INTERMITTENCY FROM NON-RANDOM CASCADES*

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A deterministic cascade model similar in many ways to the random cascade α-model is described. This model suggests the relation $a+b = 2$ for the parameters of the ($l = 2$) α-model. Some predictions for experiment following from this relation are presented.

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The experimentally observed intermittency [1] can be interpreted as due to a selfsimilar random cascade. A particular realization is the α-model [2, 3], where the experimentally measured slopes f_p are[1]

$$f_p = \frac{\log\left(\frac{b-1}{b-a}a^p + \frac{1-a}{b-a}b^p\right)}{\log 2} \qquad p = 1, 2, \ldots . \tag{1}$$

Here a and b are parameters of the model and the parameter $l = 2$, i.e. at each step of the cascade each rapidity bin splits into two.

In this note we show that in the special case

$$\frac{a+b}{2} = 1 \tag{2}$$

one can derive formula (1) from a model without randomness. Just like in the α-model, one begins with a uniform density distribution in the interval [0, 1], then one splits this interval into two equal subintervals [0, 0.5) and [0.5, 1] and obtains the density in each

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[1] We ignore for the moment complications arising for very strong intermittencies [6], when f_p calculated from this formula exceeds $p-1$.

subinterval by multiplying the density from the parent interval by a suitable factor. Instead, however, of choosing these factors at random, one chooses always the factor a for the left interval and always the factor b for the right interval. Because of condition (2) the resulting probability density remains normalized to one exactly, and not only on the average as in the α-model. At each further step of the cascade each of the intervals obtained in the previous step is again split into two subintervals of equal length. The density in the left subinterval gets an additional factor a and that in the right one a factor b. After a large number n of steps one can obtain at the bottom of the cascade a probability density, which is normalized to one and changes from a^n in the vicinity of zero to b^n in the vicinity of one. We have assumed for definiteness $b > a$. The case $a = b$ is of course trivial.

The Renyi dimensions [4, 5] for the probability distribution of particles along the rapidity axis are

$$d_p = 1 - \frac{f_p}{p-1}. \tag{3}$$

In general, the Renyi dimensions are defined by the relation [5]

$$\sum m_i^p \sim \delta^{(p-1)d_p}, \qquad \delta \to 0 \tag{4}$$

where m_i is the probability density integrated over the i-th bin, δ is the bin width and the summation extends over all the 2^n bins at the bottom of the cascade. In the present case this formula reduces to

$$2^{-np} \sum_{k=0}^{n} \binom{n}{k} a^{p(n-k)} b^{pk} = 2^{-n(p-1)d_p}. \tag{5}$$

Since the sum is the n-th power of $a^p + b^p$, this yields

$$d_p = 1 - \frac{\log \dfrac{a^p+b^p}{2}}{\log 2} \tag{6}$$

equivalent to (1).

The model can be generalized by assuming that each interval splits at each step not into two, but into N subintervals: αN subintervals getting a factor a and βN subintervals getting a factor b. Then $\alpha + \beta = 1$ and $\alpha a + \beta b = 1$. One gets the results of the α-model without condition (2), but with $l = N$.

For $a = 0$, $b = 2$ all the non-zero probability density is concentrated in the last bin whatever the length of the cascade. The sum (4) equals one for any n and p. Thus all the Renyi dimensions are zero as one should have expected for a point-like distribution. In the α-model all the Renyi dimensions are zero whenever $0 \leqslant a < 1$ and

$$b > \frac{2-a}{1-a} \tag{7}$$

This, however, is related to the fact [6] that all the averages of powers $p \geqslant 2$ of the *total* multiplicity diverge for the cascade length tending to infinity. In the present model any power of the total multiplicity is fixed.

The density in each bin can be characterized by the exponent α defined for $\delta \to 0$ by [5]

$$m_i \sim \delta^{\alpha} \tag{8}$$

where $\sim$ means equal up to a δ-independent factor. Denoting by N_α the number of bins, which have the exponent $\alpha_i = \alpha$, one can define the function $f(\alpha)$:

$$N_\alpha \sim \delta^{-f(\alpha)}. \tag{9}$$

The function $f(\alpha)$, which characterizes the density distribution, is the Legendre transform of the function $(p-1)d_p$ [5]. Therefore, since the present model has in case (2) the same dimensions d_p as the α-model, it also has the same distribution $f(\alpha)$. Consequently, the density distribution at the bottom of the cascade of the present model differs from the corresponding density distribution in any of the cascades given by the α-model only by a permutation of the 2^n intervals, except for fluctuations, which are irrelevant, i.e. do not affect the average values of the moments. Of course, the rapidity distribution from the present model, where the particle density changes almost monotonically from one end of the rapidity range to the other, is not realistic. Nevertheless, the model seems interesting, because of its close relationship with the much more complicated random cascade α-model.

Condition (2) has a simple interpretation in terms of the α-model. For $a+b \neq 2$ function $f(\alpha)$ for the α-model is negative for some values of α. This for sufficiently small δ according to (9) corresponds to $N_\alpha < 1$, which in the random cascade model just means a small probability to find a bin with density characterized by α, but cannot be realized in a deterministic cascade.

Assuming condition (2) for the α-model, one can find both a and b from one experimentally measured slope f_p. E.g. the TASSO result [7]: $f_2 = 0.023 \pm 0.003$ yields

$$a = 0.873 \qquad b = 1.127 \tag{10}$$

both close to one. Substituting

$$a = 1-\varepsilon \qquad b = 1+\varepsilon \tag{11}$$

one finds from (1) neglecting terms $O(\varepsilon^4)$ one more derivation of the popular relation

$$f_p = Cp(p-1). \tag{12}$$

Here

$$C = \frac{\varepsilon^2}{2 \log 2} = 0.0116 \tag{13}$$

This yields $f_3 = 0.070$ and $f_4 = 0.140$ to be compared with the experimental values [7] $f_3 = 0.080 \pm 0.014$ and $f_4 = 0.134 \pm 0.052$.

For the lognormal distribution [3] formula (12) is exact for all p. There, however, function $f(\alpha)$ becomes negative for $\alpha < (1-\sqrt{C})^2$ and for $\alpha > (1+\sqrt{C})^2$. Thus no non--random version of this model exists.

In Ref. [8] we have introduced the slopes $\varepsilon_{p,q}$ characterizing the fluctuations of the moments of the multiplicity distribution. From (1), (10) and formulae given in Ref. [8] one finds $\varepsilon_{p,q} = 0$ for all p, q. For the lognormal distribution, where formulae (12), (13) hold, one finds $\varepsilon_{p,q} = 0$, unless $pq^2 > C^{-1} \approx 86$.

The cascade in the present model is selfsimilar just like the cascade in the α-model. Thus, one expects the exponents f_p to be well-defined. An additional assumption is necessary only to avoid $f_p = 0$ for all p, which would mean no intermittency. In the α model this additional assumption is randomness. The present model shows that this is not the only possibility.

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REFERENCES

[1] W. Kittel, R. Peschanski, Review of intermittency in particle production, Report at the EPS Conference, Madrid 1989.
[2] A. Białas, R. Peschanski, *Nucl. Phys.* **B273**, 703 (1986).
[3] A. Białas, R. Peschanski, *Nucl. Phys.* **B308**, 857 (1988).
[4] G. Paladin, A. Vulpiani, *Phys. Rep.* **156**, 147 (1987).
[5] T. C. Halsey et al., *Phys. Rev.* **A33**, 1141 (1986).
[6] A. Białas, K. Zalewski, *Phys. Lett.* **B228**, 155 (1989).
[7] W. Braunschweig et al., Study of intermittency in electron-positon annihilation into hadrons, DESY preprint 89-092, July 1989.
[8] A. Białas, A. Szczerba, K. Zalewski, *Z. Phys. C* in print.