REPULSIVE PAIR INTERACTIONS AT THE STRONG EXTERNAL FIELD

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For the case of a pair repulsive potential, certain integral identities between conditioned correlation functions describing the grand canonical Gibbs ensemble of the Gonchar type are derived and some applications of them are described. In particular we prove the uniqueness of the grand canonical Gibbs whenever sufficiently strong external field is switched on.

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1. Introduction

The knowledge of the structure of the set of the Gibbs equilibrium states describing continuous systems of classical particles at thermal equilibrium in the region of large values of the chemical activity or at low temperatures is still very incomplete. Some recent advances include results for a class of generalized Widom-Rowlison type of models [1–3] and the charged but neutral two-component systems in which the interaction is given by a sufficiently regular function of positive type [4–9]. Recently we have proved uniqueness and analyticity of the Gibbs equilibrium states for values of the inverse chemical activity that do not belongs to the spectrum of the corresponding Kirkwood-Salsburg operator [10–11].

In the present paper we consider a very special case of the general situation analyzed in [10–11]. We consider classical gas of particles in which the interaction is given by a two body, repulsive potential V_2 and moreover there is sufficiently strong external field V_1 switched on. This corresponds to the class of systems considered before by Moraal in series of papers [12]. Selecting certain system of integral identities of the type used previously by Gonchar [13] we give a short proof of the uniqueness and analyticity of the limiting Gibbs state. This is a corollary of the results obtained in our recent papers [10, 11]. Here we intend to present an independent proof of this by a method which seems to be ideally

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well suited for the case at hands. Several other applications of those integral identities will be presented elsewhere. In the Appendix to this paper we present a quick derivation of those integral identities.

2. *Pair repulsive interactions in a strong external field*

Throughout this paper we will assume that $V_2 \geqslant 0$ and V_2 is of certain decrease at infinity (see below) and moreover V_1 is such that for every $\beta > 0$: $\exp(-\beta V_1) \in L_1(R^d)$. Let

$$\psi(dx) = \exp(-\beta V_1(x))dx. \tag{2.1}$$

Let $\mathscr{B}_1$ be from now on the Banach space of all sequences of measurable functions $f = (f_1, f_2, \ldots,)$, where each f_n is measurable on $([0, 1] \otimes R^d)^{\otimes n}$ as a function of the argument $(t, x)_n = (t_1, x_1, \ldots, t_n, x_n)$ and the norm in $\mathscr{B}_1$ is given by:

$$\|f\|_1 \equiv \operatorname*{ess\,sup}_{(t,x)} f_n(t, x)_n. \tag{2.2}$$

In the space $\mathscr{B}_1$ we define the following vectors; for any

$$\Lambda \subseteq R^d, \quad z \geqslant 0, \quad \beta > 0, \quad \omega \in \Omega^T(R^d).$$

Let:

$$E_\Lambda^\omega(z, \alpha, \beta | (t, x)_n)$$

$$\equiv \exp\left(-z \int_\Lambda d\psi(y) \left[1 - e^{-\beta\alpha \sum_{i=1}^n t_i V_2(x_i - y)} e^{-\beta \sum_{i=1}^n E((t_i, x_i)|\omega(\Lambda^c))}\right]\right), \tag{2.3}$$

where as before

$$E((t_1, x_1)|\omega(\Lambda^C)) = \sum_{z \in \omega(\Lambda^c)} t_1 V_2(x_1 - z) \tag{2.4}$$

means the energy of interaction of the configuration (t_1, x_1) with $\omega(\Lambda^C)$ by the two-body potential V_2. The parameter $\alpha \in [0, 1]$.
Then the sequence

$$E_\Lambda^\omega(z, \alpha, \beta) \equiv \{E_\Lambda^\omega(z, \alpha, \beta | (t, x)_n)\}_{n=1}^\infty \tag{2.5}$$

belongs to the space $\mathscr{B}_1$ and $\|E_\Lambda^\omega(z, \alpha, \beta)\|_1 \leqslant 1$ uniformly in all parameters.
For $\omega = \phi$ let us define

$$E_\Lambda^{\omega=\phi}(z, \alpha, \beta) = E_\Lambda(z, \alpha, \beta) \tag{2.6}$$

and for $\omega = \phi$, $\Lambda = R^d$

$$E_\Lambda^{\omega=\phi}(z, \alpha, \beta) = E_\infty(z, \alpha, \beta). \tag{2.7}$$

Certain linear operators acting in the space $\mathscr{B}_1$ will now be defined for any $\Lambda \subseteq R^d, \omega \in \Omega^T(R^d)$. Let $G_\Lambda^\omega(z,\beta)$ be defined by:

$$(G_\Lambda^\omega(z,\beta)f)_m(t,x)_m$$

$$= \beta z^2 E_\Lambda^\omega(z,1,\beta|(t,x)_m)\int_0^1 d\alpha \int_0^1 d\tau \int_\Lambda d\psi(y)\int_\Lambda d\psi(y')E_\Lambda^\omega(-z,\alpha,\beta|(t,x)_m)$$

$$\times (\sum_{i=1}^m t_i V(x_i-y))\exp(-\beta\alpha)\sum_{i=1}^m t_i V(x_i-y)V(y-y_i)\exp(-\beta\tau V(y-y'))$$

$$\times \exp(-\beta E(\tau,y)|\omega(\Lambda^C))\exp(-\beta E(y'|\omega(\Lambda^C))f_{m+2}((\alpha t,x)_m,(\tau,y),(1,y')). \tag{2.8}$$

We will use also the notation:

$$G_\Lambda^{\omega=\phi}(z,\beta) = G_\Lambda(z,\beta); \quad G_{\Lambda=R^d}^{\omega=\phi}(z,\beta) = G_\infty(z,\beta). \tag{2.9}$$

Using the following simple estimates:

$$\operatorname{ess\,sup}_{(t,x)_m}\sup_{0\leqslant\alpha\leqslant 1}\frac{E_\Lambda^\omega(z,1,\beta|(t,x)_m)}{E_\Lambda^\omega(z,\alpha,\beta|(t,x)_m)}\leqslant 1, \tag{2.10}$$

$$x\int_0^1 e^{-\alpha x}d\alpha \leqslant 1-e^{-x} \quad \text{for} \quad x>0 \tag{2.11}$$

we derive the following estimate on the norm of the operator $G_\Lambda^\omega(z,\beta)$:

$$|||G_\Lambda^\omega(z,\beta)|||_1 \leqslant 4z\beta^2\psi(\Lambda) \tag{2.12}$$

which shows that $G_\Lambda^\omega(z,\beta)$ is bounded and moreover for small $z\beta^2$ is a contraction.

Let us define also the amputated, extended m-particle correlation functions $\bar{\varrho}_\Lambda^\omega(z,\beta|(t,x)_m)$ by the following formulae:

$$\bar{\varrho}_\Lambda^\omega(z,\beta|(t,x)_m) = z^{-m}\exp[\beta E((t,x)_m|(t,x)_m \vee \omega(\Lambda^C))\varrho_\Lambda^\omega(z,\beta|(t,x)_m)], \tag{2.13}$$

where the extended m-particle, conditioned correlation functions $\varrho_\Lambda^\omega(z,\beta|(t,x)_m)$ are defined by:

$$\varrho_\Lambda^\omega(z,\beta|(t,x)_m) = (Z_\Lambda^\omega(z,\beta))^{-1}z^m$$

$$\times \sum_{n=0}^\infty \frac{z^n}{n!}\int_\Lambda d\psi(y)_n \exp[-\beta E((t,x)_m \vee (y)_n|(t,x)_m \vee (y)_n \vee \omega(\Lambda^C))], \tag{2.14}$$

where

$$Z_\Lambda^\omega(z,\beta) = \sum_{n=0}^\infty \frac{z^n}{n!}\int_\Lambda d\psi(y)_n \exp[-\beta E((x)_n|(x)_n \vee \omega(\Lambda^C))]. \tag{2.15}$$

We will use also the notation

$$\bar{\varrho}_A^{\omega=\phi} = \bar{\varrho}_A(z,\beta) = \{\bar{\varrho}_A^{\omega=\phi}(z,\beta|(t,x)_m\}_{m=1,2,\ldots}, \tag{2.16}$$

$$\bar{\varrho}_{A=R^d}^{\omega=\phi}(z,\beta) = \bar{\varrho}_\infty(z,\beta). \tag{2.17}$$

From the assumption $\psi(R^d) < \infty$ it follows easily that $\bar{\varrho}_\infty$ is a well defined vector in the space $\mathscr{B}_1$. Similarly we define

$$Z_A^{\omega=\phi}(z,\beta) = Z_A(z,\beta); \quad Z_{A=R^d}^{\omega=\phi} = Z_\infty(z,\beta). \tag{2.18}$$

It is proved in the appendix to this paper that the amputated, conditioned, extended correlation functions $\varrho_A^\omega(z,\beta)$ fulfil the following equalities:

$$\bar{\varrho}_A^\omega(z,\beta) = G_A^\omega(z,\beta)\bar{\varrho}_A^\omega(z,\beta)+E_A^\omega(z,\beta). \tag{2.19}$$

By the same arguments:

$$\bar{\varrho}_\infty^\omega(z,\beta) = G_\infty(z,\beta)\bar{\varrho}_\infty(z,\beta)+E_\infty(z,\beta). \tag{2.20}$$

The comparison of the identities (2.19) with (2.20) is the core of our method. Note the simple estimate:

$$\exp[-z\psi(A)] \leqslant \operatorname*{ess\,sup}_{(t,x)_m} |\bar{\varrho}_A^\omega(z,\beta|(t,x)_m| \leqslant 1 \tag{2.21}$$

uniformly in $\omega \in \Omega^T(R_d)$ and A. Now we redefine the norm in the space $\mathscr{B}_1$. With the help of inequality (2.21) we can define a new norm:

$$\|f\|_{\bar{\varrho}_\infty} \equiv \sup_n \operatorname*{ess\,sup}_{(t,x)_n} \frac{|f_n(t,x)_n|}{\bar{\varrho}_\infty(z,\beta|(t,x)_n}. \tag{2.22}$$

From estimate (2.21) it follows that the new norm $\|-\|_{\bar{\varrho}_\infty}$ is equivalent to the old one $\|-\|_1$

$$\|f\|_1 \leqslant \|f\|_{\bar{\varrho}_\infty} \leqslant e^{z\psi(R^d)}\|f\|_1. \tag{2.23}$$

The following simple observation is essential to our proof.

Lemma 2.1

For any $z > 0$ the following estimate is valid:

$$\|G_\infty(z,\beta)\|_{\bar{\varrho}_\infty} < 1. \tag{2.24}$$

Proof:

From the definition of the norm $\|-\|_{\bar{\varrho}_\infty}$, identities (2.20) and the estimate (2.21) it follows:

$$\sup_n \operatorname*{ess\,sup}_{(t,x)_n} \frac{|(G_\infty(z,\beta)f)|(t,x)_n}{\bar{\varrho}_\infty(z,\beta|(t,x)_n)}$$

$$\leqslant \sup_n \operatorname*{ess\,sup}_{(t,x)_n} \frac{G_\infty(z,\beta)\bar{\varrho}_\infty(z,\beta)(t,x)_n}{\bar{\varrho}_\infty(z,\beta|(t,x)_n)} \|f\|_{\bar{\varrho}_\infty}$$

$$\leqslant \sup_n \operatorname{ess\,sup}_{(t,x)_n} \frac{\bar{\varrho}_\infty(z,\beta\,|\,(t,x)_n)-E_\infty(z,1,\beta\,|\,(t,x)_n)}{\bar{\varrho}_\infty(z,\beta|\,(t,x)_m} \|f\|_{\bar{\varrho}_\infty}$$

$$\leqslant (1-e^{z\psi(R^d)})\,\|f\|_{\bar{\varrho}_\infty}.$$

Lemma 2.1 shows that the operator $G_\infty(z, \beta)$ is contractive in the space $\mathscr{B}_{\bar{\varrho}_\infty}$ for any $z > 0$ and $\beta > 0$. Therefore the Neumann-Liouville expansion

$$(1-G_\infty(z,\beta))^{-1} = \sum_{n=0}^{\infty} G_\infty^n(z,\beta) \tag{2.25}$$

is strongly convergent there. Taking into account the equivalence of the norms $\|-\|_1$ and $\|-\|_{\bar{\varrho}_\infty}$ we conclude that expansion (2.25) is also convergent in the space $\mathscr{B}_1$ (for any $z > 0$!). Thus the equation (2.20) has a unique solution $\bar{\varrho}_\infty(z, \beta)$ in the space $\mathscr{B}_1$ for any $z > 0$, $\beta \geqslant 0$ and this unique solution is given by:

$$\bar{\varrho}_\infty(z,\beta) = \sum_{n=0}^{\infty} G_\infty^n(z,\beta)E_\infty(z,\beta). \tag{2.26}$$

Similar considerations apply to the system (2.19) as well.

In the next step we check that finite volume objects, like E_Λ^ω, G_Λ^ω tend strongly to the infinite volume quantities, respectively E_∞ and G_∞ for any $\omega \in \Omega^{\mathrm{T}}(R^d)$.

Lemma 2.2

Let $(\Lambda_n)_n$ be any monotone sequence of bounded subsets of R^d which tend to R^d by inclusion. Then for any $z \geqslant 0$, $\omega \in \Omega^{\mathrm{T}}(R^d)$ we have the equalities

1. $\lim_{n\to\infty} E_\Lambda(z,\alpha,\beta) = E_\infty(z,\alpha,\beta)$ (in the norm $\|-\|_1$)

2. $\lim_{n\to\infty} G_\Lambda(z,\beta) = G_\infty(z,\beta)$ (strongly in the space $\mathscr{B}_1$).

Proof:

By the simply derived chain of estimates:

$$|E_\Lambda(z,\alpha,\beta|\,(t,x)_m - E_\infty(z,\alpha,\beta|(t,x)_n)|$$

$$\leqslant z \int_{\Lambda^c} d\psi(y)\,[1-\exp(-\beta\alpha \sum_{i=1}^{b} t_i V(x_i-y))] \tag{2.27}$$

$$\leqslant 2z\psi(\Lambda^C)$$

from which the proof of 1 follows immediately. Let us define:

$$\tilde{G}_\Lambda(z,\beta) \equiv E_\Lambda(-z,\alpha,\beta)G_\Lambda(z,\beta).$$

From point 1, if follows that $E_\Lambda(-z, \alpha, \beta)$ tends strongly to $E_\infty(-z, \beta, 1)$ as a multiplication operator. Therefore it is sufficient to show the strong convergence $\tilde{G}_{\Lambda_n} \to \tilde{G}_\infty$. For this we note (with an obviously simplified notation)

$$|((\tilde{G}_\Lambda(z,\beta)-\tilde{G}_\infty(z,\beta))f)_m|\,(t,x)_m$$

$$\leqslant \int_0^1 d\alpha \int_0^1 d\tau \int_\Lambda d\psi(y) \int_\Lambda d\psi(y')\,|E_\Lambda - E_\infty| \ldots \|f\|_1$$

$$+ \int_0^1 d\alpha \int_0^1 d\tau \int_\Lambda d\psi(y) \int_{\Lambda^c} d\psi(y')\,|E_\Lambda - E_\infty| \ldots \|f\|_1$$

$$+ \int_0^1 d\alpha \int_0^1 d\tau \int_{\Lambda^c} d\psi(y) \int_\Lambda d\psi(y')\,|(E_\Lambda - E_\infty)|\,(\ldots)\,\|f\|_1$$

$$+ \int_0^1 d\alpha \int_0^1 d\tau \int_{\Lambda^c} d\psi(y) \int_{\Lambda^c} d\psi(y')\,|(E_\Lambda - E_\infty)|\,(\ldots)\,\|f\|_1$$

$$\leqslant \{8z\psi(\Lambda^c)\psi(R^d)^2+6z\psi(\Lambda^c)^2\psi(R^d)+8z\psi(\Lambda^c)^3\}\,\|f\|_1. \qquad (2.28)$$

q.e.d.

We proceed now to the case $\omega \neq \phi$.

Lemma 2.3

Assume that $V_2 = V$ has decrease at ∞ not slower then $|x|^{-d-\varepsilon}$ for some $\varepsilon > 0$. Then for any $z \geqslant 0$, $\omega \in \Omega^T(R^d)$

1. $\lim_{n\to\infty} E^\omega_{\Lambda_n}(z,\alpha,\beta) = E_\infty(z,\alpha,\beta)$ (in the norm $\|-\|_1$)

2. $\lim_{n\to\infty} G^\omega_{\Lambda_n}(z,\beta) = G_\infty(z,\beta)$ (strongly in the space $\mathscr{B}_1$).

Here $(\Lambda_n)_n$ is an arbitrary spherical-like sequence of bounded subsets tending to R^d monotonously and by inclusion.

Proof:

The following chain of inequalities is easy to derive:

$$|E^\omega_{\Lambda_n}(z,\alpha,\beta|\,(t,x)_n)-E_\infty(z,\alpha,\beta|\,(t,x)_n)|$$

$$\leqslant z\int_{\Lambda_n} |\exp\,(-\beta\alpha \sum_{i=1}^n t_i V(x_i-y))\exp\,(-\alpha\beta \sum_{i=1}^n t_i E(x_i|\omega(\Lambda^c))$$

$$-\exp\,(-\beta\alpha \sum_{i=1}^n t_i V(x_i-y)|d\psi(y)$$

$$+z\int_{\Lambda_n{}^c} d\psi(y)\,[1-\exp\,(-\beta \sum_{i=1}^n t_i V(x_i-y))]$$

$$\leqslant 2z\psi(\Lambda_n^c)+z\int_{\Lambda_n} |\exp\,(-\beta\alpha \sum_{i=1}^n t_i E(x_i|\omega(\Lambda^c))-1|d\psi(y). \qquad (2.29)$$

Let $(Y_p)_p$ be some sequence of spherical like subsets of R^d, such that $Y_p \subset \Lambda_p$, $\text{diam}(\Lambda_p)$--diam $(Y_p) \approx r_p^{d+\varepsilon'}$ for some $\varepsilon' > 0$, where we have denoted diam $(\Lambda_p) = R_p$ and diam $Y_p = r_p$. Using such a sequence $(Y_p)_p$ we have

$$z \int_{\Lambda_p} |\exp(-\beta\alpha \sum_{i=1}^{n} t_i E(x_i|\omega(\Lambda^c)) - 1| d\psi(y)$$

$$\leqslant z\beta \int_{Y_p} d\psi(y) \sum_{i=1}^{n} E(x_i|\omega(\Lambda^c)) + 2z\psi(\Lambda_p - Y_p) \tag{2.30}$$

$$\leqslant n \text{ const } |Y_p| \sum_{N=0}^{\infty} (N+R_p)^{d-1} \ln(N+R_p)(R_p+N-r_p)^{-d-\varepsilon} + 2z\psi(\Lambda_p - Y_p)$$

$$\leqslant n \text{ const } r_p^{-\varepsilon''} + 2z\psi(\Lambda_p - Y_p)$$

for some $\varepsilon'' > 0$.
From the last estimate it follows that.

$$\limsup_{\Lambda \uparrow \infty (t,x)_n} |E_\Lambda^\omega(z, \alpha, \beta|(t, x)_n) - E_\infty(z, \alpha, \beta|(t, x)_n)| = 0. \tag{2.31}$$

Moreover taking r_p to be n-dependent, like $r_p(n) \sim n^{-1-\varepsilon''} r_p$, where r_p fulfil conditions imposed above we obtain

$$\lim_{p\to\infty} \|E_{\Lambda_p}^\omega(z, \alpha, \beta) - E_\infty(z, \alpha, \beta)\|_1 = 0. \tag{2.32}$$

Similarly we prove 2. We summarize our previous discussion in the following theorem:
Theorem 2.1

Let us assume that the pair potential V is nonnegative, $V(0) > 0$ and V is continuous at zero, $V(x)$ has fast decrease at ∞ like $|x|^{-d-\varepsilon}$ for some $\varepsilon > 0$. Let $V_1 = -1/\beta \ln \frac{d\varphi}{dx}$ be an external field such that $\exp(-\beta V_1) \in L_1(R^d)$. Then for any $z > 0$, $\beta \geqslant 0$ there exists a unique tempered, grand canonical Gibbs measure $\mu_\infty(z, \beta, \psi|d\omega)$, the correlation functions of which are given by:

$$\bar{\varrho}_\infty(z, \beta) = \bar{\varrho}_\infty(z, \beta)_{|t=1}. \tag{2.33}$$

Proof:

From the results proved above it follows that for any $\omega \in \Omega^T(R^d)$ and any $z > 0$, $\beta \geqslant 0$ we have the convergence:

$$\begin{aligned} \lim_{n\to\infty} \bar{\varrho}_{\Lambda_n}^\omega(z, \beta) &= \lim_{n\to\infty} [1 - G_{\Lambda_n}^\omega(z, \beta)]^{-1} E_{\Lambda_n}(z, \beta, 1) \\ &= [1 - G_\infty(z, \beta)]^{-1} E_\infty(z, 1, \beta) \qquad (2.34) \\ &= \bar{\varrho}_\infty(z, \beta), \end{aligned}$$

where the limit is taken in the $||-||_1$ norm and the sequence $(\Lambda_n)_n$ fulfils the assumptions of Lemma (2.3). From the definition of the set $\Omega^T(R^d)$, the assumed decay properties of V and the definition of $\bar{\varrho}_\Lambda^\omega(z, \beta)$ it follows easily that for any compact $\Xi \subset R^d$ we have the convergence:

$$\lim_{\Lambda\uparrow\infty} \sup_{(t,x)_n} \chi_\Xi^{\otimes n} \cdot \bar{\varrho}_\Lambda^\omega(z, \beta|(t, x)_n) = \chi_\Xi^{\otimes n} \cdot \bar{\varrho}_\infty(z, \beta|(t, x)_n) \tag{2.35}$$

and this is sufficient to conclude that the corresponding conditioned finite volume Gibbs measures $\mu_\Lambda^\omega(z, \beta|d\omega)$ tend to the limiting measure $\mu_\infty(z, \beta)$ in the sense of weak convergence.

Remarks:

From the assumptions made on the two-body potential V_2 in the formulation of (2.1) it follows that the interaction is superstable and R-strongly regular. In this case it is possible to prove that the corresponding "infinite volume" Kirkwood-Salsburg has zero Fredholm radius and in this case the spectrum of the corresponding Kirkwood-Salsburg operator is pure point and coincidies as a set with the inverses of zeroes of the partition functions which in our case are located outside the real line on $\{z \in \mathbf{C} | \operatorname{Im} z > 0\}$. These results follows by extending arguments of Zagrebnov [14] to the case at hand.

It is possible to switch-off the external field V_1, i.e. it is possible to control the limit $V_1 \to 0$ (in the sense of $L^1(R^d)$) at least for small $z\beta^2$ for the models at hand. For this we have to improve the bound (2.12) to obtain one uniform in Λ. This topic and several interesting criteria for the uniquencess of the limiting Gibbs states (in the limit $V_1 = 0$!) are the subject of our forthcoming paper.

APPENDIX

Let us recall that the one dimensional differential problem:

$$\frac{df}{dx} = a(x)f(x)+g(x), \quad f(x_0) = f_0$$

has a solution given by the following formula:

$$f(x) = \{f_0 + \int_{x_0}^{x} ds\, g(s) e^{-\int_{x_0}^{s} dt\, a(t)}\} e^{\int_{x_0}^{x} dt\, a(t)} . \tag{A.1}$$

This is the basic formula which we will use repeatedly to derive identities (*). Using definition (4.13) we get:

$$\begin{aligned} &\frac{\partial}{\partial\tau} \varrho_\Lambda^\omega(z, \beta|(\tau t, x)_m) \\ &= \{-2\beta\tau \sum_{i<j} t_i t_j V_2(x_i - x_j) - \beta \sum_{i=1}^{n} t_i E(x_i|\omega(\Lambda^c)\} \varrho_\Lambda^\omega((\tau t, x)_m) \\ &\quad -\beta z \int dy \sum_{i=1}^{n} t_i V_2(x_i y) \varrho_\Lambda^\omega((\tau t, x)_m \vee (1, y)) \end{aligned} \tag{A.2}$$

with

$$\varrho_\Lambda^\omega(z, \beta|(\tau t, x)_m)_{|\tau=0} = 1.$$

Application of the basic formula (A.1) then yields

$$\begin{aligned}
&\varrho_\Lambda^\omega(z, \beta|(t, x)_m) \\
&= \exp\Big(-\beta \sum_{i<j}^{n} V_2(x_i-x_j)t_it_j\Big) \exp\Big(-\beta \sum_{i=1}^{n} t_iE(x_i|\omega(\Lambda^c)\Big) \\
&\quad -\beta z \int_0^1 d\alpha \int_\Lambda d\psi(y) \exp\Big(-\beta(1-\alpha^2) \sum_{i<j} V_2(x_i-x_j)t_it_j\Big) \\
&\quad \times \exp\Big(-\beta(1-\alpha) \sum_{i=1}^{n} t_iE(x_i|\omega(\Lambda^c)\Big) \\
&\quad \times \sum_{i=1}^{n} t_iV_2(x_i-y)\big)\varrho_\Lambda^\omega((\alpha t, x)_m \vee (1, y)).
\end{aligned} \tag{A.3}$$

Proceeding similarly:

$$\begin{aligned}
&\frac{\partial}{\partial t_i} \varrho_\Lambda^\omega(z, \beta|(t, x)_m) \\
&= \Big[-\beta \sum_{i\neq j} t_jV_2(x_i-x_j) - \beta E(x_i|\omega(\Lambda^c)\Big)\varrho_\Lambda^\omega(z, \beta|(t, x)_n) \\
&\quad -\beta z \int d(x)_{m+1} V(x_i-x_{m+1})\varrho_\Lambda^\omega(z, \beta|(t, x)_n \vee (1, x_{m+1}))
\end{aligned} \tag{A.4}$$

with the initial condition:

$$\varrho_\Lambda^\omega(z, \beta|\,(t, x)_n)_{|t_i=0} = \varrho_\Lambda^\omega(z, \beta|(t, x)_{n-1}^{\prime i}),$$

where

$$(t, x)_{m=1}^{\prime i} = (t_1, x_1, \dots, t_{i-1}, x_{i-1}, t_{i+1}, x_{i+1}, \dots, t_m, x_m).$$

Integration then gives:

$$\begin{aligned}
&\varrho_\Lambda^\omega(z, \beta|(t, x)_m) \\
&= \exp\Big(-\beta \sum_{l\neq i} t_lt_iV_2(x_i-x_l\Big) \exp\Big(-\beta t_iE(x_l|\omega(\Lambda^c)\Big)\varrho_\Lambda^\omega(z, \beta|\,(t, x)_{m-1}^{\prime l}) \\
&\quad -\beta z \int_0^{t_l} d\tau_l \exp\Big(-\beta \sum_{l\neq i} t_i(t_l-\tau_l)V(x_l-x_i)\Big) \times \exp\Big(-\beta(t_l-\tau_l)E(x_l|\omega(\Lambda^c)\Big) \\
&\quad \times \int d\psi(x)_{m+1} V(x_l-x_{m+1})\varrho_\Lambda^\omega(z, \beta|\,\dots \tau_l, x_l, \dots (1, x_{m+1})).
\end{aligned} \tag{A.5}$$

Now we substitute (A.5) into (A.4) obtaining:

$$\varrho_A^{\omega}(z, \beta|(\alpha t, x)_m \vee (1, y)) = \exp(-\sum_{i\neq 1}^{m} \alpha t_i V(x_i - y))$$

$$-\beta z \int_0^1 d\tau \int_A d\psi(y_1) \exp(-\beta \sum_{l=1}^{m} \alpha t_i(1-\tau)) \exp(-\beta(1-\tau)E(y|\omega(\Lambda^c))$$

$$\times \varrho_A^{\omega}(z, \beta|(\alpha t, x)_m \vee (\tau, y) \vee (1, y_1)). \tag{A.6}$$

Identitites (A.6) are then subsitutied into (A.3), which after some simple algebra yields the following relations between amputated, extended correlation functions:

$$\bar{\varrho}_A^{\omega}(z, \beta|(t, x)_m) \equiv E_A^{\omega}(z^1, \beta|(t, x)_m)$$

$$+\beta z^2 E_A(\ldots) \int_0^1 d\alpha \int_0^1 d\tau \int_A d\psi(y) \int_A d(\psi)(y_1) E_A(-z, \alpha, \beta|(t, x)_m)$$

$$\times (\sum_{l=1}^{m} V_2(x_l - y) \exp(-\beta\alpha \sum_{l=1}^{m} t_l V_2(x_l - y)))(V_2(y - y_1) \exp(-\beta\tau V_2(y - y_1)))$$

$$\times \exp(-\beta V_2(y|\omega(\Lambda^c))) \exp(-\beta V_2(y_1|\omega(\Lambda^c)))$$

$$\times \varrho_A^{\omega}(z, \beta|(\alpha t, x)_m, (\tau, y), (1, y_1)) \tag{A.7}$$

which are exactly component-wise written identities (2.19).

REFERENCES

[1] D. Ruelle, *Statistical Mechanics. Rigorous Results*, Benjamin, New York 1969.
[2] J. Bricmont, T. Kuroda, J. L. Lebowitz, *Z. Wahrscheinlichkeits*, Verv. Geb. **67**, 121 (1984).
[3] J. Bricmont, T. Kuroda, J. Lebowitz, *Commun. Math. Phys.* **101**, 501 (1985).
[4] R. Gielerak, *Theor. Math. Phys.* **69** (2), 289 (1986).
[5] R. Gielerak, *Theor. Math. Phys.* **73** (3), 323 (1987).
[6] R. Gielerak, *Theor. Math. Phys.* **76** (3), 418 (1988).
[7] R. Gielerak, *J. Math. Phys.* **27** (12), 2892 (1986).
[8] R. Gielerak, Commun. JINR E4 /87-12, Dubna (1987).
[9] R. Gielerak, *J. Mat. Phys.* **30** (1), 125 (1989).
[10] R. Gielerak, *J. Phys. A* **22**, 71 (1989).
[11] R. Gielerak, Spectral properties of the Kirkwood-Salsburg operator and the uniqueness of the Gibbs states, BiBoS preprint 247/87, Uniwersity of Bielefeld (1987).
[12] H. Moraal, *Physica* **105A**, 469 (1981) and references therein.
[13] A. Gonchar, *Phys. Lett.* **102A**, 285 (1984).
[14] V. A. Zagrebnov, *J. Stat. Phys.* **27**, 577 (1982).