MULTIDIMENSIONAL COSMOLOGICAL MODELS WITH HYDRODYNAMICAL ENERGY-MOMENTUM TENSOR. PART I. ANALYSIS OF DYNAMICAL SYSTEMS IN FINITE DOMAINS*

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The dynamics of the full class of multidimensional cosmological models with topology FRW $\times T^D$, where T^D is a D-dimensional torus is investigated. Phase portraits show possible evolutions of FRW $\times T^D$ models with a hydrodynamical energy-momentum tensor. Typical solutions for late times are studied. The stability of solutions, with dynamical reduction and inflation as dynamical effects of extra dimensions, is also discussed.

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0. Introduction

Dimension and metric belong to fundamental properties of the physical space. The metric of our space is a pseudo-Riemannian and the dimension $N = 1+3$. This has been established, to a high degree of accuracy, within the length range of 10^{-16} cm – 10^{28} cm. However, there are no reasonable arguments available for claiming that the space-time dimension should be $N = 1+3$ also beyond this range. Presently, in connection with the Kaluza-Klein idea, it is believed that for length below 10^{-16} cm the dimension could be higher. According to this idea, one assumes that the additional dimensions, at the present epoch, are compactified and "small", i.e. their sizes are of an order of Planck's length ($l_{Pl} = 1.6 \times 10^{-33}$ cm).

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If we assume that the volume of the internal space varies with time, then the values of the fundamental physical constants should vary as well [1–3]. However, there is no experimental evidence for any dependence of the fundamental constants on time. Therefore, any theory exploiting higher dimensions should provide a machanism to make the internal space static, i.e. leading to the stable configuration of the type: (Friedman-Robertson-Walker model) × K (static internal space), [4–6]. The idea of purely gravitational dimensional reduction was first put forward by Chodos and Detweiler [7, 8]. At present, many vacuum solutions for the multidimensional cosmology, involving dynamical dimensional reduction are known [9–12]. However, the existence of this reduction depends on a particular choice of initial conditions. This is, of course, against the spirit of a "correct" reduction mechanism. One of the suggested mechanisms to solve the problem consists in taking into account the hydrodynamical energy-momentum tensor. Dynamical effects generated by such a tensor will be examined in the second part of the present work.

In Section 1, the form of Einstein's equations for the multidimensional cosmologies with a hydrodynamic energy-momentum tensor, is derived. In Section 2, the equations are reduced to the form of an autonomous dynamical system (for five different cases). In Section 3, analysis of the dynamics of the considered models, in finite domains, is performed. Conclusions are presented in Section 4.

1. Einstein's equations for homogeneous multidimensional cosmologies

The Einstein equations for multidimensional cosmologies can be derived from the variational principle, which postulates that the action I, being the sum of the geometrical action I_g and the action of matter I_m, should be stationary, i.e. $\delta I = 0$. For N-dimensional gravitation, I_g has the form

$$I_g = -\frac{1}{16\pi G}\int \sqrt{-g_N}\,(R-2\Lambda)dx^N.$$

where R is the scalar of curvature, and Λ is the cosmological constant. For I_m one has:

$$I_m = \int L\sqrt{-g_N}\,dx^N.$$

where L is a Lagrangian dependent on g_{MN} and δg_{MN}. If, after Hilbert, the energy-momentum tensor is defined as:

$$\tfrac{1}{2}\sqrt{-g_N}\,T_{MN} = \frac{\partial}{\partial g^{MN}}\sqrt{-g_N}\,L - \frac{\partial}{\partial x^\sigma}\frac{\partial}{\partial g^{MN}}\sqrt{-g_N}\,L,$$

then the Einstein equations can be reduced (by using the condition $\delta I = 0$) to the form:

$$R_{MN} - \tfrac{1}{2}\,g_{MN}R - \Lambda g_{MN} = 8\pi\bar{G}T_{MN}. \tag{1}$$

For our purposes it is convenient to transform this equation into the form which does not include the curvature scalar:

$$R_{MN} = 8\pi\bar{G}\left(T_{MN} - \frac{1}{N-2}\,g_{MN}T - \frac{2}{N-2}\,\frac{1}{8\pi\bar{G}}\,\Lambda g_{MN}\right). \tag{2}$$

The metric tensor g_{MN} for N-dimensional space-time with one time-like dimension and $n = N-1$ space like dimensions is:

$$g_{MN} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -R^2(t)\tilde{g}_{mn} & 0 \\ 0 & 0 & -r^2(t)\tilde{g}_{\mu\nu} \end{pmatrix}. \tag{3}$$

Here it is assumed that the total n-multidimensional space can be presented as a product of two maximally symmetrical space, with d and D dimensions, respectively. By $\tilde{g}_{mn}$ we shall denote the metric tensor of the maximally symmetrical d-dimensional space, and by $\tilde{g}_{\mu\nu}$ that of the maximally symmetrical D-dimensional. (Both spaces are assumed to have the unit radius.) Accordingly, $R(t)$ is the scale factor of the d-dimensional space, $r(t)$ is the scale factor of the D-dimensional space, and $M, N = 1, \ldots, n-1$; $m, n = 1, 2, \ldots, d$; $u, v = (d+1), \ldots, (d+D)$.

Following the conventions adopted by Weinberg [13], the Ricci tensor R_{MN} assumes the form

$$R_{MN} = R^{C}_{MCN},$$
$$R^{A}_{BCD} = \partial_C \Gamma^{A}_{BD} - \partial_D \Gamma^{A}_{BC} + \Gamma^{A}_{CE}\Gamma^{E}_{BD} - \Gamma^{A}_{DE}\Gamma^{E}_{BC}, \quad A, B, C, D, E = 0, 1, \ldots, N-1. \tag{4}$$

By using the diagonality of g_{MN} and the fact that $\tilde{g}_{mn}$ is independent of $\tilde{g}_{\mu\nu}$, one can readily find the non-zero Christoffel symbols:

$$g_{MN} = 0 \quad M \neq N, \quad \partial_0 \tilde{g}_{mn} = \partial_0 \tilde{g}_{\mu\nu} = 0,$$
$$\Gamma^{0}{}_{mn} = \frac{\dot{R}}{R} g_{mn}, \quad \Gamma^{0}{}_{\mu\nu} = \frac{\dot{r}}{r} g_{\mu\nu},$$
$$\Gamma^{l}{}_{m0} = \frac{\dot{R}}{R} \delta^{l}_{m}, \quad \Gamma^{\nu}{}_{\mu 0} = \frac{\dot{r}}{r} \delta^{\nu}_{\mu},$$
$$\Gamma^{l}{}_{mn} = \tilde{\Gamma}^{l}{}_{mn}, \quad \Gamma^{\nu}{}_{\mu\nu} = \tilde{\Gamma}^{\nu}{}_{\mu\nu}, \tag{5}$$

where $\tilde{\Gamma}^{l}{}_{mn}$ and $\tilde{\Gamma}^{\nu}{}_{\mu\nu}$ are the Christoffel symbols for the metrics g_{mn}, $g_{\mu\nu}$.

Basing on (5), the non-zero components of the Ricci tensor can be written as:

$$R_{00} = d\frac{\ddot{R}}{R} + D\frac{\ddot{r}}{r},$$
$$R_{mn} = \tilde{R}_{mn} - g_{mn}\left\{\frac{d}{dt}\left(\frac{\dot{R}}{R}\right) + \left[d\frac{\dot{R}}{R} + D\frac{\dot{r}}{r}\right]\frac{\dot{R}}{R}\right\},$$
$$R_{\mu\nu} = \tilde{R}_{\mu\nu} - g_{\mu\nu}\left\{\frac{d}{dt}\left(\frac{\dot{r}}{r}\right) + \left[d\frac{\dot{R}}{R} + D\frac{\dot{r}}{r}\right]\frac{\dot{r}}{r}\right\}, \tag{6}$$

R_{mn} and $R_{\mu\nu}$ are the Ricci tensors for the space of dimension d and D, respectively. In such case the tensors (6) are of the particular form [13]:

$$\tilde{R}_{mn} = -(d-1)K\tilde{g}_{mn}, \quad \tilde{R}_{\mu\nu} = -(D-1)k\tilde{g}_{\mu\nu}, \tag{7}$$

where K, k are constants characterizing the curvature of d- and D-dimensional spaces, respectively. Thus the general form of the Ricci tensor is:

$$R_{00} = d\frac{\ddot{R}}{R} + D\frac{\ddot{r}}{r},$$

$$R_{mn} = -g_{mn}\left\{\frac{(d-1)K}{R^2} + \frac{d}{dt}\left(\frac{\dot{R}}{R}\right) + \left[d\frac{\dot{R}}{R} + D\frac{\dot{r}}{r}\right]\frac{\dot{R}}{R}\right\},$$

$$R_{\mu\nu} = -g_{\mu\nu}\left\{\frac{(D-1)k}{r^2} + \frac{d}{dt}\left(\frac{\dot{R}}{R}\right) + \left[d\frac{\dot{R}}{R} + D\frac{\dot{r}}{r}\right]\frac{\dot{R}}{R}\right\}. \tag{8}$$

In the reference system comoving with matter, the energy-momentum tensor has then the following form:

$$T_{00} = \varrho, \quad T_{m0} = T_{\mu 0} = 0, \quad T_{mn} = pg_{mn}, \quad T_{\mu\nu} = p'g_{\mu\nu}. \tag{9}$$

The assumed symmetries imply that the energy density ϱ and the pressures p and p' in the individual spaces, depend only on time: $R = R(t)$, $p = p(t)$, $p' = p'(t)$. Now, the Einstein equations can be put into the following form:

$$d\frac{\ddot{R}}{R} + D\frac{\ddot{r}}{r} = -\frac{8\pi G}{N-2}[(N-3)\varrho + dp + Dp' - \varrho_\Lambda],$$

$$\frac{d}{dt}\left(\frac{\dot{R}}{R}\right) + \left[d\frac{\dot{R}}{R} + D\frac{\dot{r}}{r}\right]\frac{\dot{R}}{R} + \frac{(d-1)K}{R^2} = \frac{8\pi G}{N-2}[\varrho + (D-1)p - Dp' + \varrho_\Lambda],$$

$$\frac{d}{dt}\left(\frac{\dot{r}}{r}\right) + \left[d\frac{\dot{R}}{R} + D\frac{\dot{r}}{r}\right]\frac{\dot{r}}{r} + \frac{(D-1)k}{r^2} = \frac{8\pi G}{N-2}[\varrho + (d-1)p' - dp + \varrho_\Lambda], \tag{10}$$

where $\varrho_\Lambda = \dfrac{2\Lambda}{8\pi G}$. If, for convenience, we denote $H = \dfrac{\dot{R}}{R}$, $h = \dfrac{\dot{r}}{r}$, equations (10) assume the form (in the units $8\pi G = 1$):

$$d\dot{H} + dH^2 + D\dot{h} + Dh^2 = -\frac{1}{N-2}[(N-3)\varrho + dp + Dp' - \varrho_\Lambda],$$

$$\dot{H} + dH^2 + DHh + \frac{(d-1)K}{R^2} = \frac{1}{N-2}[\varrho + (D-1)p - Dp' + \varrho_\Lambda],$$

$$\dot{h} + Dh^2 + dHh + \frac{(D-1)K}{r^2} = \frac{1}{N-2}[\varrho + (d-1)p' - 3p + \varrho_\Lambda]. \tag{11}$$

The final form of Einstein equations for multidimensional cosmologies with topology $\mathrm{R} \times \mathrm{S}^3 \times \mathrm{S}^D$ is:

$$3\dot{H} + 3H^2 + D\dot{h} + Dh^2 = -\frac{1}{D+2}[(D+1)\varrho + 3p + Dp' - \varrho_\Lambda]$$

$$\dot{H}+3H^2+DHh+\frac{2K}{R^2} = \frac{1}{D+2}[\varrho+(D-1)p-Dp'+\varrho_\Lambda],$$

$$\dot{h}+Dh^2+3Hh+\frac{(D-1)K}{r^2} = \frac{1}{D+2}[\varrho+2p'-3p+\varrho_\Lambda]. \tag{12a}$$

The requirement that the divergence of the energy-momentum tensor should vanish yields an additional equation, describing the dependence of the energy density on the scale factor:

$$T^\mu_{\nu;\mu} = 0 \leftrightarrows \frac{d\varrho}{dt}+3H(\varrho+p)+Dh(\varrho+p') = 0. \tag{12b}$$

Equations (12) costitute a complete system of equations, which describe the evolution of multidimensional cosmological models.

2. *Multidimensional cosmological models as dynamical systems*

In this section equations (12) will be transformed into the autonomous dynamical system. Then, we will discuss models with a flat internal space. In that case, the dynamical systems which describe the dynamics of the models FRW $\times T^D$ (where $T^D = S^1 \times \ldots \times S^1$) become two-dimensional ones. This allows us to present the complete classification of admissible solutions on phase-plane diagrams.

For the sake of further discussion it is useful to distinguish five particular models A — E.

A. Vacuum model

For vacuum, $\varrho = p = p' = 0$, $\Lambda = 0$, and the Einstein equations assume the form:

$$3\dot{H}+3H^2+Dh^2+D\dot{h} = 0, \quad \dot{H}+3H^2+DHh+\frac{2K}{R^2} = 0, \quad \dot{h}+Dh^2+3Hh = 0. \tag{13}$$

From the constraint requirement we can obtain the "boundary condition":

$$\frac{3K}{R^2} = -6H^2-6DHh-D(D-1)h^2. \tag{14}$$

By substituting (14) into equations (13) one gets:

$$\frac{dH}{dt} = -H^2+DHh+\frac{D(D-1)}{3}h^2, \quad \frac{dh}{dt} = -3Hh-Dh^2,$$

$$\frac{3K}{R^2} = -6H^2-6DHh-D(D-1)h^2 \tag{15}$$

The equations have the form of an autonomous dynamical system.

B. Model with dust matter

For dust $p = p' = 0$, $\Lambda = 0$ therefore by using (12) we obtain:

$$\dot{H}+3H^2+D\dot{h}+Dh^2 = -\frac{D+1}{D+2}\varrho, \quad \dot{H}+3H^2+DHh = \frac{\varrho}{D+2}-\frac{2K}{R^2},$$

$$\dot{h}+Dh^2+3Hh = \frac{\varrho}{D+2}. \tag{16}$$

This system has the boundary condition of the form:

$$\varrho = 3H^2+3DHh+\frac{D(D-1)}{2}h^2+\frac{3K}{R^2} > 0. \tag{17}$$

After substituting (17) to (16) we obtain:

$$\dot{H} = -\frac{3(D+1)}{(D+2)}H^2-\frac{D(D-1)}{(D+2)}Hh+\frac{D(D-1)}{2(D+2)}h^2-\frac{(2D+1)}{D+2}\frac{K}{R^2},$$

$$\dot{h} = \frac{3}{D+2}H^2-\frac{6}{D+2}Hh-\frac{D(D+5)}{2(D+2)}h^2+\frac{3K}{(D+2)R^2},$$

$$\varrho = 3H^2+3DHh+\frac{D(D-1)}{2}h^2+\frac{3K}{R^2}. \tag{18}$$

It is convenient to introduce the variables $x = HR$, $y = hr$, $d\tau = \frac{1}{R}dt$. In such a case equations (18) assume the form:

$$\frac{dx}{d\tau} = -\frac{2D+1}{(D+2)}x^2-\frac{D(D-1)}{(D+2)}xy+\frac{D(D-1)}{2(D+2)}y^2-\frac{(2D+1)}{D+2}K,$$

$$\frac{dy}{d\tau} = \frac{3}{D+2}x^2-\frac{D-4}{D+2}xy-\frac{D(D+5)}{2(D+2)}y^2+\frac{3K}{(D+2)}, \tag{19}$$

and the system is determined within the domain defined by the constraint condition:

$$3x^2+3Dxy+\frac{D(D-1)}{2}y^2+3K > 0. \tag{20}$$

C. Model with a massless scalar field

In this case, $p = p' = \varrho$, $\Lambda = 0$, so we have the following system of equations:

$$\dot{H}+3H^2+DHh+\frac{2K}{R^2} = 0, \quad \dot{h}+Dh^4+3Hh = 0, \tag{21}$$

with the condition:

$$3H^2+3DHh+\frac{D(D-1)}{2}h^2+\frac{3K}{R^2} = \varrho \geqslant 0. \tag{22}$$

After transforming to the variables x, y and the new time τ, one has:

$$\frac{dx}{d\tau} = -2x^2 - Dxy - 2K, \quad \frac{dy}{d\tau} = -2xy - Dy^2, \tag{23}$$

with the condition:

$$3x^2 + 3Dxy + \frac{D(D-1)}{2} y^2 + 3K \geqslant 0. \tag{24}$$

D. Model with radiative matter

For radiation, $p = p' = \frac{\varrho}{n}$, $\Lambda = 0$. After taking this into account equations (12) become

$$\dot{H} + 3H^2 + DHh + \frac{2K}{R^2} = \frac{\varrho}{D+3}, \quad \dot{h} + Dh^2 + 3Hh = \frac{\varrho}{D+3}, \tag{25}$$

in the domain:

$$\varrho = 3H^2 + 3DHh + \frac{D(D-1)}{2} h^2 + \frac{3K}{R^2} \geqslant 0.$$

In the variables x, y, τ (25) assumes the form of the dynamical system:

$$\frac{dx}{d\tau} = -\frac{(2D+3)}{D+3} x^2 - \frac{D^2}{D+3} xy + \frac{D(D-1)}{2(D+3)} y^2 - \frac{(2D+3)K}{D+3},$$

$$\frac{dy}{d\tau} = \frac{3}{D+3} x^2 + \frac{D-6}{D+3} xy - \frac{D(D+7)}{2(D+3)} y^2 - \frac{3K}{D+3}, \tag{26}$$

E. Model with a cosmological constant

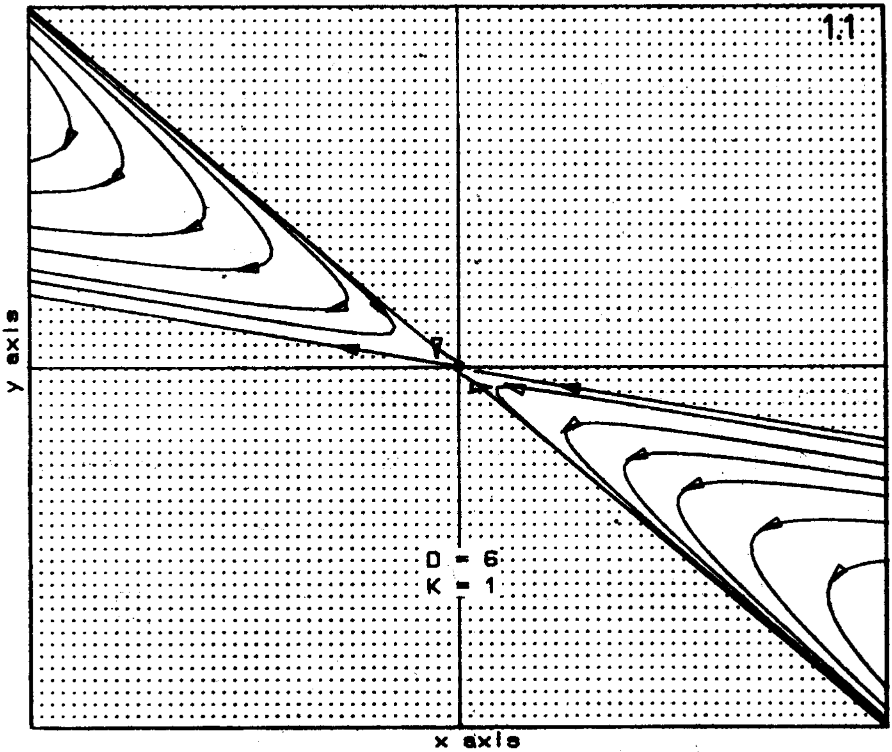
Here we have $\varrho = p = p' = 0$, $\Lambda \neq 0$. Thus, basing on (12), one obtains

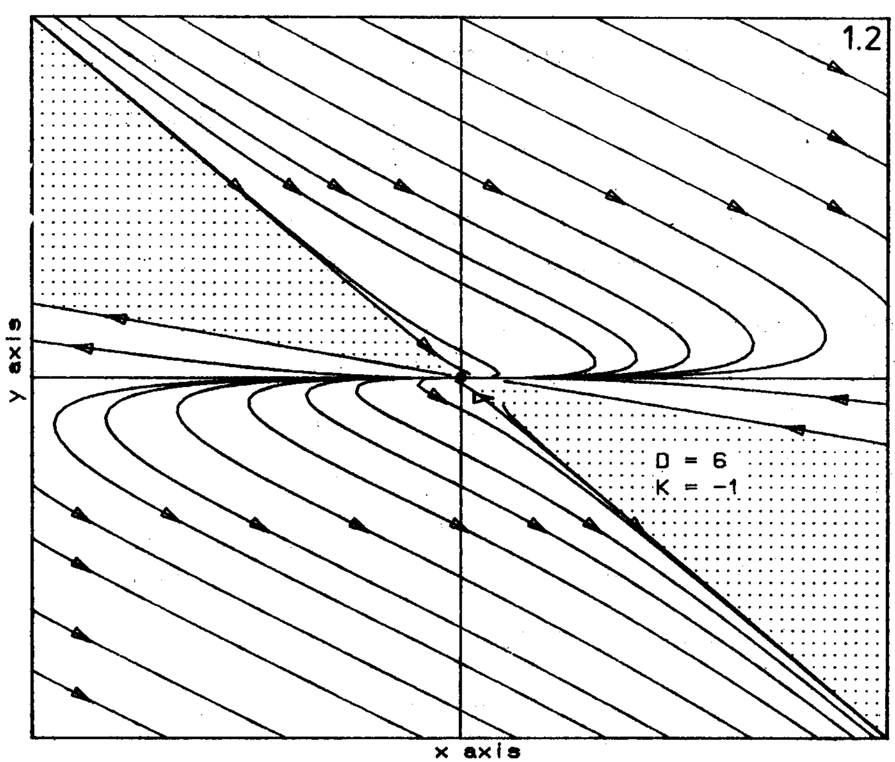
$$3\dot{H} + 3H^2 + D\dot{h} + Dh^2 = \frac{2\Lambda}{D+2},$$

$$\dot{H} + 3H^2 + DHh + \frac{2K}{R^2} = \frac{2\Lambda}{D+2}, \quad \dot{h} + Dh^2 + 3Hh = \frac{2\Lambda}{D+2}. \tag{27}$$

After simple transformations we obtain

$$\dot{H} = -H^2 + DHh + \frac{D(D-1)}{3} h^2 - \frac{2(D-1)}{3(D+2)} \Lambda, \quad \dot{h} = -3Hh - Dh^2 + \frac{2\Lambda}{D+2}, \tag{28}$$


1.1
y axis
x axis
D = 6
K = 1


1.2
y axis
x axis
D = 6
K = -1

The condition derived from the constraint equation is

$$\frac{K}{R^2} = -H^2 - DHh - \frac{D(D-1)}{6} h^2 + \frac{\Lambda}{3}.$$

3. *Qualitative methods of studying the dynamics of cosmological models FRW×T^D*

As it has been already mentioned, one of fundamental issues arising within teories with extra dimensions is the problem of a proper mechanism of the dimensional reduction. The problem of stability of solutions involving a static microspace was considered by Maeda [4], but he limited his analysis to the case of the microspace being solely a function of time. The method used in the present work allows us to investigate stability when the micro- and macrospace are anisotropic.

3.1. Dynamics of $\mathrm{FRW} \times \mathrm{T}^D$ models in finite regions of the phase plane

As we have seen, equations (13), (16), (22), (25), (27) have been transformed to the form of autonomous dynamical systems. As an example, diagrams show the phase portraits for the values of dimension $D = 1$, $D = 6$, for $K = 0, \pm 1$, and for cases A and D.

In the cases B, C, D and $K = -1$, there are two critical points $x_0 = \pm 1$, $y_0 = 0$. The point $x_0 = 1$, $y_0 = 0$ is a stable attracting point, which corresponds to the solution with a static microspace evolving linearly in time: $R \propto t$. The point $x_0 = -1$, $y_0 = 0$ represents the same solution, but it is an unstable repulsing point.

In the case $K = 0$ there is a composite critical point of the saddle-knot character at $x_0 = 0$, $y_0 = 0$, which corresponds to the solution: (Minkowski's space)×K (static space).

For $K = 1$, systems (19), (23), (26) have no critical points.

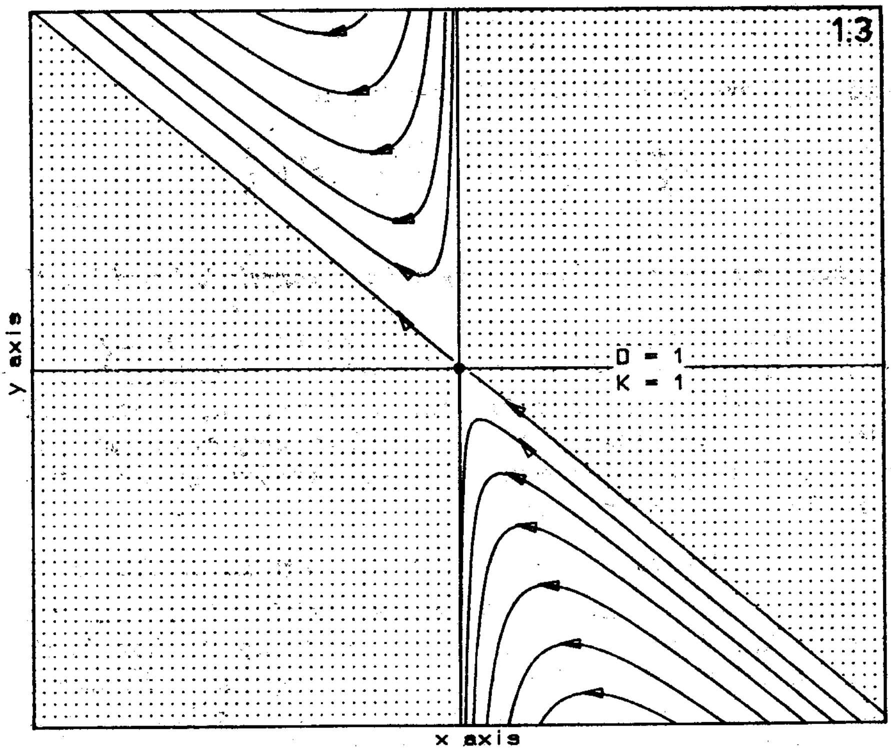
For the $\mathrm{FRW} \times \mathrm{T}^D$ model with the cosmological constant $\Lambda > 0$, there are two knots in the finite domain $H_0 = h_0 = \pm \sqrt{\frac{2\Lambda}{(D+2)(D+3)}}$. They are situated at the boundary of the physical region. When $H_0 > 0$, there is an attracting knot, and when $H_0 < 0$ — a repulsing one. Both points correspond to vacuum states. For negative values of the cosmological constant the system has no critical points.

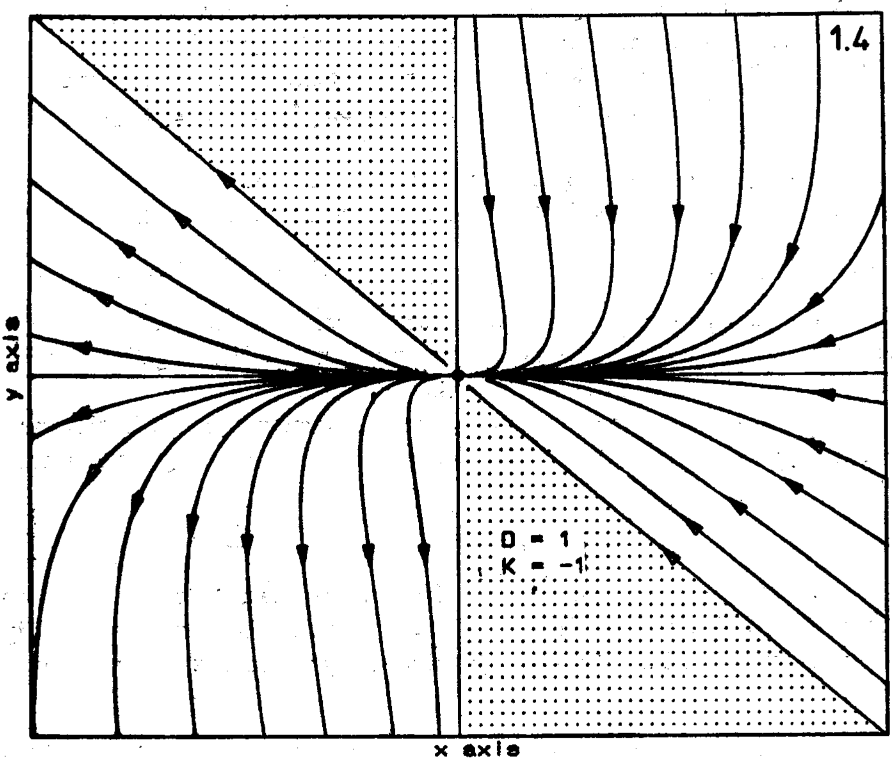
In Figs. 1.1–1.4 phase portraits are presented for vacuum models. In this case, there is one composite critical point $H_0 = h_0 = 0$ and the corresponding state is unstable.

4. *Conclusions*

In the present paper, the dynamics of the complete class of multidimensional cosmological models with topology $\mathrm{FRW} \times \mathrm{T}^D$ and a source in the form of a hydrodynamic energy-momentum tensor has been investigated by using methods of the qualitative theory

Fig. 1.1–1.2. Phase portraits for the vacuum model $\mathrm{FRW} \times \mathrm{T}^D$ in finite domains, for $D = 6$ and $K = 1, -1$


1.3
D = 1
K = 1
y axis
x axis


1.4
D = 1
K = -1
y axis
x axis

of differential equations. The phase portraits for the cases under consideration have been made with the help of the computer program "Dynamic" (see A. Łapeta, B. Łapeta, M. Szydłowski, Internal Publ. OAUJ, 1989).

In the literature, there are known only some exact vacuum solutions, namely those which can be explicitly integrated in quadratures. Application of the dynamical systems methods allows one to analyse the complete class of models and to examine their typical properties.

Our main conclusions are:

1) All typical states of the metric, for large times are situated at the boundaries of relevant constraint conditions $\varrho = 0$, $\frac{K}{R^2} = 0$, which correspond to the states for which the effects due matter or curvature $\frac{K}{R^2}$ are negligible.

2) For the FRW $(K = -1) \times T^D$ models, the typical state of the metric, as $t \to \infty$, is the model corresponding to the Milne phase of the physical space ($R \propto t$) and to a static state in the internal space. This configuration is asymptotically stable and can be attained in two ways: either a) the internal space contracts to an infinitely small size, while the physical space expands; or b) the internal space expands from a singularity to a constant size, while the physical space expands too. In the literature, for no clear reason, more attention is given to the case a). It can be demonstrated that, for models realizing case b), the horizon problem can be overcome. Barrow [17] has showed that in the inflationary models the section of the horizon problem is connected with the condition $\ddot{R} > 0$ (he has considered classical 3+1-dimensional cosmologies). For the FRW models, this condition is met only when the strong energy condition is violated $(\varepsilon + 3p) \leqslant 0$). In the FRW $\times T^D$ models, due to the expansion of the internal space to a constant size, the inequality $\ddot{R} > 0$ can be satisfied. The region in which this condition holds is indicated in the phase portrait for the world model with a radiation matter (see Fig. 3). It should be noticed that the horizon problem can be resolved only within the models with an expanding internal space.

3) Let us consider the case of models having a source in the form of a hydrodynamic energy-momentum tensor. It can be shown [18] that the quantum effects, due to a massless scalar field in an external gravitational field, at high temperatures, give rise to a hydrodynamic energy-momentum tensor of radiative matter $\left(p = \frac{\varrho}{D+3}\right)$. The effects of classical massless scalar fields correspond to the case $p = \varrho$ discussed in this paper.

4) The overall result of this paper is that the dynamical system approach can be efficiently applied to investigate the dynamics of multidimensional cosmological models. The method is especially usefull to discuss generic character of different model's properties and their dependence on initial conditions.

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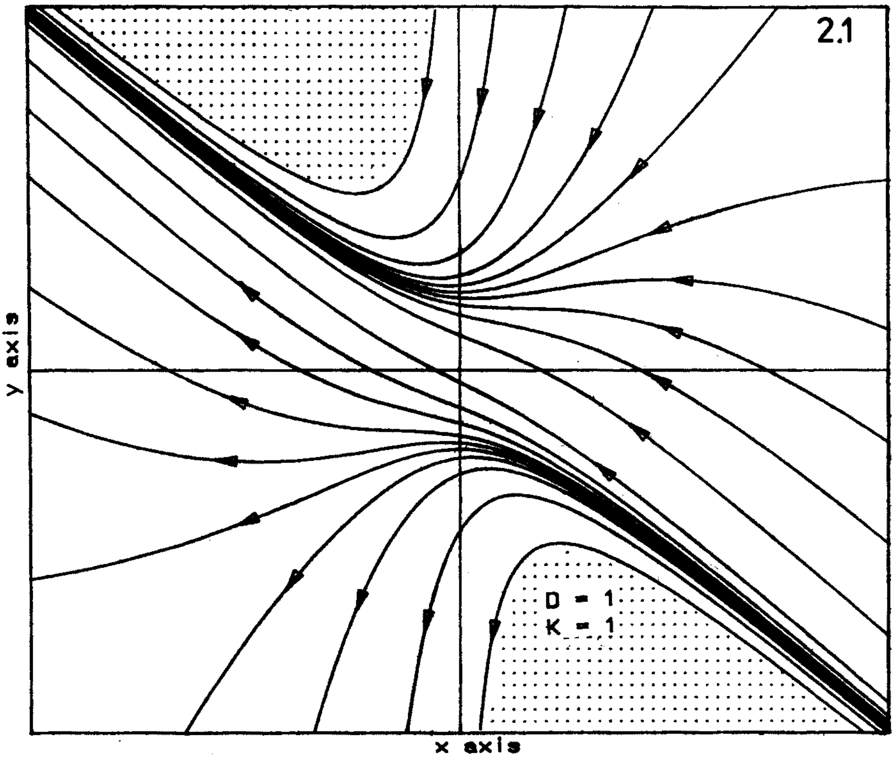
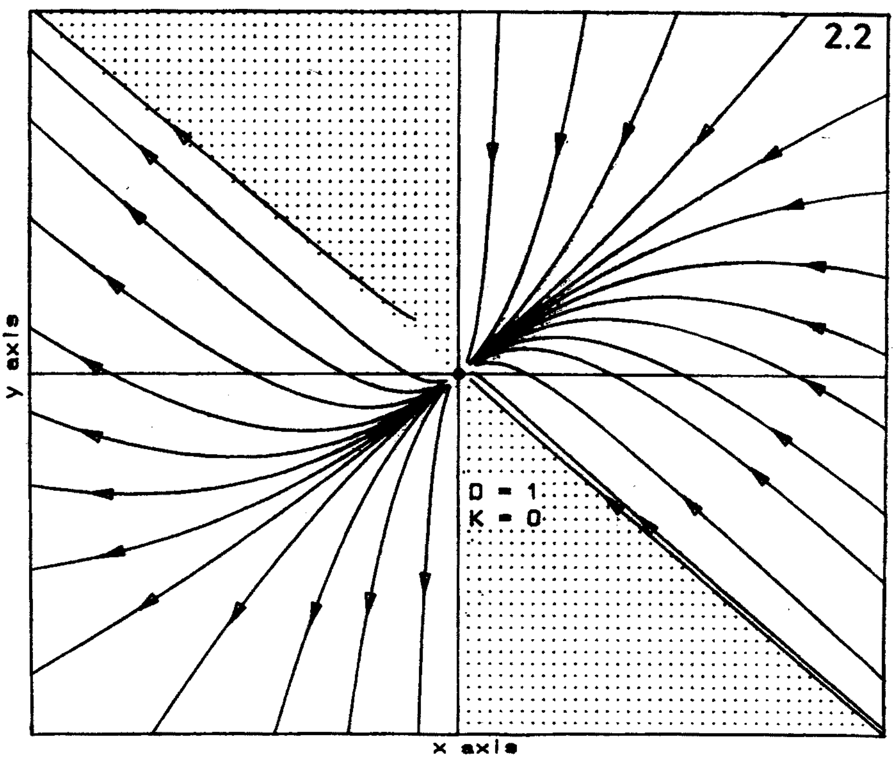
Fig. 1.3–1.4. Phase portraits for the vacuum model FRW $\times T^D$ in finite domains, for $D = 1$ and $K = 1, -1$


2.1
D = 1
K = 1
y axis
x axis


2.2
D = 1
K = 0
y axis
x axis

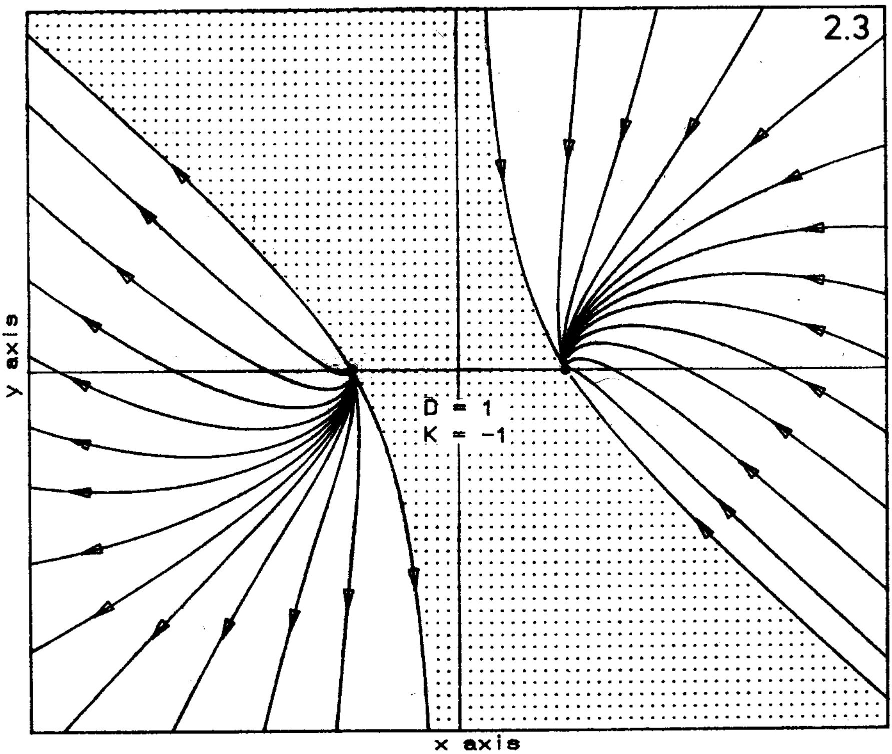


Fig. 2.1–2.3. Phase portraits for model FRW$\times$T^D with radiative matter in finite domains, for $D = 1$ and $K = 1,\ 0-1$

APPENDIX

General remarks useful in determining possible evolution patterns from phase portraits

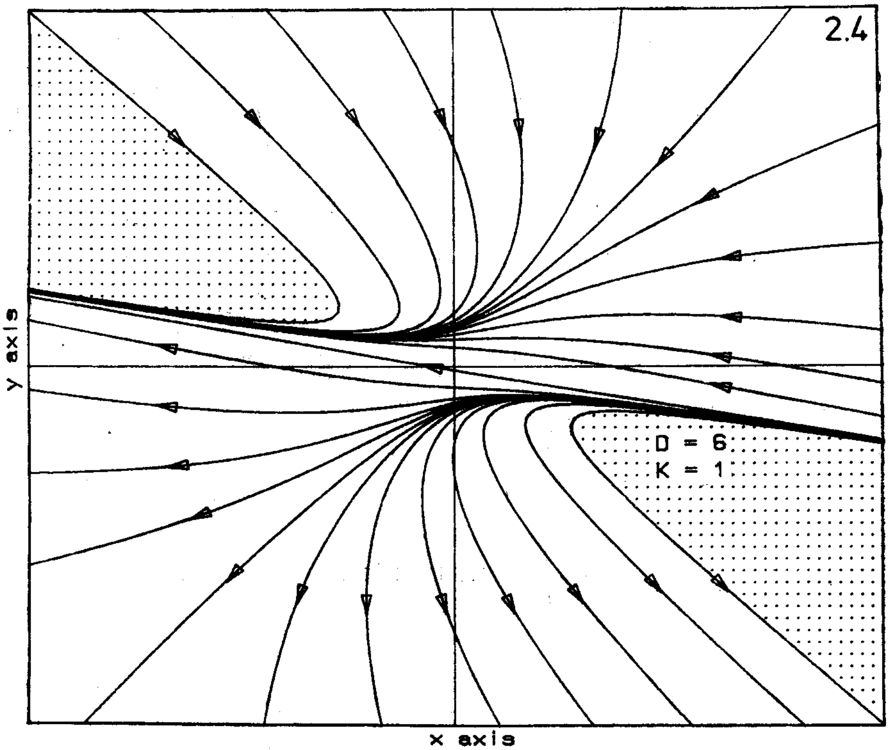
1. Phase trajectories are scaled with different time parameters, the time parameter $\tau\left(d\tau = \frac{dt}{R},\ d\tau_1 = x\,d\tau\right)$ is strictly monotonic function of the time $t\left(\frac{d\tau}{dt} = R > 0\right)$.

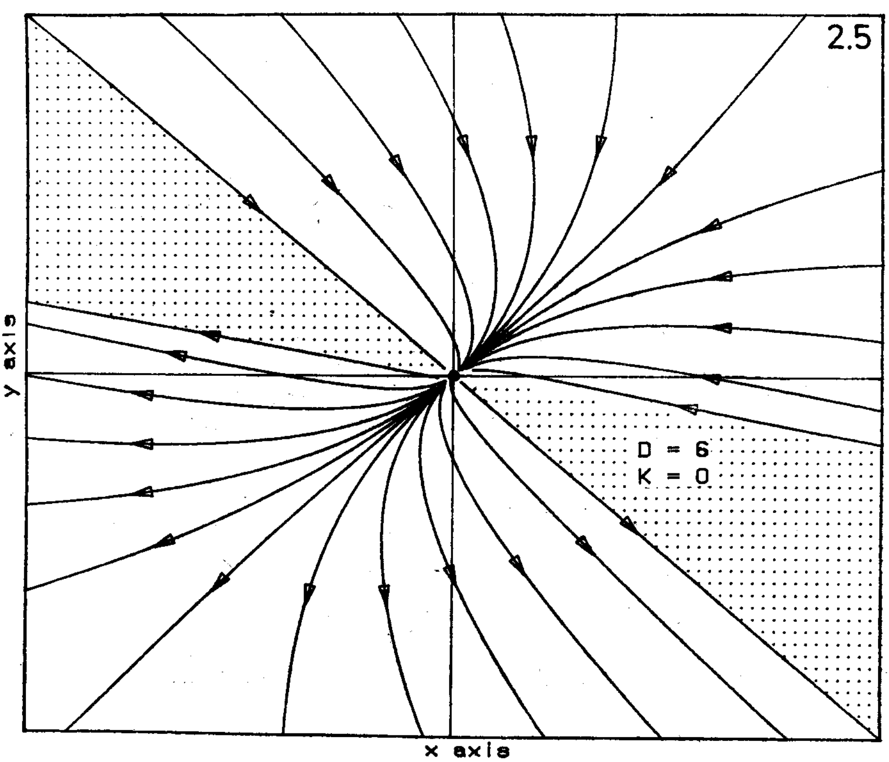
2. In asymptotic states, typical states of the metric are always situated at boundaries of the relevant constraint conditions and are represented by the Kasner asymptotics:

$$3p_1+Dp_2 = 3p_1^2+Dp_2^2 = 1, \quad p_{1\pm} = \frac{3\pm\sqrt{3D(D+2)}}{3(D+3)},$$

$$p_{2\pm} = \frac{D\pm\sqrt{3D(D+2)}}{3(D+3)}.$$

The asymptotic of corresponding solutions near the initial singularity have the form (in time t):


2.4
D = 6
K = 1
y axis
x axis


2.5
D = 6
K = 0
y axis
x axis

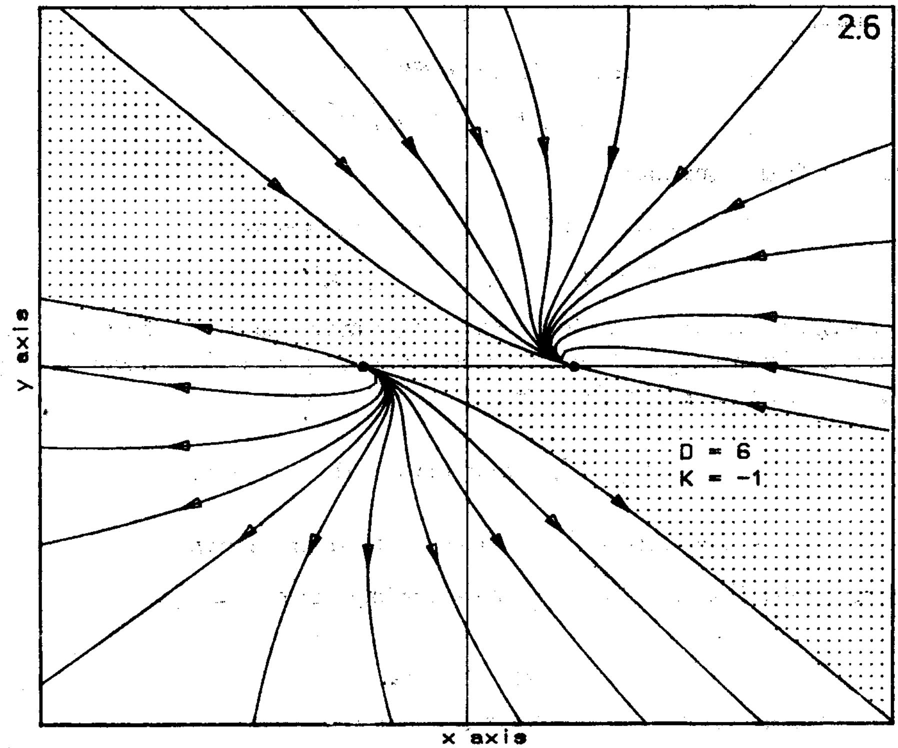


Fig. 2.4–2.6. Phase portraits for model FRE × T^D with radiative matter in finite domains, for $D = 6$ and $K = 1, 0-1$

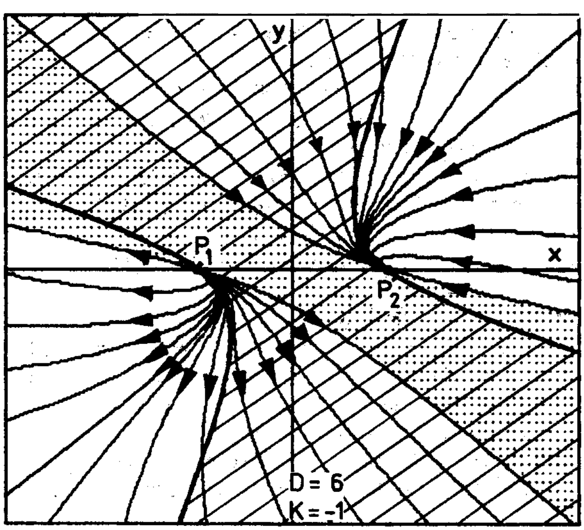


Fig. 3. Phase portraits for model FRW × T^D with radiative matter in finite domains, for $D = 6$ and $K = -1$. The region where $\ddot{R} > 0$ is represented by hatching

near the initial singularity

$$R \propto t^{p_{1}+}, \quad r \propto t^{p_{2}-} \quad t \to 0 \tag{A1}$$

$$R \propto t^{p_{1}-}, \quad r \propto t^{p_{2}+} \tag{A2}$$

and near the final singularity:

$$R \propto (t_0-t)^{p_{1}+}, \quad r \propto (t_0-t)^{p_{2}-} t \to t_0 \tag{B1}$$

$$R \propto (t_0-t)^{p_{1}-}, \quad r \propto (t_0-t)^{p_{2}+}. \tag{B2}$$

3. Asymptotic states, in which the sizes of the macrospace and the microspace are comparable in size, are represented by non-stable saddle points. For instance, asymptotic solutions for radiation are of the form:

$$R \propto r \propto (t)^{\frac{2}{D+4}}, \quad \text{near the initial singularity},$$

$$R \propto r \propto (t_0-t)^{\frac{2}{D+4}}, \quad \text{near the final singularity}.$$

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