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PHYSICS BEYOND THE STANDARD MODEL IN THE NON-PERTURBATIVE UNIFICATION SCHEME*

By D. Kapetanakis

NRCPS „Democritos", Aghia Paraskevi, Attiki, Greece

and G. Zoupanos

Max-Planck-Institut für Physik und Astrophysik, Werner-Heisenberg Institut für Physik, Föhringer Ring 6, D-8000 Munich 40, Fed. Rep. Germany

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The non-perturbative unification scenario predicts reasonably well the low energy gauge couplings of the standard model. Agreement with the measured low energy couplings is obtained by assuming certain kind of physics beyond the standard model. A number of possibilities for physics beyond the standard model is examined. The best candidates so far are a) the standard model with eight fermionic families and a similar number of Higgs doublets, b) the supersymmetric standard model with five families.

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Unification ideas have always been inspiring for particle physicists. Lately the believers in unification ideas got disappointed by the lack of observable baryon number violation [1] that was predicted by the simplest GUT of Georgi and Glashow (GG) based on SU(5) [2]. However, one should realize that most of the excitement about proton decay was triggered by overstatements of some theorists. In particular, the common particle physicist's intuition built up by the discoveries of new particles at every step towards higher energy scales has been badly violated by the "desert hypothesis" of grand unification. Here we are going to deal with a unification scheme that does not necessarily predict proton decay; a priori it requires physics beyond the standard model and has even better predictions for the low energy parameters than the GG model. This scheme was suggested by Maiani, Parisi and Petronzio (MPP) [3] and it is based on the assumption that the

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low energy couplings are the nearly infrared stable fixed points of an asymptotically non--free theory.

Let us introduce the MPP scheme by referring to some further weak points of the usual GUT's. In these theories the coupling constants obey the renormalization group equations, which to one loop order are:

$$\frac{d\alpha_i}{d \ln E} = \frac{B_i}{2\pi}\alpha_i^2, \tag{1}$$

where $\alpha_i = g_i^2/4\pi$, $i = 1, 2, 3$, are the U(1), SU(2) and SU(3) fine structure constants. From Eq. (1) we obtain the following relation

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(\Lambda)} - \frac{B_i}{2\pi}\ln\left(\frac{\mu}{\Lambda}\right) \tag{2}$$

for the coupling constants at two different scales μ and Λ. Assuming that Λ is the grand unification scale and that the theory is assymptotically free (i.e. $B_i < 0$, $i = 2, 3$), one finds from Eq. (2) that the values of the couplings at the electroweak scale μ_w arise as a result of a rather delicate balance between two large numbers. Moreover, the $\alpha_i(\mu_w)$'s depend in an crucial way on the detailed relation between the coupling constants at Λ; e.g. for SU(5):

$$\tfrac{5}{3}\alpha_1(\Lambda) = \alpha_2(\Lambda) = \alpha_3(\Lambda). \tag{3}$$

These fine tunings are disturbing given that the basic theory of elementary particles is still unknown. It is certainly much more attractive to imagine that the theory is somehow self-adjusted. This possibility is offered by the MPP scenario, which assumes that the whole theory is asymptotically non-free. i.e. $B_i > 0$, $i = 1, 2, 3$. In that case the couplings are monotonically increasing with energy until a scale Λ beyond which we can no longer trust perturbation theory. Then the term $\alpha_i(\Lambda)^{-1}$ in Eq. (2) can be neglected in a first approximation and the $\alpha_i(\mu)$ do not depend on the values $\alpha_i(\Lambda)$. The point that makes the MPP scenario very interesting is that the first guess of how to make the theory asymptotically non-free, namely to increase the number of fermion families [3, 4] gives predictions for the low energy couplings of the standard model close to their experimental values. Indeed, in the standard model we have at one loop

$$B_1 = \tfrac{20}{9}n_G + \tfrac{1}{6}n_H; \quad B_2 = \tfrac{4}{3}n_G + \tfrac{1}{6}n_H - \tfrac{22}{3}; \quad B_3 = \tfrac{4}{3}n_G - 11. \tag{4}$$

Then assuming that $\alpha_i(\Lambda) = 0.5$, $i = 1, 2, 3$, $\Lambda = 10^{16}$ GeV, $n_G = 9$, $n_H = 1$ we obtain

$$\alpha_{em}(M_w) = 0.00735; \quad \alpha_3(M_w) = 0.139; \quad \sin^2\theta_w(M_w) = 0.210 \tag{5}$$

which have to be compared with the experimental values [5]:

$$\alpha_{em}(M_w) = 0.00772: \quad \alpha_3(M_w) = 0.122 \pm 0.016; \quad \sin^2\theta_w(M_w) = 0.228 \pm 0.0044. \tag{6}$$

One can do better than that by taking into account two-loop renormalization group equations since the couplings become large at high energies. This is in fact required particularly in the cases that we consider here given that the running of α_3 is slow and two loop effects are important. Three loop contributions have been checked in some cases and turn out to be small in comparison. In addition, since the two loop renormalization group equations are coupled, one expects the strongest of the couplings to drive the others to become also strong at a common scale. Let us then examine how the predictions change when we consider two loop renormalization group equations. In that case the α_i's obey the following equations (neglecting Yukawa couplings):

$$\frac{d\alpha_i}{d \ln E} = \frac{B_i}{2\pi}\alpha_i^2 + \sum_k \frac{\alpha_i^2 \alpha_k}{8\pi^2} B_{ik}, \tag{7}$$

where B_i, $i = 1, 2, 3$, are given by Eq. (4) and

$$B_{ik} = \begin{pmatrix} \frac{95}{27} & 1 & \frac{44}{9} \\ \frac{1}{3} & \frac{49}{3} & 4 \\ \frac{11}{18} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} n_{\mathrm{G}} + \begin{pmatrix} \frac{1}{2} & \frac{3}{2} & 0 \\ \frac{1}{2} & \frac{13}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_{\mathrm{H}} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{-136}{3} & 0 \\ 0 & 0 & -102 \end{pmatrix}. \tag{9}$$

Then for $n_{\mathrm{G}} = 8, n_{\mathrm{H}} = 1, \alpha_i(\Lambda) = 1, i = 1, 2, 3, \Lambda = 4 \times 10^{17}$ GeV we obtain the following low energy couplings

$$\alpha_{\mathrm{em}}(M_{\mathrm{w}}) = 0.00773; \quad \alpha_3(M_{\mathrm{w}}) = 0.119; \quad \sin^2\theta_{\mathrm{w}}(M_{\mathrm{w}}) = 0.1973 \tag{9}$$

in better agreement with the experimental values than Eq. (5).

We consider the close agreement between Eqs. (9) and (6) as an indication that the MPP hypothesis has something to say about the way that nature likes to behave. Moreover, the slight disagreement between the values in Eqs. (8) and (6) can be considered as a hint towards physics beyond the standard model whose relics could fill the small gap between the experimental and predicted theoretical values. There exist already a number of studies which examine physics beyond the standard model in the framework of the MPP scenario [6–11]. We would like to emphasize that it is not true that whatever physics one assumes to exist beyond the standard model will provide agreement with the experimental values. Our proposal has indeed a predictive power for physics beyond the standard model. On the other hand it is true that there exists a number of quite different possibilities that give successful predictions for the low energy parameters. Then for these cases one has to impose additional criteria, for example to choose only minimal extensions of the standard model. Even doing so leaves two quite different kinds of extensions of the standard model. One has to wait for more accurate experimental data to decide what physics beyond the standard model is preferred by the MPP hypothesis.

In the following we are going to present a few examples of physics beyond the standard model. The first example intends to demonstrate that certain otherwise attractive extensions cannot be accommodated within the MPP scenario, while the other two examples provide the best and most economical candidates so far.

Standard model with high colour fermions

A popular way to avoid the hierarchy problem due to elementary Higgs fields is not to introduce them at all in the theory. Instead one might expect the breaking of $SU(2)_L \times U(1)$ to be due to fermion bound states [12] carrying appropriate quantum numbers. Here we consider a simple extension of the standard model by introducing a family of colour sextet quarks having the same electroweak couplings as ordinary quarks [13]. The breaking of the chiral symmetry of these quarks due to colour forces is expected to take place at a much higher scale than for ordinary quarks and could in principle be responsible for the spontaneous symmetry breaking of $SU(2)_L \times U(1)$ down to $U(1)_{em}$. The sextet acquires in this way a dynamical mass $\mu_6 \approx 250$ GeV. We recall that the high colour scenario found strong from the lattice calculations on the chiral symmetry breaking of quarks that belong in different representations [14]. The renormalization group equations in the various energy regions are:

$E < 2\mu_6$

$$B_1 = \tfrac{11}{9} n_q + n_L; \quad B_2 = n_q + \tfrac{1}{3} n_L - \tfrac{22}{3}; \quad B_3 = \tfrac{4}{3} n_q - 11; \tag{10}$$

$$B_{ik} = \begin{pmatrix} \frac{137}{108} & \frac{1}{4} & \frac{44}{9} \\ \frac{1}{12} & \frac{49}{4} & 4 \\ \frac{11}{18} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} n_q + \begin{pmatrix} \frac{9}{4} & \frac{3}{4} & 0 \\ \frac{1}{4} & \frac{49}{12} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_L + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{-136}{3} & 0 \\ 0 & 0 & -102 \end{pmatrix}; \tag{11}$$

$E > 2\mu_6$

$$B_1 = \tfrac{11}{9} n_q + \tfrac{22}{9} n_6 + n_L; \quad B_2 = n_q + 2n_6 + \tfrac{1}{3} n_L - \tfrac{22}{3}; \quad B_3 = \tfrac{4}{3} n_q + \tfrac{20}{3} n_6 - 11 \tag{12}$$

$$B_{ik} = \begin{pmatrix} \frac{137}{108} & \frac{1}{4} & \frac{44}{9} \\ \frac{1}{12} & \frac{49}{4} & 4 \\ \frac{11}{18} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} n_q + \begin{pmatrix} \frac{137}{54} & \frac{1}{2} & \frac{220}{9} \\ \frac{1}{6} & \frac{49}{2} & 20 \\ \frac{55}{18} & \frac{15}{2} & \frac{500}{3} \end{pmatrix} n_6$$
$$+ \begin{pmatrix} \frac{9}{4} & \frac{3}{4} & 0 \\ \frac{1}{4} & \frac{49}{12} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_L + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{-136}{3} & 0 \\ 0 & 0 & -102 \end{pmatrix}, \tag{13}$$

where n_q are the ordinary quark families, n_6 is the number of sextet quarks and n_L the number of leptons. Note that anomaly cancelation restricts $n_L = n_q + 2n_6$.

A representative example is the following using $n_q = 7$, $n_6 = 1$, $\Lambda = 7 \times 10^{15}$ and $\alpha_i(\Lambda) = O(1)$, $i = 1, 2, 3$, we obtain

$$\alpha_{em}(M_w) = 0.00773; \quad \alpha_3(M_w) = 0.0288; \quad \sin^2\theta_w(M_w) = 0.218. \tag{14}$$

We see that this kind of physics beyond the standard model is not compatible with the MPP hypothesis. We should mention that similar ideas about dynamical symmetry breaking which are due to technicolour interactions can give better values for the low energy parameters [10]. We should also recall that the high colour scenario is compatible with perturbative unification [15].

Multi-fermion-Higgs families in the standard model [6, 7]

In this case one introduces new fermion families as well as extra Higgs doublets. The renormalization group equations were already given in Eqs. (4), (7), (8). We find that for eight fermion families and an approximately equal number of Higgs doublets there is agreement of between the low energy couplings and their experimental values. A good example is $n_G = n_H = 8$. Starting with $\alpha_i(\Lambda) = 1$, $i = 1, 2, 3$, $\Lambda = 1.2 \times 10^{16}$ we find

$$\alpha_{em}(M_w) = 0.00772; \quad \alpha_3(M_w) = 0.128: \quad \sin^2\theta_w(M_w) = 0.2284.$$

This case is of particular interest given that it can also be achieved in perturbative unification. Indeed the coupling constants meet at $5\alpha_1(\Lambda)/3 = \alpha_2(\Lambda) = \alpha_3(\Lambda) = 0.3$, $\Lambda = 10^{15}$ and the corresponding low energy values are:

$$\alpha_{em}(M_w) = 0.00779; \quad \alpha_3(M_w) = 0.11; \quad \sin^2\theta_w(M_w) = 0.230.$$

Supersymmetric standard model [8]

In this case $N = 1$ supersymmetry is introduced in the standard model, which has to be broken at some scale M_S given that the low energy spectrum is not supersymmetric. For E less than the supersymmetry breaking scale M_S, the B_i's and B_{ik}'s of Eq. (7) are:

$$B_1 = \tfrac{20}{9} n_G + \tfrac{1}{2} n_H; \quad B_2 = \tfrac{4}{3} n_G + \tfrac{1}{2} n_H - \tfrac{22}{3}; \quad B_3 = \tfrac{4}{3} n_G - 11 \tag{15}$$

and

$$B_{ik} = \begin{pmatrix} \frac{95}{27} & 1 & \frac{44}{9} \\ \frac{1}{3} & \frac{49}{3} & 4 \\ \frac{11}{18} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} n_G + \begin{pmatrix} \frac{3}{4} & \frac{9}{4} & 0 \\ \frac{3}{4} & \frac{25}{4} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_H + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{-136}{3} & 0 \\ 0 & 0 & -102 \end{pmatrix}. \tag{16}$$

For $E > M_S$ they are:

$$B_1 = \tfrac{10}{3} n_G + \tfrac{1}{2} n_H; \quad B_2 = 2n_G + \tfrac{1}{2} n_H - 6; \quad B_3 = 2n_G - 9 \tag{17}$$

and

$$B_{ik} = \begin{pmatrix} \frac{190}{27} & 2 & \frac{88}{9} \\ \frac{2}{3} & 14 & 8 \\ \frac{11}{9} & 3 & \frac{68}{3} \end{pmatrix} n_G + \begin{pmatrix} \frac{1}{2} & \frac{3}{2} & 0 \\ \frac{1}{2} & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_H + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix}. \tag{18}$$

The results one finds are: for $\alpha_i(\Lambda) = 1$, $i = 1, 2, 3$, at $\Lambda = 10^{17}$ GeV and $M_S = 10^4$ GeV with five generations the low energy couplings are

$$\alpha_3(M_w) = 0.1313; \quad \alpha_{em}(M_w) = 0.00771; \quad \sin^2\theta_w(M_w) = 0.234$$

in quite good agreement with Eq. (6).

There are two more points that we would like to emphasize. The first has to do with the stability of the low energy predictions against changes in their values at Λ. Indeed we have allowed $\alpha_1(\Lambda) \neq \alpha_2(\Lambda) \neq \alpha_3(\Lambda)$ for the successful cases discussed above and we found that the low energy couplings still agree with the experimental values. This demonstrates the natuıality of the MPP scenario. The second point is to see whether MPP works assuming in addition that Λ is equal to the Planck mass, which is the natural scale at which all couplings including gravity become O(1). The best results for the multi--fermion-Higgs extension of the standard model are, using $n_G = 7$, $n_H = 5$,

$$\alpha_{em}(M_w) = 0.00776; \quad \alpha_3(M_w) = 0.263; \quad \sin^2\theta_w(M_w) = 0.1894$$

while for the case of the supersymmetric standard model using $n_G = 5$, $n_H = 2$

$$\alpha_{em}(M_w) = 0.00769; \quad \alpha_3(M_w) = 0.247; \quad \sin^2\theta_w(M_w) = 0.2101.$$

These results exclude an immediate connection of Λ with the Planck scale.

In conclusion, the assumption that the theory which describes the interaction of elementary particles is asymptotically non-free and that coupling constants become large at some large scale Λ leads to quite successful predictions of the low energy parameters. Complete agreement with the experimental values is obtained if one assumes physics beyond the standard model. So far the most economical ways to realize the MPP hypothesis are: a) extension of the standard model with eight fermionic families and a similar number of Higgs doublets and b) a supersymmetric standard model with five fermionic families. Despite the recent discoveries of three light neutrinos families [17] one should keep in mind these interesting possibilities which are certainly compatible if the extra neutrinos are heavy.

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