THE GAUGE MODEL OF THE FIFTH FORCE

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Gauge fields of the translation group are suggested in order to describe nongravitational deviations from Newton's gravitational law.

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The hypothesis of the fifth fundamental interaction has arised because of current verifications of Newton's gravitation potential [1, 2]. The translation group gauge model of this interaction was suggested [3, 4]. Our aim here is to consider its geometric aspects and Lagrangians.

Let AX be the fibre bundle of affine repers over a space-time manifold X^4. It is the principal fibre bundle with the affine structure group $A(4, \mathbf{R})$. For the sake of simplicity, AX is believed to be trivial. Provide the total space $P = \text{tl}\, AX$ of AX with coordinates $\{x^\mu, u^a, S^\mu_a\}$. Here x^μ are coordinates in X^4, u^a are parameters of the translation subgroup T_4 of $A(4, \mathbf{R})$, and S^μ_a are coordinates of the reper $\{St_a\}$ with respect to the holonomic reper $\{\partial_\mu\}$, where $\{t_a\}$ is the fixed basis for T_4 and S is an element the subgroup $GL(4, \mathbf{R})$. On P, the group $A(4, \mathbf{R})$ acts by the law

$$(G^a_b, g^a): \{x^\mu, u^a, S^\mu_a\} \to \{x^\mu, (G^{-1})^a_b u^b + g^a, S^\mu_b G^b_a\}.$$

Note that $\{x^\mu, u^\mu = S^\mu_a u^a\}$ are coordinates in the total space of the affine tangent bundle $AT(X)$.

Let a general affine connection be given in AX. Given the above mentioned coordinates, its connection form ω and the corresponding horizontal fields τ^h (i.e. $\omega(\tau^h) = 0$) on P read

$$\omega = (S^{-1})^a_\varepsilon (dS^\varepsilon_b + \Gamma^\varepsilon_{\mu\alpha}(x) S^\alpha_b dx^\mu) I^a_b + (du^a - B^a_\mu(x) dx^\mu) T_a,$$

$$\tau^h = \tau^\mu(x) \left(\frac{\partial}{\partial x^\mu} + B^a_\mu(x) \frac{\partial}{\partial u^a} - \Gamma^\varepsilon_{\mu\alpha}(x) S^\alpha_b \frac{\partial}{\partial S^\varepsilon_b} \right). \tag{1}$$

Here I^a_b, T_a are generators of the group $A(4, \mathbf{R})$, $\Gamma^\varepsilon_{\alpha\mu}$ are coefficients of a linear connection, and $B^\varepsilon_\mu = S^\varepsilon_a B^a_\mu$ are gauge fields of the translation group T_4. Physical treatment of B faces difficulties because matter fields are sections of fibre bundles with linear structure groups, not affine ones. The way out, in our opinion, is pointed by the successful physical utilization of spatial translation gauge fields in the theory of continuous media [5].

Treating u like displacement vectors, let us consider the following mapping ϱ of the space P onto the total space $Q = \mathrm{tl}\, LX$ of the linear frame fibre bundle LX over X^4:

$$\varrho:(x^\mu, u^a, S^\mu_a) \to (\gamma^\mu(x^\varepsilon, u^\varepsilon(x) - S^\varepsilon_a u^a, 1), 0, S^\mu_a)|_{S_a{}^\varepsilon u^a = u^\varepsilon(x)} = (x^\mu, 0, S^\mu_a).$$

Here $\gamma(x, u, S)$ is the geodesic defined by the linear connection Γ through x in the direction u, and $u(x)$ is some section of the fibre bundle $AT(X)$. The map $\partial\varrho$ of the tangent bundle $T(Q)$ over Q transforms horizontal fields (1) on P into the fields

$$\tau^h_Q = \tau^\mu(x)\,(\delta^\nu_\mu + D_\mu u^\nu(x))\left[\frac{\partial}{\partial x^\nu} - \Gamma^\varepsilon_{\mu\alpha}(x) S^\alpha_b \frac{\partial}{\partial S^\varepsilon_b}\right] \tag{2}$$

on Q where

$$D_\mu u^\varepsilon(x) = \partial_\mu u^\varepsilon(x) + \Gamma^\varepsilon_{\mu\alpha} u^\alpha(x) - B^\varepsilon_\mu(x) = \sigma^\varepsilon_\mu(x) \tag{3}$$

is the covariant derivative of fields $u(x)$. Remark that fields (2) are horizontal with respect to the linear connection Γ.

Comparing a general case of ϱ and $\partial\varrho$ with those in the particular case $B = 0$, $u(x) = 0$ we may say that, in a sense, these mappings define deformation of a manifold X^4 characterized by a dislocation field $B(x)$ and a displacement field $u(x)$. A field u however is always removed by gauge transformations. Therefore, only its covariant derivatives (3) can make the physical sense. For instance, $\sigma^\varepsilon_\mu = -B^\varepsilon_\mu$ under the gauge condition $u = 0$.

Let φ be some tensor field on X^4 and f_φ the corresponding tensorial function on Q. We shall say that φ is defined on the deformed manifold X^4 if differentiaton of φ is given by the expression

$$(D\varphi)(\tau) = (df_\varphi)(\tau^h_Q),$$

where τ^h_Q is the horizontal lift (2) of a field $\tau = \tau^\mu(x)\partial_\mu$. It follows that, in the field theory, deformation of a space-time manifold can be described by replacement of familiar covariant derivatives D_μ in the exterior differential $dx^\mu D_\mu$ by the quantities

$$\tilde{D}_\mu = (\delta^\alpha_\mu + \sigma^\alpha_\mu) D_\alpha = H^\alpha_\mu D_\alpha.$$

For example, the Lagrangian of a scalar field φ takes the form

$$L_{(\varphi)} = \tfrac{1}{2}\left[g^{\mu\nu} H^\alpha_\mu H^\beta_\nu \partial_\alpha\varphi\partial_\beta\varphi - m^2\varphi^2\right].$$

The action functional and the equation of motion of a point mass read

$$S = -m_0 \int (g_{\alpha\beta} H^\alpha_\mu H^\beta_\nu u^\mu u^\nu)^{1/2} ds,$$

$$\frac{du^\mu}{ds} + \tilde{\Gamma}^\mu_{\alpha\beta} u^\alpha u^\beta = 0, \tag{4}$$

where quantities $\tilde{\Gamma}$ look like Christoffel symbols of the "metric" $\tilde{g}_{\mu\nu} = H^{\alpha}_{\mu}H^{\beta}_{\nu}g_{\alpha\beta}$, but ds is defined by the true metric g. A Lagrangian $L_{(A)}$ of an electromagnetic field is constructed by means of a modified tensor of strength $H^{\alpha}_{\mu}H^{\beta}_{\nu}F_{\alpha\beta}$, and a Lagrangian of a gravitational field $L_{(g)}$ is constructed by means of a modified curvature tensor $H^{\varepsilon}_{\mu}H^{\beta}_{\nu}R^{ab}_{\varepsilon\beta}$.

A Lagrangian $L_{(\sigma)}$ of translation gauge fields B^{ε}_{μ} cannot be built in the Yang-Mills form because Lie algebras of affine groups admit no invariant nondegenerate bilinear forms. To construct $L_{(\sigma)}$ one can apply quantities σ^{ν}_{μ} and $D_{\alpha}\sigma^{\nu}_{\mu}$ where, in view of the definition of σ^{ν}_{μ}, the connection Γ acts only on the upper index of σ^{ν}_{μ}. Only the combination $F^{\alpha}_{\nu} = D_{[\nu}\sigma^{\alpha}_{\mu]}$ is then possible. The general form of $L_{(\sigma)}$ is

$$L_{(\sigma)} = \tfrac{1}{2}(a_1 F_{\mu\nu\mu}F^{\alpha\nu\alpha} + a_2 \ {}_{\mu\nu\sigma}F^{\mu\nu\sigma} + a_3 F_{\mu\nu\sigma}F^{\nu\mu\sigma}$$
$$+ a_4 \varepsilon^{\mu\nu\sigma\gamma}F^{\varepsilon}_{\mu\varepsilon}F_{\gamma\nu\sigma} - \mu\sigma^{\mu}_{\nu}\sigma^{\nu}_{\mu} + \lambda\sigma^{\mu}_{\mu}\sigma^{\nu}_{\nu}),$$

where $\sigma_{\mu\nu} = g_{\mu\alpha}\sigma^{\alpha}_{\nu}$. It seems natural to require that the component $T^{00}_{(\sigma)}$ of a metric energy-momentum tensor of σ should be positive. This requirement imposes the following constraints

$$d = 0, \quad a_1 \geqslant 0, \quad a_2 \geqslant 0, \quad a_3 + 2a_2 = 0, \quad \mu \geqslant 0, \quad \lambda \leqslant \tfrac{1}{4}\mu$$

on constants of $L_{(\sigma)}$. The Lagrangian $L_{(\sigma)}$ then reads

$$L_{(\sigma)} = \tfrac{1}{2}[a_1 F_{\mu\nu\mu}F^{\alpha\nu\alpha} + a_2 F_{\mu\nu\sigma}(F^{\mu\nu\sigma} - 2F^{\nu\mu\sigma}) - \mu\sigma^{\mu}_{\nu}\sigma^{\nu}_{\mu} + \lambda\sigma^{\mu}_{\mu}\sigma^{\nu}_{\nu}].$$

Matter sources of a field σ are the following: a short canonical energy-momentum tensor of matter fields

$$\frac{\delta L_{(m)}}{\delta\sigma^{\mu\nu}} = (H^{-1})_{\nu\beta}D_{\mu}\varphi\frac{\partial L_{(m)}}{\partial D_{\beta}\varphi} = (H^{-1})_{\nu\beta}(t^{\beta}_{(m)\mu} + \delta^{\beta}_{\mu}L_{(m)}),$$

a short metric energy-momentum tensor of an electromagnetic and Yang–Mills fields, and a curvature tensor $-\kappa^{-1}H^{\nu}_{\mu}R^{\mu\gamma}_{\nu\varepsilon}$ of a gravitational field. We can however, replace this gravitation term by the term in the right hand side of Einstein's equation. The equation for σ then takes the form

$$\frac{\delta L_{(\sigma)}}{\delta\sigma^{\mu\nu}} = \left[-\frac{\delta L_{(m)}}{\delta\sigma^{\mu\nu}} + (H^{-1})_{\nu\beta}(T^{\beta}_{(m)\mu} - \tfrac{1}{2}\delta^{\beta}_{\mu}T_{(m)})\right]$$
$$-(H^{-1})_{\nu\mu}L_{(A)} + (H^{-1})_{\nu\beta}(T^{\beta}_{(\sigma)\mu} - \tfrac{1}{2}\delta^{\beta}_{\mu}T_{(\sigma)}). \tag{5}$$

We restrict ourselves to a weak field σ, that is, we neglect the gravitational field and the torsion on the left-hand side of equation (5) and the field σ on the right hand side of this equation. We have

$$\frac{\delta L_{(\sigma)}}{\delta\sigma^{\mu\nu}} = a_1(\eta_{\mu\nu}\partial^{\varepsilon}F^{\alpha}_{\alpha\varepsilon} - \partial_{\mu}F^{\alpha}_{\alpha\nu}) + 2a_2\partial^{\varepsilon}(F_{[\mu\nu]\varepsilon} + F_{\varepsilon\mu\nu}) - \mu\sigma_{\mu\nu} + \lambda\eta_{\mu\nu}\sigma^{\alpha}_{\alpha}.$$

In the case of a free field σ, taking into account the relation

$$\partial^{\nu}\frac{\delta L_{(\sigma)}}{\partial\sigma^{\mu\nu}} = -\mu\partial^{\nu}\sigma_{\mu\nu} + \lambda\partial_{\mu}\sigma = 0, \tag{6}$$

one can bring equation (5) into the

$$4a_2\partial^\varepsilon(\omega_{\mu\varepsilon,\nu}+\omega_{\nu\mu,\varepsilon}-\omega_{\nu\varepsilon,\mu})+2a_1\omega_{\alpha[\nu,\mu]\alpha}-\mu\omega_{\mu\nu} = 0, \tag{7}$$

$$a_1\left[\frac{\lambda}{\mu}-1\right][\eta_{\mu\nu}\Box e-e_{,\mu\nu}]+2a_1\omega_{\alpha(\nu,\mu)\alpha}-\mu e_{\mu\nu}+\lambda\eta_{\mu\nu}e = 0, \tag{8}$$

where $e_{\mu\nu} = \frac{1}{2}\sigma_{(\mu\nu)}$ and $\omega_{\mu\nu} = \frac{1}{2}\sigma_{[\mu\nu]}$ $(e = \sigma^\alpha_\alpha)$ are symmetric and antisymmetric parts of σ respectively. It seems natural to choose the solution $\omega = 0$ of equation (7). Equation (8) then can be written in the form

$$e_{\mu\nu} = \frac{\mu-\lambda}{3\mu}\left(\eta_{\mu\nu}e-m^{-2}\left(\frac{\mu-4\lambda}{\mu-\lambda}\right)e_{,\mu\nu}\right),$$

$$\Box e+m^2e = 0, \qquad m^2 = \frac{\mu(\mu-4\lambda)}{3a_1(\mu-\lambda)}. \tag{9}$$

This equation admits plane wave solutions

$$e_{\mu\nu} = \frac{\mu-\lambda}{3\mu}\left(\eta_{\mu\nu}+\left(\frac{\mu-4\lambda}{\mu-\lambda}\right)\frac{p_\mu p_\nu}{p^2}\right)a(p)e^{ipx}, \qquad p^2 = m^2.$$

Note that, in general case, the divergence (6) is not equal to zero and is not a gradient quantity. It follows then that $\mu \neq 0$.

Let a matter source of σ be a motionless point mass M. In this case, the right-hand side of equation (5) is

$$-\tfrac{1}{2}\eta_{\mu\nu}M\delta(r),$$

and this equation admits a static spherically symmetric solution given by the following expressions

$$e_{rr} = -\frac{1}{\mu-\lambda}(3\lambda e_{00}+\tfrac{1}{2}M\delta(r)), \qquad e_{\theta\theta} = -e_{00}r^2, \qquad e_{\varphi\varphi} = -e_{00}r^2\sin^2\theta,$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}e_{00}-m^2e_{00} = -\tfrac{1}{6}\frac{\mu}{a_1(\mu-\lambda)}M\delta(r),$$

$$e_{00} = -\frac{\mu M}{6a_1(\mu-\lambda)}\frac{e^{-mr}}{r}.$$

Substituting this solution into equation (4), we obtain the modification of Newton's gravitational potential

$$\tilde{\varphi} = \varphi+e_{00} = -\frac{\kappa M}{8\pi r}\left(1-\frac{\kappa^{-1}\mu}{3a_1(\mu-\lambda)}e^{-mr}\right).$$

Note that, to contribute to standard gravitational effects, the fifth fundamental interaction must be as universal as gravity, so its matter source can be represented by a mass or other parts of the energy-momentum tensor. This interaction must be described by a massive classical field, though its mass may be unusually small. The translation field gauge model fits all these conditions. For example, the mass (9) is expressed by means of constants of the Lagrangian $L_{(\sigma)}$ where μ and λ make the sense of coefficients of "elasticity" of a space-time [3, 4].

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