STRUCTURE OF SPACETIME AT THE PLANCK SCALE*

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Mathematics underlying the structure of spacetime at experimentally available energy range should be based on the field of real numbers. At the Planck scale we are unable to examine experimentally the geometry of spacetime. If the concept of spacetime has any physical meaning in the Planckian region, its mathematical model might be based on the adelic number system. Physical motivation is given for such proposal.

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Presently, many theoretical physicists believe that all known interactions in physics, including gravity, can be unified at the Planck scale. The best candidate for such a theory is probably the theory of strings [1]. This theory is far from being completed. One of the main problems is that we still do not fully understand what the underlying principles and symmetries of this theory are. Further development may depend here on better understanding what the structure of spacetime at the Planck scale is.

One usually assumes that the spacetime has a structure of a *real* manifold. Why is the mathematics underlying the structure of spacetime based on the field of real numbers, R? All the results of the measurements, including the measurements of space and time intervals, are given by rational numbers. However, the field of rational numbers, Q, is not sufficient for theoretical analysis: some Cauchy sequences do not converge in Q and some polynomial equations with coefficient in Q do not have solutions in Q. Theoretical physics "likes" mathematics based on the number field which is complete and algebraically closed[1]. There are only *two* non-equivalent ways [2] we can complete Q: with respect to the usual absolute value norm which leads to R and with respect to the so called p-adic

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[1] For simplicity, we will only consider the complete number fields.

norm [2] which gives the p-adic field, Q_p. Here $p = 2, 3, 5, 7, 11,\ldots$ denotes the prime numbers. (Note, that we have as many Q_p as many prime numbers). Why do we prefere R to Q_p? Is it only convention? No, the best choice is R. The field R is a unique model for the Archimedean geometry. One can say that the Archimedean geometry and the field R are two equivalent ways of expressing the same thing. The crucial point is that spacetime obeys all axioms of the Archimedean geometry in the available energy range. By the Archimedean geometry we mean here the geometry which obeys four groups of axioms of basic geometry [3]: axioms of incidence, of order, of congruence, and the axioms of continuity.

The Archimedean geometry was constructed as a result of dealing with *macroscopic* distances but its formulations does not depend on this fact; the macroscopic and microscopic intervals are treated equally. When we extend this model of spacetime to microphysics, we tacitly assume that this geometry is still applicable. This assumption works well at experimentally accessible energies but seems to be doubtful when we go to higher energies. Here, the parallel existence of quantum mechanics and general relativity impose restrictions on the notions of spacial distance and time intervals. Suppose we want to measure a position and momentum of a particle with a *very high* precision. It results from Heisenberg's uncertainty principle that we have

$$\Delta x \geqslant \hbar/(2\Delta p) \simeq \hbar c/(2\Delta E) = \hbar/(2c\Delta m), \tag{1}$$

where $\hbar = h/2\pi$, h is Planck's constant, and c is the speed of light; Δx, Δp, and ΔE are the uncertainties in the particle position, momentum, and energy, respectively. During this process of measurement "we introduce" the mass Δm in the interval Δx. On the other hand, Δm is a source of a gravitational field. According to general relativity, the metric near Δm can be Schwarzschildian; distance and time retain their meanings for

$$\Delta x/2 > 2G\Delta m/c^2 = r, \tag{2}$$

where G is Newton's constant and r is the Schwarzschild radius for Δm. Solving Eqs. (1) and (2) gives that the smallest distance having an ordinary meaning is $\Delta x \simeq (\hbar G/c^3)^{1/2} = l_P$, called Planck's distance. Mass corresponding to such Δx is $\Delta m \simeq (\hbar c/G)^{1/2} = m_P$, and is called Planck's mass. One can also carry on this reasoning for time intervals, Δt, with the result that Δt has an ordinary meaning only for $\Delta t \gtrsim t_P = l_P/c$; t_P is called Planck's time. When we recall what the properties of the event horizon of a black hole are, we come to a very pessimistic conclusion: *It is impossible to get information on the structure of spacetime in the Planckian region experimentally.*

One can replace our heuristic reasoning by a more precise one; the conclussion would be the same [4].

Our considerations were only based on *classical* gravity and the uncertainty principle. Therefore, one can doubt whether our conclusion is true. Correct considerations should be based on *quantum* gravity. The problem is that presently we are not sure that we have the final version of quantum gravity at our disposal. However, let us make use of available results related to our problem. If we assume that it makes sense to quantize pure classical gravity, we can use results of Ref. [5] concerning the spherically symmetric gravitational collapse. This paper shows that instead of event horizon, we have the so called apparent horizon which causes a gigantic but finite delay of outgoing light signals. The apparent horizon turns into the event horizon when $m_P/m \to 0$, where m is the mass of collapsing matter. Applying this result to our problem, we can speculate that very ambitious measurements of space or time intervals (which introduce a very large Δm) last unreasonably long time. The more ambitious we are the longer our measurement lasts and in the limit $m_P/\Delta m \to 0$ it is simply impossible. Now, let us apply the results of the string theory. Here, we can use results concerning the high energy behaviour of string scattering amplitudes [6, 7]. Ref. [6] shows that at the Planck scale the minimal observable length is of the order of the string size. In Ref. [7] one explores the distances using strings as local probes; the size of strings at the time of collision is identified with the size of explored distances. At energies low when compared to m_P, we gain in spacial resolution as we increase the energy, but for energies higher than m_P strings themselves expand with increasing energy. The minimal length that can be probed this way is of order of l_P.

Ref. [5] suggests that the scale at which spacetime cannot be penetrated experimentally is not identical to the Planck scale. However, the general feeling is that gravitational interaction unifies with all the other interactions at the scale where quantum effects come into play. Therefore, we find the string theory results to be more significant and we will stick to the Planck scale.

The analysis based on quantum gravity seem to conform the conclusion of low energy theories.

Recently, Volovich [8] put forward the hypothesis that in the Planck region the Archimedean geometry is not correct as a mathematical model for physical spacetime. The reason is that at the Planck scale the so called Archimedean axiom[2] is not compatible with the physical process of measurement. If this is true, the use of the field R is also incorrect. He claims that R should be replaced by the field Q_p. We are going to show that confrontation of nonpenetrability of spacetime with the axioms of Archimedean geometry does not lead necessarily to such a conclusion [9]. Let us consider the Archimedean axiom. It says that any given segment on a straight line can be surpassed by the successive addition of a smaller one along the same line [3]. If we insist that an order of points on a straight line should be discernible by physical methods, then the order of points in the Planckian region and therefore the Archimedean axiom are meaningless. In fact, give two Planckian segments a and b we have no way to determine which one is shorter. Thus, the relation of being shorter does not hold neither for a and b nor for b and a. Therefore, the relation of being shorter is no longer an order relation and the use of the term "Archimedean axiom" in connection with such relation is the abuse of the language. Thus, in such a context the Archimedean axiom is meaningless. However, if we admit that the order and the addition of segments are only abstract mathematical notions which need not have direct physical counterparts, then the Planck scale is not an argument against the validity of the Archimedean axiom. If we, however, insist on the correspondence between mathe-

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[2] This axiom belongs to the continuity axioms in the Hilbert axiom system.

matical notions and actual physical operations, we run into difficulties with much more basic axioms in Hilbert's axiom system. For example, the first axiom of congruence which says that it is always possible to construct a segment equal to a given one [3], is equally meaningless in the Planckian region. In fact, we can go even further and say that we do not know if *any axiom* of the Archimedean geometry can be obeyed at the Planck scale. The reason is that we are unable to perform experimentally any test due to physical nonpenetrability of spacetime.

The above arguments do not prove that the Archimedean geometry (the field R) does not provide the correct mathematical model for spacetime at the Planck scale. We are simply in the position of being unable to carry on any proof. Which mathematics should we take? First of all, we can be conservative and still use R. Next candidate is the whole family of Q_p' s (we have no reason to make a given prime preferable). However, the most general case is when we take all the available completions of Q, namely R and all Q_p' s. Here, mathematicians offer us the so called adelic structure [10] which combines R and Q_p' s. It has been shown by Manin [11] in the context of string theory that mathematical expressions based on adelic numbers have a strong tendency to be *simpler* than corresponding real formulae. The hope, however, is that formulation of the theory of the Planck scale physics in terms of more general number set than R can make *visible* some new symmetries of this theory.

We have found that the choice of mathematics underlying the structure of spacetime is not unique. This might signal the possibility that the very concept of spacetime has no physical meaning at the Planck scale. Such interpretation supports a growing feeling that spacetime should be a derived quantity in the final formulation of string theory; spacetime would emerge only in its low energy approximation [12].

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