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SCATTERING OF PROTONS, ANTIPROTONS AND PIONS ON ^{12}C AND ^{16}O NUCLEI

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The elastic scattering differential cross sections of intermediate energy protons, antiprotons and charged pions on ^{12}C and ^{16}O nuclei and the elastic scattering polarization observables of protons on these nuclei have been calculated on the basis of the α-particle model with dispersion and multipole diffraction scattering theory. The results of these calculations contain no fitting parameters and are in agreement with the experimental data.

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1. Introduction

Different properties of light nuclei can be described by the cluster model [1]. In many cases an α-particle that possesses high stability and symmetry can be chosen as a cluster. Various approaches are used to explain the properties of light nuclei on the basis of the cluster model [2]. But even the simplest α-particle model describes reasonably satisfactorily different nuclear characteristics. In order to study the interaction of the intermediate energy particles with ^{12}C and ^{16}O nuclei the α-particle model can be applied. We assume that these nuclei consist of three and four α-particles arranged at the vertices of an equilateral triangle (^{12}C) and tetrahedron (^{16}O).

A correct α-particle model should take into account the possibility of exchange of nucleons between α-clusters and include antisymmetrization of nuclear wave functions over all filled nucleon states [1]. But at sufficiently high energies of the incident particles the time of interaction of the projectiles with nuclei is small as compared with that of the intranuclear motion. Therefore, the influence of the nucleon exchange between α-clusters and antisymmetrization of nuclear wave functions on the value of the cross section is small. In other words, the incident particle does not have enough time "to feel" the nucleon exchange between α-clusters during the interaction of the projectiles with nuclei.

The correlation effects between the nucleons due to the exclusion principle are not

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significant because the energy of the incident particle is much higher than the Fermi energy. However, even at high energies of the projectiles one cannot neglect the possibility of displacement of α-particles from their most probable positions of equilibrium. These displacements lead to the increase of the effective sizes of α-particles in nuclei.

In the framework of the α-particle model, our work is concerned with intermediate energy protons, antiprotons and pions interacting with ^{12}C and ^{16}O nuclei. In order to study the scattering of these particles on the α-cluster nucleus the multiple scattering theory [3, 4] constructed by using the scattering amplitudes of a projectile on α-particles can be used. The scattering amplitude of an incident particle can be chosen with good accuracy as a scattering amplitude of a projectile on a free α-particle, because the energy of the scattered particle is much higher than the characteristic energy of the motion of an α-particle in the nucleus.

In first approximation ^{12}C and ^{16}O nuclei can be considered as composed of α-particles which are fixed relative to each other at vertices of an equilateral triangle (^{12}C nucleus) and tetrahedron (^{16}O nucleus) [5]. A more realistic model is the α-particle model with dispersion [6]. In this model α-particles can be displaced from their most probable positions of equilibrum.

In Section 2 we introduce the model densities of the α-particle distribution in ^{12}C and ^{16}O nuclei with the dispersion parameter Δ which characterises the probability of the α-particle displacement from their most probable positions at the vertices of the equilateral triangle or tetrahedron. The density parameters are derived from a comparison of the calculated and measured charge form factors of ^{12}C and ^{16}O nuclei. The introduction of the dispersion parameter Δ does not change the qualitative behaviour of the form factors and leads to the same effect as the increase of effective sizes of α-particles.

The p-α amplitude used in this model is obtained from the experimental data on the p-^{4}He elastic scattering. That is why in Section 3 we use a simple parametrization of the p-α amplitude approximating these data in the range of energies considered. The parameters of the p-α amplitude for different energies are derived from a comparison of the calculated and measured cross sections and polarization observables for protons elastically scattered on free ^{4}He nuclei.

In Section 4 the differential cross sections and polarization observables for elastic scattering of protons on ^{12}C and ^{16}O nuclei are calculated on the basis of the α-particle model with dispersion and the multiple scattering theory. The differential cross sections for elastic scattering of antiprotons on ^{12}C and ^{16}O nuclei are calculated in the same way. This approach allows us to describe a complete set of observables necessary for the investigation of the elastic scattering of particles on atomic nuclei without any additional fitting parameters.

In Section 5 the α-particle model with dispersion was used to analyze the elastic scattering of pions on ^{12}C and ^{16}O nuclei. The parameters of the π-α amplitude were taken from Ref. [7].

The calculations show that the α-particle model with dispersion correctly describes the set of experimental data on the elastic scattering of different particles on ^{12}C and ^{16}O nuclei.

2. Nuclear density and form factors

If the positions of α-particles are fixed at the vertices of an equilateral triangle (^{12}C nucleus) and tetrahedron (^{16}O nucleus) the α-particle densities are determined in the CMS of ^{12}C and ^{16}O nuclei by

$$\varrho_0^{(C)}(\xi, \eta) = \frac{1}{4\sqrt{3}\,\pi^2 d^2}\,\delta(\xi-d)\delta\left(\eta-\frac{\sqrt{3}}{2}d\right)\delta(\xi\eta), \tag{1}$$

$$\varrho_0^{(O)}(\xi, \eta, \zeta) = \frac{1}{(4\pi)^2}\,\delta(\xi-d)\delta\left(\eta-\frac{\sqrt{3}}{2}d\right)\delta(\zeta-\sqrt{\tfrac{2}{3}}\,d)\delta(\xi\eta)\delta(\xi\zeta)\delta(\eta\zeta), \tag{2}$$

where d is the distance between two α-particles.

The Jacobi coordinates ξ, η, ζ are related to the α-particle coordinates r_1, r_2, r_3, r_4 through

$$\xi = \tau_2-\tau_1, \quad \eta = \tau_3-\tfrac{1}{2}(\tau_1+\tau_2), \quad \zeta = \tau_4-\tfrac{1}{3}(\tau_1+\tau_2+\tau_3). \tag{3}$$

The densities $\varrho_0^{(C)}(\xi, \eta)$ and $\varrho_0^{(O)}(\xi, \eta, \zeta)$ are normalized to unity.

Now we introduce the distribution of α-particles which allows their displacement from the vertices of the equilateral triangle (^{12}C) and tetrahedron (^{16}O). In the α-particle model with dispersion these densities are determined by

$$\varrho_\Delta^{(C)}(\xi, \eta) = \int \varrho_0^{(C)}(\xi', \eta')\Phi_\Delta^{(C)}(\xi-\xi', \eta-\eta')d^3\xi' d^3\eta', \tag{4}$$

$$\varrho_\Delta^{(O)}(\xi, \eta, \zeta) = \int \varrho_0^{(O)}(\xi', \eta', \zeta')\Phi_\Delta^{(O)}(\xi-\xi', \eta-\eta', \zeta-\zeta')d^3\xi' d^3\eta' d^3\zeta'. \tag{5}$$

The smearing functions $\Phi_\Delta^{(C)}(\xi, \eta)$ and $\Phi_\Delta^{(O)}(\xi, \eta, \zeta)$ do not violate the invariance of the densities relative to the rearrangement of any pair of α-particles. These functions are transformed into $\delta(\xi)\,\delta(\eta)$ and $\delta(\xi)\delta(\eta)\delta(\zeta)$, respectively, if $\Delta = 0$. The functions $\Phi_\Delta^{(C)}(\xi, \eta)$ and $\Phi_\Delta^{(O)}(\xi, \eta, \zeta)$ are normalized to unity. We choose the smearing functions in the Gaussian forms

$$\Phi_\Delta^{(C)}(\xi, \eta) = \frac{1}{(\sqrt{3}\,\pi\Delta^2)^3}\exp\left(-\frac{\xi^2+\frac{4}{3}\eta^2}{2\Delta^2}\right), \tag{6}$$

$$\Phi_\Delta^{(O)}(\xi, \eta, \zeta) = \frac{1}{8(\pi\Delta)^9}\exp\left(-\frac{\xi^2+\frac{4}{3}\eta^2+\frac{3}{2}\zeta^2}{2\Delta^2}\right). \tag{7}$$

We can readily see that the densities $\varrho_\Delta^{(C)}(\xi, \eta)$ and $\varrho_\Delta^{(O)}(\xi, \eta, \zeta)$ are symmetrical relative to the rearrangement of any pair of α-particles. This means an invariance relative to transformations

$$\xi \to -\xi, \tag{8}$$

$$\xi \to \tfrac{1}{2}\xi-\eta, \quad \eta \to -\tfrac{3}{4}\xi-\tfrac{1}{2}\eta, \tag{9}$$

$$\xi \to \tfrac{1}{2}\xi-\tfrac{1}{3}\eta-\zeta, \quad \eta \to -\tfrac{1}{4}\xi+\tfrac{5}{6}\eta-\tfrac{1}{2}\zeta, \quad \zeta \to -\tfrac{2}{3}\xi-\tfrac{4}{9}\eta-\tfrac{1}{3}\zeta. \tag{10}$$

The parameters of the densities $\varrho_\Delta^{(C)}(\xi, \eta)$ and $\varrho_\Delta^{(O)}(\xi, \eta, \zeta)$ can be determined from comparison of the calculated and measured charge form factors of ^{12}C and ^{16}O nuclei. The form factors of ^{12}C and ^{16}O nuclei as systems of three and four α-particles can be written in the form

$$F^{(C)}(q) = F_\alpha(q) \int \exp\left(\tfrac{2}{3}\, iq\eta\right) p_\Delta^{(C)}(\xi, \eta) d^3\xi d^3\eta, \tag{11}$$

$$F^{(O)}(q) = F_\alpha(q) \int \exp\left(\tfrac{3}{4}\, iq\zeta\right) \varrho_\Delta^{(O)}(\xi, \eta, \zeta) d^3\xi d^3\eta d^3\zeta, \tag{12}$$

where q is a transferred momentum, $F_\alpha(q)$ is the α-particle form factor which can be taken in the Gaussian form [8]

$$F_\alpha(q) = \exp\left(-\tfrac{1}{6}\, q^2 \langle r^2 \rangle_\alpha\right). \tag{13}$$

In this formula RMS radius of an α-particle is $\langle r^2 \rangle_\alpha^{1/2} = 1.61$ fm.

Using equations (4)–(7) we obtain

$$F^{(C)}(q) = F_\alpha(q) S^{(C)}(q) g_\Delta^{(C)}(q), \tag{14}$$

$$S^{(C)}(q) = \int \exp\left(\tfrac{2}{3}\, iq\eta\right) \varrho_0^{(C)}(\xi, \eta) d^3\xi d^3\eta = j_0\left(\frac{qd}{\sqrt{3}}\right), \tag{15}$$

$$g_\Delta^{(C)}(q) = \int \exp\left(\tfrac{2}{3}\, iq\eta\right) \Phi_\Delta^{(C)}(\xi, \eta) d^3\xi d^3\eta = \exp\left(-\tfrac{1}{6}\, q^2 \Delta^2\right), \tag{16}$$

$$F^{(O)}(q) = F_\alpha(q) S^{(O)}(q) g_\Delta^{(O)}(q), \tag{17}$$

$$S^{(O)}(q) = \int \exp\left(\tfrac{3}{4}\, iq\zeta\right) \varrho_0^{(O)}(\xi, \eta, \zeta) d^3\xi d^3\eta d^3\zeta = j_0\left(\sqrt{\tfrac{3}{8}}\, qd\right); \tag{18}$$

$$g_\Delta^{(O)}(q) = \int \exp\left(\tfrac{3}{4}\, iq\zeta\right) \Phi_\Delta^{(O)}(\xi, \eta, \zeta) d^3\xi d^3\eta d^3\zeta = \exp\left(-\tfrac{3}{16}\, q^2 \Delta^2\right). \tag{19}$$

Fig. 1a shows the calculated ^{12}C form factor [6] with the experimental data taken from Ref. [9]. The form factor of the ^{16}O nucleus together with the experimental data taken from Ref. [10] is shown in the Fig. 1b. From the comparison of calculated by (14)–(19) and measured charge form factors we obtain the following values of parameters: $d = 2.98$ fm, $\Delta = 0.346$ fm for ^{12}C nucleus and $d = 3.16$ fm, $\Delta = 0.643$ fm for ^{16}O nucleus. Fig. 1 shows that the calculated and measured form factors are in agreement up to the values of transferred momenta $q \sim 3$ fm^{-1}.

As it is seen from equations (14)–(19), the introduction of the smearing functions $\Phi_\Delta^{(C)}(\xi, \eta)$ and $\Phi_\Delta^{(O)}(\xi, \eta, \zeta)$ into the densities (4) and (5) results in the occurence of the additional factors $g_\Delta^{(C)}(q)$ and $g_\Delta^{(O)}(q)$ which does not change the qualitative behaviour of $F^{(C)}(q)$ and $F^{(O)}(q)$ and leads to a more rapid decrease of the form factor values with increasing transferred momentum.

Notice that in Ref. [11] the ^{16}O nucleus was considered on the basis of the α-particle model. It was assumed that the α-particles arranged at the vertices of the tetrahedron also have a form of the tetrahedron with the nucleons at the vertices. On the basis of this model the elastic and inelastic form factors of the oxygen nucleus were calculated. The results of these calculations are in agreement with the experimental data.

The Fourier transformations of the functions $F^{(C)}(q)$ and $F^{(O)}(q)$ yield the expressions for the charge distributions for the ^{12}C and ^{16}O nuclei:

$$\varrho(r) = \left(\frac{3}{8\pi^3}\right)^{1/2} \frac{1}{abr} \exp\left(-\frac{3}{2a^2}(r^2+b^2)\right) \mathrm{sh}\left(\frac{3rb}{a^2}\right), \tag{20}$$

where $a^2 = \langle r^2\rangle_\alpha + \Delta^2$, $b^2 = d^2/3$ for ^{12}C nucleus and $a^2 = \langle r^2\rangle_\alpha + 9\Delta^2/8$, $b^2 = 3d^2/8$ for ^{16}O nucleus. Then the RMS radii of these nuclei are determined by

$$\langle r^2\rangle_C = \langle r^2\rangle_\alpha + \tfrac{1}{3}d^2 + \Delta^2, \quad \langle r^2\rangle_O = \langle r^2\rangle_\alpha + \tfrac{3}{8}d^2 + \tfrac{9}{8}\Delta^2. \tag{21}$$

As can be seen from expressions (20) and (21), the presence of the dispersion parameter Δ is equivalent to the increase of the effective sizes of α-particles and leads to the increase of the RMS radii of the ^{12}C and ^{16}O nuclei. The obtained values of the parameters d and

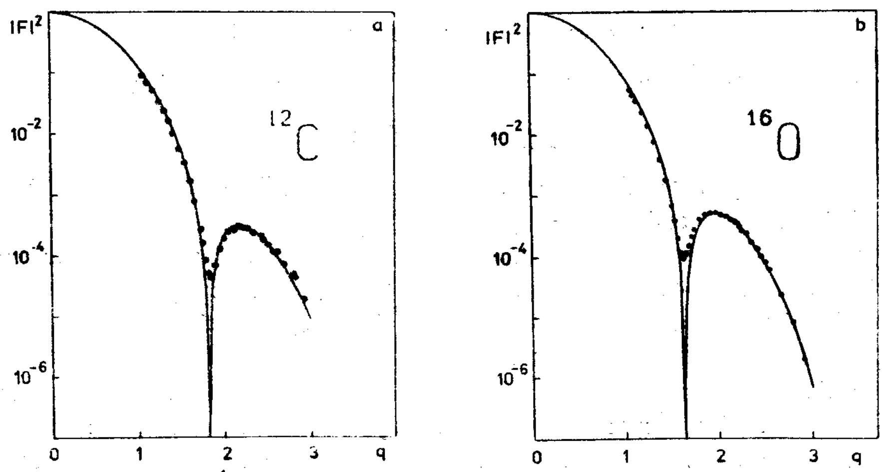

Fig. 1. The ^{12}C (a) and ^{16}O (b) charge form factors as a functions of the momentum transferred q(fm^{-1})

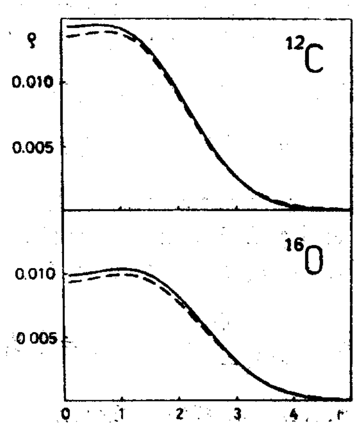

Fig. 2. The ^{12}C and ^{16}O charge distributions ϱ(fm^{-3}) as a functions of the radius r(fm^{-1})

Δ yield the RMS radii of the ^{12}C and ^{16}O nuclei: $\langle r^2 \rangle_C^{1/2} = 2.38$ fm and $\langle r^2 \rangle_O^{1/2} = 2.61$ fm, which are in agreement with those experimentally measured: $\langle r^2 \rangle_C^{1/2} = 2.38$ fm [8], $\langle r^2 \rangle_O^{1/2} = 2.65$ fm [9] and $\langle r^2 \rangle_O^{1/2} = 2.73$ fm [10].

The charge distributions for the ^{12}C and ^{16}O nuclei calculated with (20) together with the densities obtained from model-independent analysis [12, 13] of the experimental data on electron elastic scattering are shown in Fig. 2. From Fig. 2 one can see that the densities obtained from our model are in agreement with the densities obtained by the model-independent analysis. Some discrepancies in these densities in the region of small r can be associated with the differences of RMS radii of ^{12}C and ^{16}O nuclei used in our model and in model-independent analysis: $\langle r^2 \rangle_C^{1/2} = 2.46$ fm [12], $\langle r^2 \rangle_O^{1/2} = 2.74$ fm [13].

We have obtained the parameters of the densities d and Δ which will be used to describe the efiperimental data on the elastic scattering of protons, antiprotons and pions on ^{12}C and ^{16}O nuclei.

3. Nucleon — ^{4}He scattering amplitude

The elastic p-^{4}He scattering amplitude may be written in the form

$$f_{p\alpha}(q) = f_c(q) + f_s(q)\sigma n \tag{22}$$

where σ is a spin operator of an incident proton, $n = [k, k']/|[k, k']|$, k, k' are the wave-vectors of the incident and scattered protons.

At present there is a set of experimental data of differential cross sections and polarization observables for p-^{4}He elastic scattering in the region of energies under study. We can determine the amplitudes $f_c(q)$ and $f_s(q)$ from the analysis of these data by employing some approximations for the amplitudes.

Notice that in Ref. [6, 14] $f_c(q)$ and $f_s(q)$ amplitudes are approximated by the Gaussian function of the transferred momentum. But this approximation of the p-α amplitude fits the experimental data on p-^{4}He elastic scattering only in the region of small transferred momenta.

In order to more accurately determine the p-α amplitude we approximate the central $f_c(q)$ and the spin-orbit $f_s(q)$ parts of the amplitude by the sum of two Gaussians:

$$f_c(q) = k(G_1 e^{-\beta_1 q^2} + G_2 e^{-\beta_2 q^2}), \tag{23}$$

$$f_s(q) = kq(G_3 e^{-\beta_3 q^2} + G_4 e^{-\beta_4 q^2}). \tag{24}$$

In equations (23), (24) the complex parameters G_1, G_3, β_1, β_3 are determined by fitting the differential cross section and polarization observables of protons scattered by ^{4}He nuclei. The parameters G_2, G_4, β_2, β_4 are related to G_1, G_3, β_1, β_3 by the expressions

$$G_2 = \frac{3iG_1^2}{32\beta_1}, \quad G_4 = \frac{3iG_1G_3\beta_1}{8(\beta_1+\beta_3)^2}, \quad \beta_2 = \tfrac{1}{2}\beta_1, \quad \beta_4 = \frac{\beta_1\beta_3}{\beta_1+\beta_3}. \tag{25}$$

The relation between the parameters (25) is chosen so that the terms with G_2 and G_4 in (23), (24) correspond to the leading terms of the double p-α scattering amplitude obtained in the multiple diffraction scattering model with the Gaussian one-particle density of the ^{4}He nucleus.

The results of the calculations of the differential cross section $d\sigma/dt$ and polarization P of the 800 MeV p-α elastic scattering are shown in Fig. 3a together with the experimental data [15]. The solid and dashed curves in Fig. 3a correspond to the choice of the p-α amplitude in the form of one and two Gaussian functions, respectively.

Fig. 3b shows the differential cross section, the polarization and the Wolfenstein parameter R calculated and measured experimentally [16] for the p-^{4}He elastic scattering at 500 MeV.

The parameters of the p-α amplitude used in the calculations are shown in Table I. Notice that the parameters of the p-α amplitude at 650 MeV (see Table I) are obtained by interpolating these values at 500 and 800 MeV. Then these values of the parameters are corrected by fitting the experimental data [17] for the differential cross section of the p-^{4}He elastic scattering at 650 MeV. This choise of the parameters of the p-α amplitude is explained

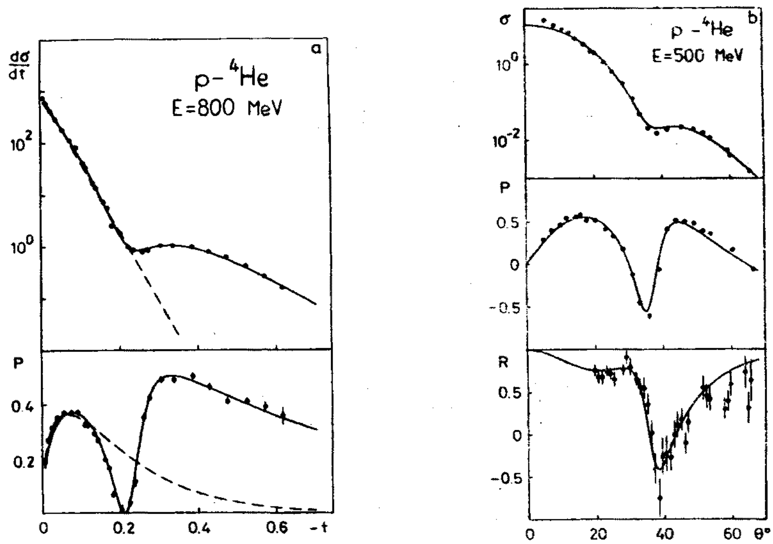

Fig. 3a. Differential elastic scattering cross section $d\sigma/dt$ (mb/(GeV/c^2)) and polarization P for 800 MeV protons on ^{4}He as a functions of the square 4-momentum transferred $-t$((GeV/c)^2)

Fig. 3b. Differential elastic scattering cross section σ (mb/sr), polarization P and Wolfenstein parameter R for 500 MeV protons on ^{4}He

TABLE I

Particle	E, MeV	β_1, fm^2	β_3, fm^2	G$_1$, fm^2	G_3, fm^3
p	500	$0.393-i\ 0.079$	$0.464+i\ 0.076$	$-0.217+i\,1.042$	$0.194+i\ 0.413$
p	650	$0.439-i\ 0.094$	$0.503+i\ 0.014$	$-0.269+i\,1.144$	$0.174+i\ 0.347$
p	800	$0.424-i\ 0.025$	$0.490+i\ 0.052$	$-0.330+i\,1.258$	$0.177+i\ 0.295$
$\bar{p}$	179	$0.852-i\ 1.041$	$0.081+i\ 0.295$	$-1.184+i\,2.789$	$0.099+i\ 0.110$

by the lack of the experimentally measured polarization observables for the p-^{4}He elastic scattering at 650 MeV which makes the exact determination of the spin-orbit part of the amplitude impossible.

The p-α scattering amplitude we have chosen will be used to calculate the differential cross sections and the polarization observables for the elastic scattering of protons and antiprotons on ^{12}C and ^{16}O nuclei.

4. *Scattering of protons and antiprotons on the ^{12}C and ^{16}O nuclei*

Consider the interaction of intermediate energy protons and antiprotons on the ^{12}C and ^{16}O nuclei on the basis of the approach presented above. We assume that the interaction of the incident protons with α-particles constituting ^{12}C and ^{16}O nuclei is the same as that with free α-particles. Therefore, we can use the multiple diffraction scattering theory [3, 4] with p-α amplitude in the form (22)–(25). The assumption made is based on the smallness of time of flight of sufficiently high energy projectiles through the nucleus as compared to the characteristic time of the α-particle motion in these nuclei. That is why the internal structure of these nuclei does not actually change during the time of their interaction with the scattered particle, i.e. we can use the multiple diffraction scattering theory in order to describe the scattering process of a particle on the α-cluster nucleus.

According to this model the scattering amplitudes of protons (antiprotons) on ^{12}C and ^{16}O nuclei have the forms

$$F^{(C)}(q) = \frac{ik}{2\pi}\int d^2b d^3\xi d^3\eta \varrho_A^{(C)}(\xi, \eta) e^{iqb}$$

$$\times\left[1-\prod_{i=1}^{3}\left(1-\frac{1}{2\pi ik}\right) e^{-iq'(b-r_i)} f_{p\alpha}(q')d^2q'\right)\Bigg], \tag{26}$$

$$F^{(O)}(q) = \frac{ik}{2\pi}\int d^2b d^3\xi d^3\eta d^3\zeta \varrho_A^{(O)}(\xi, \eta, \zeta) e^{iqb}$$

$$\times\left[1-\prod_{i=1}^{4}\left(1-\frac{1}{2\pi ik}\int e^{-iq'(b-r_i)} f_{p\alpha}(q')d^2q'\right)\right]. \tag{27}$$

Here the impact parameter b lies in the plane perpendicular to the incident beam.

Taking into account the spin-orbit interaction, the amplitudes $F^{(C)}(q)$ and $F^{(O)}(q)$ can be presented in the form

$$F(q) = A(q)+B(q)\sigma n. \tag{28}$$

The expressions for the amplitudes of elastic scattering of protons (antiprotons) on ^{12}C and ^{16}O nuclei are presented in the Appendix.

To calculate these amplitudes we used the effective deformation approximation [18]. This approximation is equivalent to substituting the mean values for $\xi_\perp^2$, $\eta_\perp^2$, $\zeta_\perp^2$, $\xi_\perp \eta_\perp$, $\xi_\perp \zeta_\perp$, $\eta_\perp \zeta_\perp$ in the integrals in equations (26), (27):

$$\xi_\perp^2 \to \langle \xi_\perp^2 \rangle = \tfrac{2}{3} d^2, \quad \eta_\perp^2 \to \langle \eta_\perp^2 \rangle = \tfrac{1}{2} d^2, \quad \zeta_\perp^2 \to \langle \zeta_\perp^2 \rangle = \tfrac{4}{9} d^2,$$

$$\xi_\perp \eta_\perp, \quad \xi_\perp \zeta_\perp, \quad \eta_\perp \zeta_\perp \to \langle \xi_\perp \eta_\perp \rangle = \langle \xi_\perp \zeta_\perp \rangle = \langle \eta_\perp \zeta_\perp \rangle = 0, \tag{29}$$

where $\xi_\perp$, $\eta_\perp$, $\zeta_\perp$ are the projections of the vectors ξ, η, ζ onto the plane perpendicular to the incident beam. The results obtained in this approach do not differ significantly from those calculated by the exact formula and allow us to obtain the amplitude $F^{(C)}(q)$ and $F^{(O)}(q)$ in the obvious form.

The scattering amplitudes calculated in this approach contain no fitting parameters. All the values of the parameters included in the amplitudes $F(q)$ are determined by fitting the charge form factors of ^{12}C and ^{16}O nuclei, as well as by fitting of the differential cross sections and polarization observables of proton — ^4He elastic scattering.

The differential cross section $\sigma(\theta)$, polarization (asymmetry) $P(\theta)$, spin-rotation functions $Q(\theta)$ and $S(\theta)$, and the spin-rotation angle $\beta(\theta)$ for the elastic scattering of incident particles on nuclei are determined by the formulae

$$\sigma(\theta) = |A(\theta)|^2 + |B(\theta)|^2, \tag{30}$$

$$P(\theta) = 2\,\mathrm{Re}\,(A(\theta)B^*(\theta))/\sigma(\theta), \tag{31}$$

$$Q(\theta) = 2\,\mathrm{Im}\,(A(\theta)B^*(\theta))/\sigma(\theta), \tag{32}$$

$$S(\theta) = (|A(\theta)|^2 - |B(\theta)|^2)/\sigma(\theta), \tag{33}$$

$$\sin\beta(\theta) = \frac{Q(\theta)}{\sqrt{Q^2(\theta)+S^2(\theta)}}, \quad \cos\beta(\theta) = \frac{S(\theta)}{\sqrt{Q^2(\theta)+S^2(\theta)}}. \tag{34}$$

The differential cross section and two of these polarization observables form a complete set of the functions of the scattering angle necessary for description of the elastic scattering of nucleons on zero-spin nuclei.

In Fig. 4a and 4b the results of the theoretical calculations of $\sigma(q)$, $P(q)$ and $\beta(q)$ for elastic scattering of protons on ^{16}O nuclei at 500 and 650 MeV are shown along with the experimental data from Ref. [19, 20]. From these figures we can see that the calculated and measured quantities are in agreement for all three observables in the region of not too large transferred momenta.

Fig. 5a shows the results of the calculations of $\sigma(\theta)$, $P(\theta)$ and $Q(\theta)$ together with the experimental data from Ref. [21] for the 800 MeV p-^{16}O elastic scattering. The solid curves correspond to the choice of p-α amplitude (22)–(25) in the form of two Gaussian functions and the dashed curves correspond to that in the form of one Gaussian function ($G_2 = G_4 = 0$). As can be seen from Fig. 5a, making use of the p-α amplitude with two Gaussian functions is much better than with one Gaussian function. This distinction is pronounced in the cross section in the region of the third maximum and in the spin-rotation function, where the values of $Q(\theta)$ differ even qualitatively.

The results of the calculations of $\sigma(\theta)$, $P(\theta)$ and $Q(\theta)$ for p-^{12}C elastic scattering at 800 MeV are presented in Fig. 5b. Our calculations of the differential cross section and polarization are in agreement with the experimental data [22, 23].

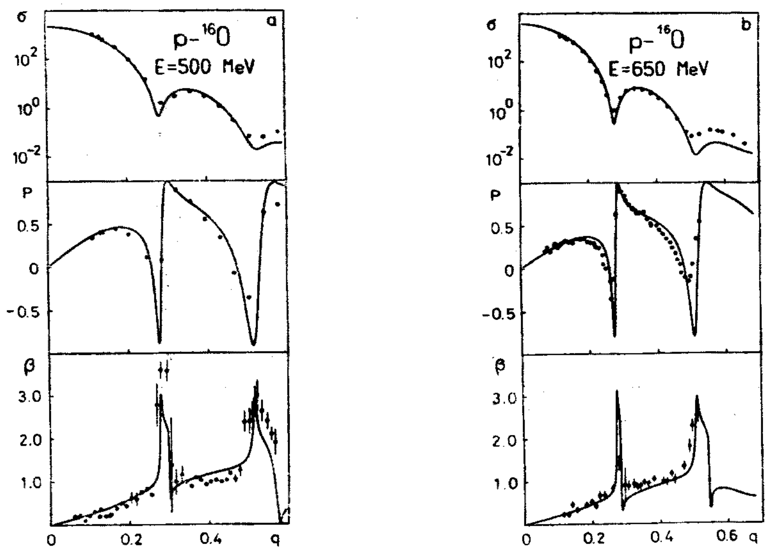

Fig. 4. Differential cross sections σ (mb/sr), polarizations P and spin-rotation angles β of 500 MeV (a) and 650 MeV (b) protons elastically scattered on ^{16}O as a functions of the momentum transferred q(GeV/c)

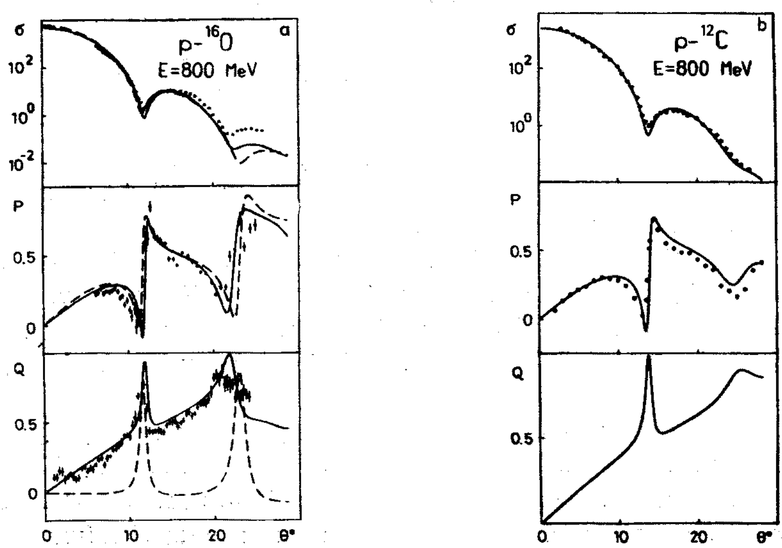

Fig. 5. Differential elastic scattering cross sections σ (mb/sr), polarizations P and spin-rotation functions Q of 800 MeV protons on ^{16}O (a) and ^{12}C (b)

We apply the above developed approach to describe the measured differential cross sections of $\bar{p}$-^{12}C and $\bar{p}$-^{16}O elastic scattering at 179 MeV. In this region of energies we can use the multiple diffraction scattering theory for antiproton elastic scattering. This fact is related with a hihg probability of annihilation and consequently with a high probability of absorption of antiprotons by nuclei. Therefore, the scattering has a clearly pronounced diffraction character and $\bar{p}$-α amplitude has a sharp maximum at small transfered momenta [24].

We have calculated the differential cross sections of the antiproton elastic scattering on ^{12}C and ^{16}O nuclei at 179 MeV. The lack of experimental data on $\bar{p}$-^4He scattering and the data on the polarization characteristics in the elastic $\bar{p}$-^{12}C and $\bar{p}$-^{16}O scattering at this energy does not allow us to reconstruct the $\bar{p}$-α amplitude with enough precision. Therefore, we considered the parameters of the $\bar{p}$-^4He amplitude as a ffitting ones when we calculated the elastic $\bar{p}$-^{12}C scattering amplitude (see Table I). The calculation of the differential cross section of elastic $\bar{p}$-^{12}O scattering does not contain any fitting parameters. The results of our calculations are presented in Fig. 6 along with the experimental data [25, 26]. Fig. 6 shows that the approach developed allows us to describe simultaneously the differential cross sections of elastic scattering of 179 MeV antiprotons by ^{12}C and ^{16}O nuclei.

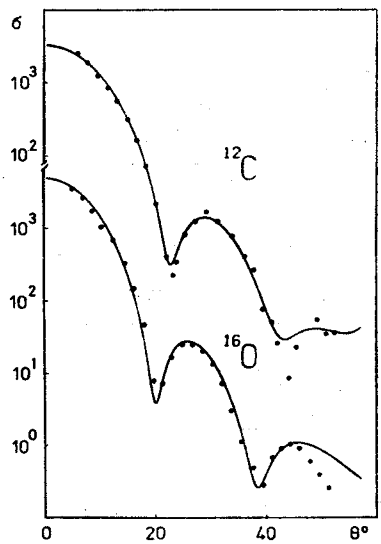

Fig. 6. Differential elastic scattering cross sections σ (mb/sr) for 179 MeV antiprotons on ^{12}C and ^{16}O

5. *Elastic scattering of pions on ^{12}C and ^{16}O nuclei*

In order to describe the elastic scattering of high energy pions on ^{12}C and ^{16}O we can also apply the α-particle model with dispersion. As the probability of pion absorption by an α-particle is much higher than by a single nucleon of the nucleus [27, 28], then the multiple diffraction scattering theory constructed by means of the scattering amplitudes of the incident pion on an α-particle may be used. The attractiveness of this approach is in

TABLE II

E, MeV	β, fm^2	f(0), fm^2	τ_1, fm^2	τ_2, fm^2
114	0.473	$1.077+i\,1.931$	$0.598-i\,0.157$	$0.151+i\,0.039$
120	0.473	$1.077+i\,1.931$	$0.567-i\,0.149$	$0.243+i\,0.110$
150	0.253	$0.377+i\,2.545$	$0.422-i\,0.079$	$0.140+i\,0.035$
163	0.277	$0.307+i\,2.562$	$0.384-i\,0.053$	$0.147+i\,0.035$
170	0.281	$0.051+i\,2.566$	$0.366-i\,0.046$	$0.147+i\,0.034$
180	0.286	$-0.072+i\,2.576$	$0.349-i\,0.038$	$0.147+i\,0.034$
200	0.265	$-0.043+i\,2.280$	$0.299-i\,0.008$	$0.146+i\,0.020$
220	0.282	$-0.170+i\,2.121$	$0.258-i\,0.007$	$0.126+i\,0.018$
230	0.283	$-0.192+i\,1.962$	$0.246-i\,0.007$	$0.120+i\,0.017$
240	0.296	$-0.382+i\,1.910$	$0.227-i\,0.013$	$0.116+i\,0.013$
260	0.300	$-0.597+i\,1.899$	$0.204-i\,0.018$	$0.110+i\,0.010$
280	0.347	$-0.633+i\,1.541$	$0.183-i\,0.016$	$0.099+i\,0.009$
343	0.376	$-1.504+i\,1.074$	0.214	$0.088+i\,0.001$

the fact that the π-α amplitude as a elementary "brick" of the model includes automatically the effects of the pion absorption, nucleon-nucleon correlations, antisymmetrization of the nucleon wave function and the processes of many-body interaction of pions with nucleons of the nucleus.

We construct the elastic scattering amplitudes of pions on ^{12}C and ^{16}O nuclei from the π-α amplitudes on the basis of the multiple diffraction scattering theory. As the energy of the incident pion is much higher than the characteristic energy of α-cluster motion inside the nucleus, then we can choose the π-α amplitude as a scattering amplitude of pion on a free α-particle [29, 30]:

$$f(q) = kf(0)(1-\tau_1 q^2)(1-\tau_2 q^2)e^{-\beta q^2}, \tag{35}$$

where the parameters $f(0)$, τ_1, τ_2, β, taken from Ref. [7] are given in Table II.

The scattering amplitude of pions on ^{12}C nucleus is equal to

$$F_e^{(C)}(q) = 3F_1^{(C)}(q) - 3F_2^{(C)}(q) + F_3^{(C)}(q), \tag{36}$$

$$F_1^{(C)}(q) = f(q)S^{(C)}(q)g_\Delta^{(C)}(q), \tag{37}$$

$$F_2^{(C)}(q) = -\frac{ik}{\tilde{\gamma}}f^2(0)\exp\left(-\tfrac{1}{24}(\tilde{\gamma}+8\tilde{\beta})q^2 - \frac{d^2}{3\tilde{\gamma}}\right)j_0\left(\frac{qd}{2\sqrt{3}}\right), \tag{38}$$

$$F_2^{(C)}(q) = -\frac{4k}{3\tilde{\gamma}^2}f\,(0)\exp\left(-\tfrac{1}{3}\tilde{\beta}q^2 - \frac{2d^2}{3\tilde{\gamma}}\right) \tag{39}$$

where $\tilde{\beta} = \beta+\tau_1+\tau_2$, $\tilde{\gamma} = 4\tilde{\beta}+\Delta^2$ and $S^C(q)$ and $g_\Delta^C(q)$ are described by the formulae (15), (16).

The pion-^{16}O scattering amplitude is determined by

$$F_e^{(O)}(q) = 4F_1^{(O)}(q) - 6F_2^{(O)}(q) + 4F_3^{(O)}(q) - F_4^{(O)}(q), \tag{40}$$

$$F_1^{(O)}(q) = f(q)S^{(O)}(q)g_\Delta^{(O)}(q), \tag{41}$$

$$F_2^{(O)}(q) = -\frac{ik}{\tilde{\gamma}} f^2(0) \exp\left(-\tfrac{1}{16}(\tilde{\gamma}+4\tilde{\beta})q^2 - \frac{d^2}{3\tilde{\gamma}}\right) j_0\left(\frac{qd}{2\sqrt{2}}\right), \tag{42}$$

$$F_3^{(O)}(q) = -\frac{4k}{3\tilde{\gamma}^2} f^3(0) \exp\left(-(\tilde{\gamma}+12\tilde{\beta})q^2 - \frac{2d^2}{3\tilde{\gamma}}\right) j_0\left(\frac{qd}{2\sqrt{6}}\right), \tag{43}$$

$$F_4^{(O)}(q) = i\frac{2k}{\tilde{\gamma}^3} f^4(0) \exp\left(-\tfrac{1}{4}\tilde{\beta}q^2 - \frac{d^2}{\tilde{\gamma}}\right). \tag{44}$$

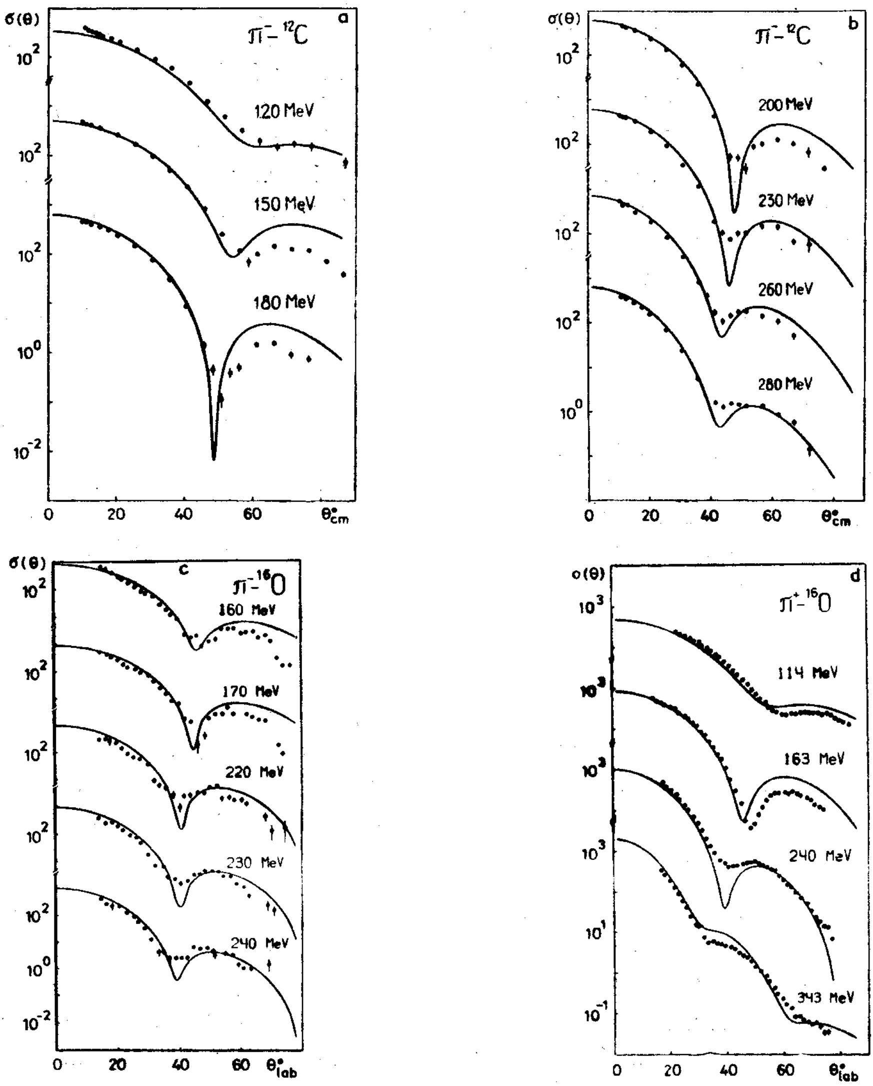

Fig. 7. Differential elastic scattering cross sections σ (mb/sr) for pions on the nuclei: a) $\pi^- - {}^{12}C$ et 120, 150 and 180 MeV; b) $\pi^- - {}^{12}C$ at 200, 230, 260 and 280 MeV; c) $\pi^- - {}^{16}O$ at 160, 170, 220, 230 and 240 MeV; d) $\pi^+ - {}^{16}O$ at 114, 163, 240 and 343 MeV

Here $S^{(O)}(q)$ and $g_A^{(O)}(q)$ are given by equations (18) and (19).

In formula (36)–(44) the amplitudes $F_1^{(C)}(q)$ and $F_1^{(O)}(q)$ are calculated exactly. In the calculations of amplitudes (38), (39) and (42)–(44) we again used the effective deformation approximation and replaced the π-α amplitude by the expression

$$f(q) = Kf(0)\exp(-\tilde{\beta}q^2) \tag{45}$$

which is valid in the region of small q.

On the basis of expressions (36)–(44) we calculated the differential cross sections of π^--^{12}C, π^--^{16}O and π^+-^{16}O elastic scattering at different energies. The results of our calculations are presented in Fig. 7a-7d along with the experimental data [30–32]. Fig. 7a-7d show that the calculated cross sections are in agreement with the experimental data. The best agreement is observed at higher energies and in the region of not too large scattering angles ($\theta < 40° - 50°$). For the scattering of pions on ^{12}C nucleus at 230, 260 and 280 MeV and on ^{16}O nucleus at 220, 230, 240 and 343 MeV the calculated and measured cross sections are in agreement up to angles $\theta \sim 80°$. The deep minima in the angle distributions in the region of (3,3) resonance are due to a strong absorption of pions at these energies [27]. As one can see from Fig. 7a-7d this effect is described by the model.

Notice that in Ref. [33, 34] the interaction of pions with ^{12}C and ^{16}O nuclei was studied on the basis of the optical potential which was constructed by means of an α-particle model. The interaction of π^+ and π^- with ^{12}C nucleus was cosidered in Ref. [35]. It was shown that the differential cross sections of π^+ and π^- elastic scattering are distinguished significantly in the region of minima. The differential cross section for π^+ elastic scattering lies higher than for π^- elastic scattering. This difference in the behaviour of the elastic scattering cross sections is caused by the influence of Coulomb interaction [36].

It follows from the above calculations that the α-particle model with dispersion gives a good description of the elastic scattering both π^- and π^+ by ^{12}C and ^{16}O nuclei.

6. Conclusions

We have carried out the calculations of charge form factors and densities of ^{12}C and ^{16}O nuclei, the differential cross sections of elastic scattering of protons, antiprotons and charged pions on these nuclei and the polarization observables for elastic scattering of protons on ^{12}C and ^{16}O nuclei. We have obtained a good agreement of the calculated values with the experimental data available. The obtained results point out a strong α-clusterization in these nuclei.

To achieve a good agreement between the calculated and measured observables it is necessary to choose correctly the scattering amplitude of the incident particle on an α-cluster. We have demonstrated that this choice for the proton and antiproton amplitudes can be made by approximating the corresponding experimental data by the sum of two Gaussians.

The agreement between the theory and experiment becomes worse in the region of large transferred momenta, where microscopic model calculations seem necessary. Moreover, in the region of large transferred momenta a significant role can be played by the nucleon exchange between α-clusters.

The α-particle model with dispersion provides a description of the complete set of experimental data for the hadron elastic scattering on ^{12}C and ^{16}O nuclei without fitting parameters. Taking account of dispersion in the α-particle model leads to the same effect as increasing the α-particle size in the model of rigid triangle (^{12}C) or tetrahedron (^{16}O).

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APPENDIX

The amplitudes of proton (antiproton) elastic scattering on ^{12}C and ^{16}O nuclei have the forms

$$F^{(C)}(\mathbf{q}) = 3F_1(\mathbf{q}) - 3F_2(\mathbf{q}) + F_3(\mathbf{q}),$$

$$F^{(O)}(\mathbf{q}) = 4F_1(\mathbf{q}) - 6F_2(\mathbf{q}) + 4F_3(\mathbf{q}) - F_4(\mathbf{q}).$$

The quantities $F_1(\mathbf{q})$, $F_2(\mathbf{q})$, $F_3(\mathbf{q})$, $F_4(\mathbf{q})$ are the single, double, triple and quadruple amplitudes of incident particle scattering on α-cluster nuclei that take into account the spin-orbit interaction.

$$F_1(\mathbf{q}) = f_{p\alpha}(\mathbf{q})S(q)g_\Delta(q),$$

$$F_2(\mathbf{q}) = \left\{\sum_{i,k=1}^{2} + 2\sum_{i,k=3}^{4}\left(\frac{\partial}{\partial \gamma_1} + \frac{q^2}{8}\right)\right\} G_i G_k S_2(i, k)$$

$$+ \frac{2}{q}\sum_{i=1}^{2}\sum_{k=3}^{4}\left(\frac{\partial}{\partial \alpha} + \frac{q^2}{2}\right) G_i G_k S_2(i, k)\,(\boldsymbol{\sigma n}),$$

$$F_3(\mathbf{q}) = \{\sum_{i,k,l=1}^{2} + 3\sum_{i=3}^{4}\sum_{k,l=1}^{2} D_2(i, k, l)\} G_i G_k G_l S_3(i, k, l)$$

$$+\{\sum_{i,k=1}^{2}\sum_{l=3}^{4} D_1(i, k, l) + \sum_{i,k,l=3}^{4} D_1(i, k, l) D_2(i, k, l)\} G_i G_k G_l S_3(i, k, l)\,(\boldsymbol{\sigma n}),$$

$$F_4(\mathbf{q}) = \{\sum_{i,k,l,m=1}^{2} + 6\sum_{i,k=3}^{4}\sum_{l,m=1}^{2} D_4(i, k, l, m)$$

$$+ \sum_{i,k,l,m=3}^{4} D_4(i, k, l, m) D_5(i, k, l, m)\} G_i G_k G_l G_m S_4(i, k, l, m)$$

$$+\{4\sum_{i,k,l=1}^{2}\sum_{m=3}^{4} D_3(i, k, l, m) + 4\sum_{i,k,m=3}^{4}\sum_{l=1}^{2} D_3(i, k, l, m)$$

$$\times D_4(i, k, l, m)\} G_i G_k G_l G_m S_4(i, k, l, m)\,(\boldsymbol{\sigma n}),$$

$$S_2^{(0)}(i,k) = -\frac{iK}{\gamma_1}\exp\left[-\left(\frac{\gamma_1}{8}+\frac{\alpha_1}{4}-\frac{\alpha^2}{\gamma_1}\right)\frac{q^2}{2}-\frac{d^2}{3\gamma_1}\right]j_0(\chi\varphi_1).$$

$$S_3^{(0)}(i,k,l) = -\frac{4K}{3\gamma_1\gamma_2}\exp\left[-\left(\frac{\Delta^2}{48}+g_1\right)q^2-g_2d^2\right]j_0(\chi\varphi_2),$$

$$S_4^{(0)}(i,k,l,m) = i\frac{3K}{8\gamma_1\gamma_2\gamma_3}\exp\left[-\left(\tfrac{1}{4}\sum_{j=1}^{4}\beta_j-\frac{\alpha^2}{2\gamma_1}+\tfrac{3}{2}\gamma_2\chi_2\right.\right.$$

$$\left.\left.-\frac{\alpha_7^2}{\gamma_3}\right)\frac{q^2}{4}-\left(g_2+\frac{1}{16\gamma_3}+\frac{9\chi_1^2}{128\gamma_3}+\frac{2\chi_2}{9\gamma_3}\right)d^2\right]j_0(\chi\varphi_3),$$

$$S_2^{(C)}(i,k) = -\frac{iK}{\gamma_1}\exp\left[-\left(\frac{\gamma_1}{12}+\frac{\alpha_1}{3}-\frac{\alpha^2}{\gamma_1}\right)\frac{q^2}{2}-\frac{d^2}{3\gamma_1}\right]j_0(\chi'\varphi_4),$$

$$S_3^{(C)}(i,k,l) = -\frac{4K}{3\gamma_1\gamma_2}\exp\left[-g_1q^2-g_2d^2\right]j_0(\chi'\varphi_5),$$

$$D_1(i,k,l) = \frac{3}{q}\left(\frac{q^2}{9}+\frac{\partial}{\partial\alpha_3}\right),$$

$$D_2(i,k,l) = \frac{q^2}{9}+2\frac{\partial}{\partial\gamma_1}-\tfrac{2}{3}\frac{\partial}{\partial\gamma_2}-\frac{\partial}{\partial\alpha_3},$$

$$D_3(i,k,l,m) = \frac{3}{4q}\left(\frac{q^2}{3}-\frac{\partial}{\partial\alpha_7}\right),$$

$$D_4(i,k,l,m) = \frac{q^2}{16}+2\frac{\partial}{\partial\gamma_1}-\tfrac{2}{3}\frac{\partial}{\partial\gamma_2}-\tfrac{1}{16}\frac{\partial}{\partial\gamma_3}+\tfrac{1}{6}\frac{\partial}{\partial\alpha_6}+\tfrac{1}{8}\frac{\partial}{\partial\alpha_7}$$

$$D_5(i,k,l,m) = \frac{q^2}{16}+\tfrac{3}{16}\frac{\partial}{\partial\gamma_3}-\tfrac{1}{6}\frac{\partial}{\partial\alpha_6}-\tfrac{1}{8}\frac{\partial}{\partial\alpha_7},$$

$$\alpha = \beta_i-\beta_k,\quad \alpha_1 = \beta_i+\beta_k,\quad \alpha_2 = \beta_i+\beta_k+4\beta_l,\quad \alpha_3 = \beta_i+\beta_k-2\beta_l,$$

$$\alpha_4 = \beta_i+\beta_k+\beta_l+9\beta_m,\quad \alpha_5 = \beta_i+\beta_k+\beta_l-3\beta_m,\quad \alpha_6 = \frac{\alpha^2}{3\gamma_1}-\frac{\alpha_3}{6},$$

$$\alpha_7 = \tfrac{1}{8}\alpha_5+\frac{\alpha^2}{4\gamma_1},$$

$$\gamma_1 = 2\alpha_1+\Delta^2,\quad \gamma_2 = \Delta^2-\frac{4\alpha^2}{3\gamma_1}+\tfrac{2}{3}\alpha_2,\quad \gamma_3 = \tfrac{3}{16}\Delta^2+\tfrac{1}{16}\alpha_4-\frac{\alpha^2}{8\gamma_1}-\frac{3\alpha_6^2}{2\gamma_2},$$

$$\chi = \frac{qd}{2\sqrt{2}}, \quad \chi' = \frac{qd}{2\sqrt{3}}, \quad \chi_1 = -\frac{8\alpha_6}{3\gamma_2}, \quad \chi_2 = \frac{2\alpha}{3\gamma_1}\left(1+\frac{4\alpha_6}{\gamma_2}\right),$$

$$\chi_3 = \frac{3\alpha_7}{8\gamma_3}, \quad \chi_4 = -\tfrac{3}{4}\chi_1\left(1+\frac{\alpha_7}{2\gamma_3}\right), \quad \chi_5 = \tfrac{3}{4}\chi_2\left(1+\frac{\alpha_7}{2\gamma_3}\right),$$

$$\varphi_1 = \sqrt{1+\frac{8\alpha^2}{\gamma_1^2}}, \quad \varphi_2 = \sqrt{1+18\chi_1^2+24\chi_2^2}, \quad \varphi_3 = \sqrt{\chi_3^2+18\chi_4^2+24\chi_5^2},$$

$$\varphi_4 = \sqrt{1+\frac{12\alpha^2}{\gamma_1^2}}, \quad \varphi_5 = \sqrt{9\chi_1^2+12\chi_2^2},$$

$$g_1 = \tfrac{1}{9}\sum_{j=1}^{3}\beta_j - \frac{2\alpha^2}{9\gamma_1} - \tfrac{3}{8}\gamma_2\chi_2^2, \quad g_2 = \frac{1}{3\gamma_1}+\frac{1}{3\gamma_2}+\frac{4\alpha^2}{9\gamma_1^2\gamma_2}$$

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