the form

$$\lambda^\pm = (\lambda^{\pm\gamma\delta}_{\alpha\beta,\varepsilon\sigma}) = (\delta_{\alpha\gamma}\delta_{\beta\varepsilon}\delta_{\sigma\delta}\pm\delta_{\gamma\varepsilon}\delta_{\beta\delta}\delta_{\alpha\sigma}). \tag{48}$$

The nilpotency condition $Q^2 = 0$ is satisfied when $x_{n+1}x_n = 0$. This actually always is the case when one chooses ϱ_k in the form (33).

For finite dimensional case (21) the operation of integration may be defined as scalar multiplication of ψ by vector $T = (0, 0, 0, \delta_{\alpha\beta})$, i.e. projecting and tracing

$$\int \psi = T\psi = T_A\psi_A = \operatorname{Tr} D. \tag{49}$$

In this case conditions (4) and (5) are given by

$$T_A Q_{AB} = 0, \tag{50}$$

$$T_A \Lambda^B_{AC} = (-1)^{|B||C|} T_A \Lambda^C_{AB}. \tag{51}$$

By direct calculation it is easy to verify that the matrix operations Λ, Q, T given in the graded space ψ satisfy the system of axioms of Witten's algebra.

3. *Matrix conditions of action invariance*

The matrix form of the action reads:

$$S_W = \int \{\phi\beta\phi + 2/3 g\phi\Gamma\phi\phi\}, \tag{52}$$

where

$$\beta = (\beta^C_{AB}) = (\Lambda^A_{CD} Q_{DB}) = \Lambda Q, \tag{53}$$

$$\Gamma = (\Gamma^D_{ABC}) = (\Lambda^E_{DA} \Lambda^C_{EB}) = \Lambda\Lambda, \tag{54}$$

$$\Phi = P_1 \psi, \quad P_p \psi = \psi^p, \quad P_p^2 = P_p. \tag{55}$$

The gauge transformation takes the form similar to (17), i.e.:

$$\delta\phi_A = 1/g Q_{AB}\varepsilon_B + \Pi^C_{AB}\phi_B\varepsilon_C, \quad \Pi = \Lambda - \tilde{\Lambda}. \tag{56}$$

Here $\tilde{\Lambda}^C_{AB} = (-1)^{|B||C|}\Lambda^B_{AC}$, ε is the arbitrary vector from the subspace ψ^0.

From the vanishing of action (52) variation under gauge transformation (56) it follows the system of matrix invariance conditions:

$$(\alpha + \tilde{\alpha})\hat{Q} = 0, \tag{57}$$

$$(\alpha + \tilde{\alpha})N + \tilde{N}(\alpha + \tilde{\alpha}) + (\sum_\pi M)\hat{Q} = 0, \tag{58}$$

$$\sum_\pi \{(\sum_\pi M)N\} = 0, \tag{59}$$

where

$$\alpha = TP_1\beta P_1, \quad M = TP_1\Gamma P_1 P_1,$$

$$\hat{Q} = QP_0, \quad N = \Pi P_1 P_0.$$

The structure of matrix conditions (57)–(59) coincides with that for gauge field theories (18)–(20). Hence analyzing the relationships involved one may use our methods of matrix analysis of gauge field theories [5, 6].

4. *Matrix realization of axioms of B-algebra*

Making use of matrix realization of the system of axioms of Witten's algebra one may also find the expression for the system of axioms of *B*-algebra. For this purpose we use the connection between the operations ($*$, Q, $\int$) and those of *B*-algebra ($\circ$, Q, $(\ldots, \ldots)$)

according to [2]

$$\psi \circ \varphi = \psi * \varphi - (-1)^{|\psi||\varphi|} \varphi * \psi, \tag{60}$$

$$(\psi, \varphi) = \int \psi * \varphi. \tag{61}$$

From formulas (60), (61) and relationships (38), (45), (49) we derive the following matrix expressions for the operations of B-algebra in the space of the vectors (36)

$$\psi \circ \varphi = \Pi \psi \varphi, \quad \Pi^C_{AB} = \Lambda^C_{AB} - (-1)^{|B||C|} \Lambda^B_{AC}, \tag{62}$$

$$(\psi, \varphi) = \psi \eta \varphi, \quad \eta_{AB} = T_C \Lambda^B_{CA}. \tag{63}$$

Then the action takes the form

$$S_B = (\phi \eta Q \phi) + \tfrac{1}{3} g(\phi \eta \Pi \phi \phi), \tag{64}$$

and the gauge transformation is

$$\delta\phi = Q\varepsilon + g\Pi\phi\varepsilon. \tag{65}$$

The invariance conditions of action (64) under the transformations (65) may be also represented in the form of matrix relationships (57)–(59) connecting the matrix operations α, M, $\hat{Q}$, N, where

$$\alpha = P_1 \eta Q P_1, \quad M = P_1 \eta \Pi P_1 P_1,$$
$$\hat{Q} = Q P_0, \quad N = \Pi P_1 P_0. \tag{66}$$

5. *Conclusion*

We have shown that in the case of "truncated Virasoro algebra" all the operations of Witten's- and B-algebras may be realized in the explicit form as some matrix operations in finite dimensional vector space of all different ghost states.

The expression for the action of this model in terms of invariant cubic matrix forms has been obtained. The matrix conditions of action gauge invariance, which may be considered as a representation of the system of axioms of string algebras have been also suggested.

REFERENCES

[1] E. Witten, *Nucl. Phys.* **B263**, 253 (1986).

[2] I. Ya. Aref'eva, I. V. Volovich, *Teor. Mat. Fiz.* **67**, 460 (1986).

[3] H. Hata, K. Itoh, T. Kugo, H. Kunitomo, K. Ogawa, *Phys. Lett.* **B172**, 186, 195 (1986); **B175**, 138 (1986)

[4] I. Ya. Aref'eva I. V. Volovich, *Phys. Lett.* **B182**, 312 (1986).

[5] V. I. Kuvshinov, F. I. Fedorov, *Izv. Akad. Nauk. BSSR, Ser. Fiz.-Mat. Nauk* **2**, 69 (1970); A. A. Babich, L. F. Babichev, V. I. Kuvshinov, F. I. Fedorov, Preprint No. 461 Inst. Phys. BSSR Acad. Sci., Minsk 1987.

[6] A. A. Babich, V. I. Kuvshinov, F. I. Fedorov, *Dokl. Akad. Nauk. SSSR* **257**, 1093 (1982).

[7] F. Jimenez, G. Sierra, *Phys. Lett.* **B202**, 58 (1988).

[8] I. Ya. Aref'eva, I. V. Volovich, Proc. XXII Winter School, Karpacz 58 (1986).