NEUTRON MATTER AND SKYRME FORCES

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The properties of neutron matter are studied using several existing parametrizations of the Skyrme interaction SK. Relations for energy per particle, effective mass, magnetic susceptibility, ... etc. for neutron matter are derived using the Skyrme interaction. It is found that the ratio $\chi_{\rm F}/\chi$ of the magnetic susceptibility of a Fermi gas of non-interacting neutrons to that of a neutron matter increases with density up to $k_{\rm f} \approx 1.1~{\rm fm^{-1}}$ and then decreases. In order to obtain a similar behaviour to that of realistic interactions we varied the SK parameters to produce values of $\chi_{\rm F}/\chi$ which increase steadily with $k_{\rm f}$. The thermal properties of neutron matter are calculated also using SK force. The temperature dependences of the volume and spin pressure are determined. The results show a similar trend as previous theoretical estimates using realistic forces.

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1. Introduction

It is currently believed that pulsars are rapidly spinning neutron stars. In the normal nuclear density region the neutron star matter is believed to consist mainly of neutrons, with a small admixture of protons, electrons, and muons. As a first approximation pure neutron matter appears to be a good approximation of the real neutron star matter near and above nuclear density. Several calculations have been made on neutron matter. It was found that neutron matter is unbound. The thermal and dynamical properties of neutron matter are of interest in a variety of problems and applications of current interest, e.g. heavy and superheavy nuclei, high energy heavy-ion collisions and neutron stars.

The binding energy per particle, effective mass, single particle potential and magnetic susceptibility were calculated using different potentials [1-3]. Also, the equation of state for hot neutron matter has been considered by several authors [4-6]. The Skyrme type forces lead to particularly simple calculations. Skyrme forces have been used for example in the calculation of the equation of state of neutron star matter at densities below the standard nuclear matter density [7]. Skyrme forces have also been used to calculate the properties of hot, dense matter encountered in the gravitational collapse of massive stars [6, 8, 9].

The purpose of the present work is to study the properties of neutron matter such as energy per particle, single particle potential, effective mass and magnetic susceptibility using several available parametrizations of the Skyrme forces. Our objective is to provide a better Skyrme parametrization of realistic calculations of neutron matter.

In the next section we explain the theory where we show how the energy per particle, single particle potential, effective mass, magnetic susceptibility, the pressure and the velocity of sound can be calculated. The numerical results obtained using various parametrizations of the Skyrme forces are discussed in Section 3. We compare the Skyrme force results using a new parametrization with estimates based on a realistic nucleon-nucleon interaction [4]. Also we compare the Skyrme force results for Fermi liquid parameters with previous theoretical calculations.

2. Theory

2.1. Energy per particle and compressibility

The Skyrme interaction [10, 11] can be written as the sum of a two-body term and a three-body term, namely:

$$V_{12} = t_0 (1 + x_0 P^{\sigma}) \delta(\vec{r}_1 - \vec{r}_2) + \frac{1}{2} t_1 [\delta(\vec{r}_1 - \vec{r}_2) k^2 + k'^2 \delta(\vec{r}_1 - \vec{r}_2)] + t_2 \vec{k}' \cdot \delta(\vec{r}_1 - \vec{r}_2) \vec{k},$$
(1)

where $\vec{k} = (\vec{\nabla}_1 - \vec{\nabla}_2)/2i$ acts on the right, while $\vec{k}' = -(\vec{\nabla}_1 - \vec{\nabla}_2)/2i$ acts on the left. For the three-body term, we shall use the equivalent two-body density dependent term:

$$V_{12}^{(3)} = \frac{t_3}{6} (1 + x_3 P^{\sigma}) \varrho^d \delta(\vec{r}_1 - \vec{r}_2), \tag{2}$$

where ϱ is the density of neutron matter, d=1 for the seven forces SKI-SKVI [12, 13] and SKLR [6], d=1/3 for the two forces SKA and SKB [14], and d=1/6 for the two forces SKM [15] and SKM* [16].

Using the above equations of the Skyrme interaction the binding energy of neutron matter can be written as [3]

$$E/N = \varepsilon_{\rm vol} + \frac{1}{2} \varepsilon_{\sigma} \alpha_{\sigma}^2, \tag{3}$$

where $\alpha_{\sigma} = (N \uparrow - N \downarrow)/N$, ε_{vol} is the volume energy and ε_{σ} is the spin symmetry energy. Terms higher than quadratic in α_{σ} are neglected in equation (3).

The expressions of $\varepsilon_{\rm vol}$ and ε_{σ} can be expressed as a function of the Fermi momentum $k_{\rm f}$ and the result is

$$\varepsilon_{\text{vol}} = \frac{3}{10} \frac{k_{\text{f}}^2}{M_{\text{N}}} + \frac{(t_0 - t_0 x_0)}{12\pi^2} k_{\text{f}}^3 + \frac{(t_1 + 3t_2)}{40\pi^2} k_{\text{f}}^5 + \frac{t_3 (1 - x_3)}{72\pi^2 (3\pi^2)^4} k_{\text{f}}^{3+3d} \tag{4}$$

and

$$\varepsilon_{\sigma} = \frac{k_{\rm f}^2}{3M_{\rm N}} - \frac{(t_0 - t_0 x_0)}{6\pi^2} k_{\rm f}^3 - \frac{(t_1 - 3t_2)}{18\pi^2} k_{\rm f}^5 - \frac{t_3(1 - x_3)}{36\pi^2 (3\pi^2)^d} k_{\rm f}^{3+3d},\tag{5}$$

where M_N is the neutron mass and the compressibilities can be given by:

$$K_{\mathbf{v},\sigma} = k_{\mathbf{f}}^2 \frac{\partial^2 \varepsilon_{\mathbf{v},\sigma}}{\partial k_{\mathbf{f}}^2}.$$
 (6)

As the interaction potential here is density dependent one has to add the contribution due to rearrangement [1, 3] namely,

$$\Delta \varepsilon_{\sigma R} = -\frac{dt_3(1-x_3)}{216\pi^2(3\pi^2)^d} k_f^{3+3d}. \tag{7}$$

2.2. Single particle potential and effective mass

In an infinite neutron matter the average potential can be defined as

$$U = \sum_{i \leq m} \left[\langle ij | V | ij \rangle - \langle ij | V | ji \rangle \right] + U_{\mathbf{R}}, \tag{8}$$

where $U_{\mathbb{R}}$ is the rearrangement potential [3]. This potential is added only if the two-body interaction V is dependent on density or if second and higher-order terms in the perturbation series are taken into account.

For SK force we have at a momentum k

$$U(k, N\uparrow) = U_V + \alpha_\sigma U_\sigma \uparrow$$

$$= \frac{t_0}{6\pi^2} (1 - x_0) k_f^3 + \frac{t_1}{8\pi^2} \left(\frac{k_f^5}{5} + \frac{k^2 k_f^3}{3} \right) + \frac{t_2}{8\pi^2} \left(\frac{3}{5} k_f^5 + k^2 k_f^3 \right)$$

$$+ \frac{t_3 (1 - x_3)}{36\pi^2 (3\pi^2)^d} k_f^{3+3d} + \alpha_\sigma \left[-\frac{t_0 (1 - x_0)}{6\pi^2} k_f^3 + \frac{(t_2 - t_1)}{24\pi^2} (k_f^5 + k^2 k_f^3) + \frac{t_3 (x_3 - 1)}{36\pi^2 (3\pi^2)^d} k_f^{3+3d} \right]$$

$$+ \frac{t_3 (x_3 - 1)}{36\pi^2 (3\pi^2)^d} k_f^{3+3d}$$

$$(9)$$

and a similar expression for $U(k, N\downarrow)$ but with the minus sign for the α_{σ} term. For the rearrangement potential we obtain

$$U_R = \frac{dt_3(1-x_3)}{72\pi^2(3\pi^2)^d} k_{\rm f}^{3+3d}.$$
 (10)

From the above equations one can obtain the expression for the effective mass. At $\alpha_{\sigma} = 0$ and $M_{\rm N_1} = M_{\rm N_2}$ we obtain

$$\frac{M_{\rm N}^*}{M_{\rm N}} = 1 / \left[1 + \frac{2M_{\rm N}}{h^2} \frac{(t_1 + 3t_2)}{24\pi^2} k_{\rm f}^3 \right]. \tag{11}$$

2.3. Magnetic susceptibility

Following Haensel [3] and Behera and Satpathy [1] one can write an expression for the ratio of the magnetic susceptibility of the Fermi gas of noninteracting neutrons and that of neutron matter as

$$\chi_F/\chi = 3\varepsilon_\sigma/2\varepsilon_n, \tag{12}$$

where ε_n is the Fermi energy of an unpolarized neutron matter with $N\uparrow=N\uparrow=\frac{1}{2}N$. Adding the rearrangement contribution we then obtain

$$\chi_{\rm F}/\chi = \frac{k_{\rm f}^2}{3M_{\rm N}} - \frac{t_0(1-x_0)}{6\pi^2} k_{\rm f}^3 - \frac{(t_1-3t_2)}{18\pi^2} k_{\rm f}^5 - \frac{(6+d)t_3(1-x_3)}{216\pi^2(3\pi^2)^d} k_{\rm f}^{3+3d} \bigg] / (k_{\rm f}^2/3M_{\rm N}). \quad (13)$$

2.4. Pressure and velocity of sound

In the case of polarized neutron matter the pressure can be expanded up to second order in α_{σ} to have the form [17, 18]

$$P = \varrho^2 \left(\frac{\partial f}{\partial \varrho}\right)_{\rm T} = P_{\rm V} + \frac{1}{2} \alpha_{\sigma}^2 P_{\sigma}$$
 (14)

where f is the free energy per particle.

Hence, for SK interaction we get

$$P_{V} = \frac{k_{f}^{4}}{9\pi^{2}} \left[\frac{3}{5} \frac{k_{f}}{M_{N}} + \frac{(t_{0} - t_{0}x_{0})}{4\pi^{2}} k_{f}^{2} + \frac{(t_{1} + 3t_{2})}{8\pi^{2}} k_{f}^{4} + \frac{t_{3}(1+d)(1-x_{3})}{24\pi^{2}(3\pi^{2})^{d}} k_{f}^{2+3d} + \frac{\pi^{2}}{2k_{f}} \frac{(KT)^{2}}{\mu_{0}} - \frac{6}{k_{f}} \frac{23\pi^{4}}{1920} \frac{(KT)^{4}}{\mu_{0}^{3}} - \dots \right]$$

$$(15)$$

and

$$P_{\sigma} = \frac{k_{\rm f}^4}{9\pi^2} \left[\frac{2}{3} \frac{k_{\rm f}}{M_{\rm N}} - \frac{t_0(1-x_0)}{2\pi^2} k_{\rm f}^2 - \frac{5}{18\pi^2} (t_1 - 3t_2) k_{\rm f}^4 \right] - \frac{(d+1)(d+6)t_3(1-x_3)}{72\pi^2 (3\pi^2)^d} k_{\rm f}^{2+3d} - \frac{2\pi^2}{36k_{\rm f}} \frac{(KT)^2}{\mu_0} - \frac{6\pi^4}{k_{\rm f}} \frac{23}{1920} \frac{(KT)^4}{\mu_0^3} + \dots \right], \quad (16)$$

where K is Boltzmann's constant, $\mu_0 = \hbar^2 k_{\rm f}^2 / 2M_{\rm N}$ and the sound velocity (in units of c) at zero temperature is given by [4]

$$S = \sqrt{\partial P/\partial \varepsilon_{,}} \tag{17}$$

where

$$\varepsilon = \varrho(f + M_{\rm N}c^2). \tag{18}$$

3. Results and discussion

The Skyrme interactions are pure phenomenological nucleon-nucleon interactions, which were determined to fit some nuclear properties. Due to the phenomenological origin of this model, one cannot expect reliable predictions for properties of neutron matter which were not fitted in the original model especially for higher k_f values. In a previous work [18, 19] it is observed that the sets of parameters SKII, SKIII and SKLR give a non-realistic antipairing force at large k_f values which spoils also the values calculated for the magnetic susceptibility. Here we report that a similar result is obtained for the other sets of parameters, i.e. for SKI, SKIV, SKV, SKVI, SKA, SKB, SKM and SKM*. Therefore, it is desirable to search for a new set of parameters in order to obtain better values for the spin symmetry energy. For the above sets of parameters we find that the ratio χ_F/χ increases with k_f up to $k_f \approx 1.1$ fm⁻¹ and then decreases continuously. In order to obtain a similar behaviour to that of realistic potentials we varied the parameters t_0 , t_1 ,... etc. of the SK force to produce values for χ_F/χ which increase steadily with k_f . Our results are then compared with the realistic interaction calculations discussed previously by Friedmann and Pandharipande [4].

The new set obtained is $t_0 = -1673.293 \text{ MeV fm}^3$, $t_1 = 97.709 \text{ MeV fm}^5$, $t_2 = 37.43 \text{ MeV fm}^5$, $t_3 = 10625.107 \text{ MeV fm}^6$, $x_0 = 0.403$, $x_3 = 0.692$ and d = 1/3.

Using this new set of parameters we obtain a value of $\varepsilon_{\rm vol} = -15.41$ MeV at $k_{\rm f} = 1.291$ fm⁻¹ in the nuclear matter case. In this case the incompressibility of nuclear matter is 200 which is better than the large values obtained by the old sets of SK parameters. Also we obtained a value of 25.708 MeV for the isospin symmetry energy to be compared with the value 25.6 MeV calculated previously by Sjöberg [20].

The volume energy per particle calculated by using equation (4) is given in Table I along with its corresponding value for the realistic interaction of Ref. [4] for different values of $k_{\rm f}$. If we take Ref. [4] as the bench-line for comparison we notice that the best agreement is obtained for a value of the parameter d=1/3. For the three values d=1, 1/3 and 1/6 the agreement is good until a value of $k_{\rm f}=1.6$ fm⁻¹. At higher $k_{\rm f}$ values the calculated values of $\varepsilon_{\rm vol}$ go high above those of Ref. [4] for the case d=1 while exactly the opposite occurs for the case d=1/6. From Table I we notice that the values obtained for $\varepsilon_{\rm vol}$ are comparable with those obtained by realistic forces for a good range of $k_{\rm f}$ values.

The single particle potential $U_{\rm V}$ as well as the rearrangement correction $U_{\rm R}$ at $k=k_{\rm f}$ are displayed in Table II as a function of $k_{\rm f}$. It is clear that the potential $U_{\rm V}$ has the same trend as those calculated by Behera and Satpathy [1]. After passing through a minimum it increases steadily with $k_{\rm f}$. The results of our calculation and other previous works for the effective mass are displayed in Table III which shows a steady decrease of $M_{\rm N}^*/M_{\rm N}$ with the increase of density as expected. Here again the results are good in comparison with those of the realistic calculations of Ref. [4].

In Table IV the ratio χ_F/χ of the magnetic susceptibility of a Fermi gas of noninteracting neutrons to that of neutron matter is given as a function of k_f . From the table we see that χ_F/χ increases monotonically with the values of k_f with a behaviour which is similar to that of realistic potentials. The results obtained by using the sets of parameters

TABLE 1 The volume energy $\varepsilon_{\rm vol}$ in MeV as a function of $k_{\rm f}; \varepsilon_{\rm vol}({\rm MeV})$

$k_{\rm f}({ m fm}^{-1})$	Present work	Ref. [4]
0.5	2.165	2.057
0.6	2.891	2.747
0.7	3,650	3,503
0.8	4.428	4.322
0.9	5.219	5.177
1.0	6.027	6.027
1.1	6.864	6.873
1.2	7.751	7.744
1.3	8.723	8.699
1.4	9.822	9.822
1.5	11.105	11.220
1.6	12.640	13.010
1.7	14.509	15.260
1.8	16.806	17.690
1.9	19.643	20.730
2.0	23.144	24.670

TABLE II

The single particle potential and rearrangement correction in MeV as a function of k_f

k _f (fm ⁻¹)	$U_{\mathbf{R}}(MeV)$	$U_V(\text{MeV})$	
0.1	4.97 × 10 ⁻⁶	- 0.01656	
0.5	0.031	-1.878	
1.0	0.497	-12.471	
1.4	1.908	-27.213	
2.0	7.948	-41.882	
2.5	19.404	-8.647	
3.0	40.236	130.618	

Effective mass M^*/M as a function of Fermi momentum

TABLE III

$k_{\rm f}({\rm fm}^{-1})$	Present work	Ref. [4] $T = 3 \text{ MeV}$	Ref. [4] $T = 10 \text{ MeV}$	Ref. [1]	Ref. [22]
0.6	0.991	0.993	0.993	. 0.975	
0.8	0.979	0.979	0.979	0.946	0.94
1.0	0.959	0.953	0.967	0.907	
1.2	0.931	0.920	0.945	0.860	
1.4	0.895	0.880	0.907	0.811	0.78
1.6	0.851	0.833	0.860	0.763	
1.8	0.800	0.787	0.800	0.719	
2.0	0.745	0.727	0.727	0.681	0.59

The quotient χ_F/χ as a function of k_f

$k_{\rm f}({ m fm}^{-1})$	Present work	Ref. [22]	Ref. [24]	Ref. (25]	Ref. [3]	Ref. [26]
0.5	1.554				į	1.602
0.6	1.652					
0.7	1.745	-				
0.8	1.834	1.62	İ			
0.9	1.918					
1.0	1.999				1.74	1.935
1.1	2.075	-				
1.2	2.147				1.85	
1.25			1.99	1.90		
1.3	2.215					
1.4	2.279	1.81			1.94	1
1.5	2.339	1				2.258
1.6	2.394				2.06	
1.7	2.446					
1.8	2.494				2.25	
1.89			2.25	2.22		
1.9	2.538					
2.0	2.578	2.24			2.42	2.581

SKI-SKVI, SKLR, SKA, SKB, SKM and SKM* show a different behaviour where χ_F/χ increases with k_f up to a value of $k_f \approx 1.1$ fm⁻¹ and then decreases. We note also that for the force SKRATP [21] (d=1/5 in this case) which has been adapted to astrophysical considerations a similar behaviour to the above sets of Skyrme type forces is observed, i.e. the values of χ_F/χ increase with k_f and then they decrease.

Tables V and VI show the values of $P_{\rm V}$ and P_{σ} against $k_{\rm f}$ at different temperatures for SK force and they reproduce the same feature as the corresponding ones obtained by the realistic forces of Friedmann and Pandharipande [4]. The results obtained here are reliable for low temperatures only but at higher temperatures one has to use the exact Fermi integrals and take into account the dependence of the Skyrme parameters on density, which depends on temperature.

In Tables VII and VIII we show the calculated values for Landau parameters F_0^s and F_1^s as a function of k_f . The results show a similar trend as those calculated by Nitsch [22] and Bäckman et al. [23] (see Ref. [22] for notations).

The sound velocity (in units of c) in neutron matter is given in Table IX as a function of ϱ along with the corresponding values calculated by Friedmann and Pandharipande [4]. The values obtained are very close to their results.

From the results obtained for the Skyrme force and its agreement with the realistic calculations we conclude that this interaction gives satisfactory results for polarized neutron matter over the range of $k_{\rm f} \leq 2 \, {\rm fm}^{-1}$. For higher $k_{\rm f}$ values one should add the rela-

The volume pressure in MeV fm⁻³ as a function of $k_{\rm f}$ at different temperatures; $P_{\rm F}({
m MeV~fm^{-3}})$

	T=0	0	T=3 MeV	MeV	T=6 MeV	MeV	T = 10 MeV) MeV
kf (fm ⁻¹)	Present work	Ref. [4]	Present work	Ref. [4]	Present work	Ref. [4]	Present work	Ref. [4]
0.6	0.01087	0.01065	0.02204	0.02202			. 1.	
0.7	0.02081	0.02115	0.03562	0.03427	0.05511	0.06091		
0.8	0.03619	0.03871	0.0541	0.05541	0.0911	0.09264		
6.0	0.05897	0.06331	0.0797	0.08465	0.1302	0.01355	0.1795	0.2199
1.0	0.09225	0.09506	0.1157	0.1187	0.1773	0.1799	0.2722	0.2891
1.1	0.14134	0.1402	0.1673	0.1667	0.2389	0.2392	0.3700	0.3761
1.2	0.21547	0.2101	0.244	0.2399	0.3247	0.3234	0.4861	0.49
1.3	0.33017	0.3287	0.3612	0.3606	0.4504	0.4524	0.6385	0.6443
1.4	0.51056	0.5375	0.5441	0.5710	0.6415	0.6688	0.8537	0.8824
1.5	0.79577	0.8973	0.8318	0.9353	0.9371	1.048	1.1718	1.298
1.6	1.24461	1.437	1.283	1.518	1.3962	1.639	1.6519	1.922
1.7	1.9426	2.408	1,9835	2.454	2.1043	2.59	2.3803	2.895
1.8	3,0106	3.570	3.0539	3.607	3.1823	3.749	3.4779	4.052
1.9	4.616	5.230	4.6617	5.276	4.7976	5.401	5.1123	5.72
0.0	6 9843	7 744	7.0324	7.788	7.1758	7.927	7.5093	8.297

TABLE VIII

TABLE VI The spin pressure in MeV fm⁻³ as a function of $k_{\rm f}$ at different temperatures; $P_{\rm o}({\rm MeV~fm^{-3}})$

$k_{\mathrm{f}}(\mathrm{fm}^{-1})$	T = 0	T=3 (MeV)
0.6	0.0469	0.0453
0.0	0.1079	0.1060
0.8	0.2224	0.2202
0.9	0.4207	0.4183
1.0	0.7439	0.7413
1.1	1.2444	1.2414
1.2	1.9880	1.9848
1.3	3.0544	3.0509
1.4	4.539	4.5352
1.5	6.5513	6.5473
1.6	9.218	9.2137
1.7	12.681	12.6764
1.8	17.097	17.0922
1.9	22.634	22.6289
2.0	29.476	29.4706

TABLE VII The interaction coefficient $F^s_{
m o}$ as a function of $k_{
m f}$

$k_{\rm f}$ (fm ⁻¹)	Present work	Ref. [22]	Ref. [23]
0.8	-0.684	-0.695	-0.271
1.0	-0.729	-0.716	-0.242
1.2	-0.712	-0.653	-0.189
1.6	-0.498	-0.379	0.00
2.0	-0.102	0.00	0.379

The interaction coefficient F_1^s as a function of k_f

$k_{\rm f}({\rm fm}^{-1})$	Present work	Ref. [22]	Ref. [23]
0.8	-0.063	-0.2	0.0
1.0	-0.123	-0.368	-0.091
1.2	-0.207	-0.526	-0.179
1.6	-0.447	-0.842	-0.389
2.0	-0.765	-1.158	-0.558

TABLE IX

ρ(fm ⁻³)	Present work	Ref. [4]	Ref. [4]
0.5	0.4463	0.32	0,53
1.0	0.7314	0.6	0.99
1.5	0.9062	0.9	1.02
2.0	1.015	1.00	1.03

The sound velocity v/c in neutron matter is a function of ρ

tivistic corrections and more care should be taken to study the magnetic properties at high densities.

In his original work Skyrme fixed the numerical values of the parameters by fitting the binding energy E/A and the density ϱ of nuclear matter and also binding energies and mass differences of some light nuclei calculated with oscillator wave functions. In the present work the parameters were adjusted to fit the neutron and nuclear matter calculations using realistic interactions which are in turn related to the phase shifts. An important test of an effective interaction is that it can account for (among other things) the ground-state binding energies of finite nuclei. In the present work we restrict ourselves to the double-closed-shell N=Z nuclei ¹⁶O and ⁴⁰Ca. For light nuclei the binding energy calculated with harmonic-oscillator wave functions is given by Skyrme's formula [10, 12] (d=1)

$$E = A \frac{\hbar^2}{2mb^2} + \frac{6}{(2\pi b^2)^{3/2}} \left(B_0 t_0 + B_1 t_1 \frac{3}{2b^2} + B_2 t_2 \frac{5}{2b^2} \right) + \frac{4B_3 t_3}{(\pi b^2 \sqrt{3})^3}, \tag{19}$$

where b denotes the oscillator parameter $(\hbar/m\omega)^{1/2}$.

Equation (19) is used to calculate the ground-state energies of two double-closed-shell nuclei ¹⁶O and ⁴⁰Ca in the harmonic oscillator basis without any further adjustment of the parameters t_0 , t_1 , t_2 and t_3 . The total energy is minimized as a function of the oscillator length b.

In the present work d=1/3 and hence the density dependence is different for the t_3 term. Here we have calculated the t_3 term in equation (19) by direct integration over the density using an equation similar to Eq. (15) of Ref. [12]. For convenience, we used the two-parameter Fermi form for the nuclear densities with the parameters given in Ref. [27]. The results of our calculations are shown in Table X. These results although calcu-

TABLE X Ground state energies in MeV. Comparison of the present work and empirical results. The empirical results and E_{Coul} are taken from Refs. [28] and [29]

Nucleus	E _{emp} . – E _{Coul}	$b=(h^2/m\omega)^{\frac{1}{2}}$	E Present work
¹⁶ O	139.5	1.7339	-139.3
⁴⁰ Ca	410.8	1.971	-411.3

lated using oscillator wave functions as single particle wave functions, are in good agreement with experimental values. The results of our calculations for the binding energies give the semiempirical mass formula for the binding energies of

$$E/A = -14.688 + 15.077 A^{-1/3}. (20)$$

The quality of the agreement obtained here raises in fact the question of the existence of a relation between the parameters of Skyrme interaction and those of realistic forces. We would like to emphasize that the Skyrme force parameters presented here are by no means a final one. A more detailed analysis using Skyrme forces is needed to obtain a better fit for a wider physical quantities for the nuclear and neutron matter as well as for finite nuclei.

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