GAUGE BREAKING REGULARIZATION OF THE NAMBU–JONA-LASINIO MODEL WITH VECTOR MESONS*

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Two regularizations of the Nambu–Jona-Lasinio model with vector mesons are compared: the proper-time method which preserves the gauge invariance of the quark loop term and a 4-momentum cutoff method which breaks gauge invariance. When meson masses are substituted for the coupling constants, the action has only a logarithmic dependence on the cutoff, and the two methods yield similar results. The binding of $\bar{q}q$ excitations is discussed as well as the importance of calculating on-shell meson masses rather than using a gradient expansion of the action. The vector mesons are expected to reduce significantly the modification of a nucleon in nuclear matter. A consistent way to compare the model predictions with experiment is proposed.

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1. Introduction

A low energy effective theory should yield observables which do not depend too much on the way high momenta are treated. At first sight, this does not appear to be the case when vector meson degrees of freedom are introduced into the Nambu–Jona-Lasinio model of hadronic matter. In this model, the vector fields are minimally coupled to quarks thereby making the quark loop gauge invariant. So far all the applications of this model have

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used gauge invariant regularization of the fermion determinant [9, 25, 26] such as, for example, the proper-time regularization of Schwinger [27].

When a gauge invariant regularization is used, the fermion determinant has only a logarithmic dependence on the cutoff. In other regularizations it develops a quadratic dependence.

Thus, if the model really required a gauge invariant regularization of the fermion determinant and if it would yield different results when, say, the momenta are sharply cut off at some value, it would be a model in which the high momenta play a crucial role in the dynamics. This contradicts our intuition of what a low energy effective theory should do. Our purpose is to show that the Nambu–Jona-Lasinio model does not require gauge invariant regularization and that it can be formulated in a way which does not depend critically on how the high momenta are cut off. The observed vector mesons are massive and only the a_1 and f_1 acquire mass by a Higgs mechanism. The ϱ and the ω do not. Phenomenology therefore does not compel us to use gauge invariant effective actions.

Before showing this explicitly it may be useful to state to this mixed audience what our aim is when we use such a model. As the French saying goes, *la plus belle fille du monde ne peut pas donner plus qu'elle a.* Let us explain what we are *not* attempting to do. We are obviously not attempting to formulate a theory of strongly interacting particles. We only calculate the mesons in order to fix the parameters of the model and to make sure that we have a correct description not only of the vacuum but also of its elementary excitation modes. We also calculate the baryons because we want to know how they get modified in dense baryonic matter. Indeed, we use the model with the aim of calculating hadronic matter in extreme conditions of high density or temperature and to investigate chiral symmetry restoration and the possible formation of exotic phases [1–6] at high density. So far lattice calculations have not helped much in this search. They have not been able so far to deal with dense matter. They indicate a phase transition at high temperature but, oddly, more effort appears to be spent on determining the (first or second) order of the phase transition than the nature of the other phase.

We are also not doing QCD. The latter may need a word of explanation. All too many papers begin with statements such as "we believe that QCD is the fundamental theory of strong interactions". The paper then usually goes on with its bag, constituent quark or whatever model and gluons are only introduced towards the end in order to account for whatever corrections are still needed. The insistence on the QCD relevance of low energy Lagrangians has probably done more harm than good. All too often a seemingly elaborate theory is formulated to express nothing more than a possibly modified form of gluon exchange. This is not to say that one should not attempt to make the link between QCD and low energy hadronic phenomena but one should not force the issue and claim *urbi et orbi* that one is doing QCD when, in fact, we simply do not know what the nature of the QCD ground state is.

It is as if we knew that metals are conductors but did not know the underlying crystalline structure of the solid. In such a situation, to invoke QED and to insist that the electrons in the metal interact with a possibly modified photon exchange would be misleading. Indeed the properties of the metal are dominated by the electron-phonon interaction which is due to the crystalline structure of the solid.

Ideas are needed, not statements. Several ideas have been put forward. Diakonov and Petrov [7] have suggested that the QCD vacuum is an instanton liquid. This has, so far, neither been verified nor disproved. Witten [8] has suggested that Skyrmion physics may be justified by the $1/N_c$ expansion. But he, nor anyone else, has never come close to deriving a Skyrme Lagrangian from QCD. Other models, such as the Nambu–Jona-Lasinio model, have no trouble in reproducing the correct N_c behavior of observables. Color dielectric models [11] have features taken from lattice QCD calculations. But whether they are relevant to low energy phenomena such as spontaneous chiral symmetry breaking is not known. Effective Lagrangians have also been derived by guessing that the two-point function dominates the dynamics [9, 10]. This leads to one-gluon exchange between quarks with a modified gluon propagator and it is not confirmed by experiment.

2. The Nambu–Jona-Lasinio action

The essential assumption of the Nambu–Jona-Lasinio model is that the low energy properties of the vacuum and of hadrons are governed by quark dynamics. (That quark dynamics determine the structure of hadrons is amply demonstrated by the successful spectroscopy of constituent quark models [12].) Dirac sea quarks cause the spontaneous breakdown of chiral symmetry in the physical vacuum as Nambu–Jona-Lasinio showed in their original paper [13]. The model attempts to reconcile chiral symmetry and constituent quark models.

When vector mesons are present, the Lagrangian can be written in the form:

$$\mathcal{L} = \overline{\psi}(-i\partial)\psi - \frac{1}{2a^2}\left(\left(\overline{\psi}\Gamma_a\psi\right)^2 - \left(\overline{\psi}\Gamma_a\gamma_5\psi\right)^2\right)$$

$$-\frac{1}{2b^2}\left(\left(\overline{\psi}\Gamma_a\gamma_\mu\psi\right)^2 + \left(\overline{\psi}\Gamma_a\gamma_\mu\gamma_5\psi\right)^2\right). \tag{2.1}$$

We are using a Euclidean metric in which $x_\mu = x^\mu = (\tau, \vec{r})$ and $\gamma_\mu = \gamma^\mu = (i\beta, \vec{\gamma})$ and we assume SU(2) quark flavors with $\Gamma_a \equiv (1, \vec{\tau})$. The Lagrangian (2.1) is equivalent to the form:

$$\mathcal{L} = \overline{\psi}(-i\partial + S + iP\gamma_5 + V + A\gamma_5)\psi + \frac{a^2}{2}\left(S_a^2 + P_a^2\right) + \frac{b^2}{2}\left(V_{\mu a}^2 + A_{\mu a}^2\right) , \quad (2.2)$$

which is written in terms of 8 scalar and pseudoscalar fields S_a and P_a and 8 vector and axial fields V_a and A_a :

$$S \equiv S_a \Gamma_a \;, P \equiv P_a \Gamma_a \;, V_\mu \equiv V_{\mu a} \Gamma_a \;, A_\mu \equiv A_{\mu a} \Gamma_a . \quad (2.3)$$

By integrating out the quark degrees of freedom we obtain an effective action in the form:

$$I = -\mathrm{Tr}\ln(-i\partial + S + iP\gamma_5 + V + A\gamma_5) + \int d^4x \left(\frac{a^2}{2}\left(S_a^2 + P_a^2\right) + \frac{b^2}{2}\left(V_{\mu a}^2 + A_{\mu a}^2\right) \right) . \quad (2.4)$$

The coupling of the fields S, P, V and A to the quarks allows us to determine their quantum numbers and to associate them to the observed mesons. The scalar and pseudoscalar fields are thus:

Field	Couples to	Isospin I	G-parity $G = e^{i\pi I_2} C$	Spin J	Parity P	Charge conj. C	$I^G\,(J^{PC})$	Meson
S^0	$\overline{\psi}\psi$	0	+	0	+	+	$0^+\,(0^{++})$	σ
$\vec{S}$	$\overline{\psi}\vec{\tau}\psi$	1	−	0	+	+	$1^-\,(0^{++})$	$\vec{a}_0$ *ex* $\vec{\delta}$ 983 MeV
P^0	$\overline{\psi}\gamma_5\psi$	0	+	0	−	+	$0^+\,(0^{-+})$	η 548.8 MeV 957.5 MeV
$\vec{P}$	$\overline{\psi}\gamma_5\vec{\tau}\psi$	1	−	0	−	+	$1^-\,(0^{-+})$	$\vec{\pi}$ 138 MeV

(2.5a)

The vector fields are:

Field	Couples to	Isospin I	G-parity $G = e^{i\pi I_2} C$	Spin J	Parity P	Charge conj. C	$I^G\,(J^{PC})$	Meson
V_μ^0	$\bar{\psi}\gamma_\mu\psi$	0	$-$	1	$-$	$-$	$0^-\,(0^{--})$	ω 783 MeV
$\vec{V}_\mu$	$\bar{\psi}\gamma_m u\vec{\tau}\psi$	1	$+$	1	$-$	$-$	$1^+\,(1^{--})$	$\vec{\rho}$ 770 MeV
A_μ^0	$\bar{\psi}\gamma_\mu\gamma_5\psi$	0	$+$	1	$+$	$+$	$0^+\,(1^{++})$	f_1 *ex* D 1283 MeV
$\vec{A}_\mu$	$\bar{\psi}\gamma_\mu\gamma_5\vec{\tau}\psi$	1	$-$	1	$+$	$+$	$1^-\,(1^{++})$	$\vec{a}_1$ *ex* $\vec{A}_1$ 1260 MeV

(2.5b)

The field P_0 cannot be properly identified with the observed η mesons unless strange quarks are included. It is only listed for completeness. Furthermore, the pion turns out to be a mixture of the $\vec{P}$ and $\vec{A}_\mu$ fields.

For the sake of simplicity, we assume exact chiral symmetry (neglecting the pion mass) as well as $U_5(1)$ symmetry (neglecting the U(1) anomaly). We also assume the O(4) symmetry of the vector fields [14] which makes the (ω, ϱ) and the (a_1, f_1) degenerate.

The action (2.4) can be regularized by introducing a cutoff Λ of the 4-momenta, thereby defining the trace in the quark loop term to be:

$$-\mathrm{Tr}\ln D \equiv -\sum_{k<\Lambda} \mathrm{tr}\langle k|\ln D|k\rangle\,, \qquad (2.6)$$

where $D \equiv -i\partial + S + iP\gamma_5 + V + A\gamma_5$ is the Dirac operator and where tr is the trace over the discrete Dirac, flavor and color variables. Such a regularization breaks the gauge invariance of the quark loop term.

In the popular proper-time regularization, the real part $(1/2)\mathrm{Tr}\,\ln D^\dagger D$ of the quark loop term is regularized introducing a cutoff Λ in the proper-time integral, there by defining the trace in the quark-loop term to be:

$$-\frac{1}{2}\mathrm{Tr}\,\ln D^\dagger D = \frac{1}{2}\mathrm{Tr}\int_{1/\Lambda^2}^{\infty}\frac{ds}{s}\exp[-sD^\dagger D]\,. \qquad (2.7)$$

This form preserves the gauge invariance of the quark loop term, whereas the form (2.6) with a sharp 4-momentum cutoff does not. The two regularizations will be compared in detail below.

The Lagrangian (2.1) represents, hopefully, a first approximation to quark dynamics in the QCD vacuum. When quarks propagate in the physical vacuum, whatever its structure is, they interact and the Nambu–Jona-Lasinio model assumes *faute de mieux*, a point interaction. Instead of viewing the Lagrangian (2.1) as low energy QCD, it may be more useful to view it in the same way as the Landau theory of Fermi liquids, in which an effective zero range interaction between particles is also assumed and in which the coupling constants are fitted to experiment. We will fit the 3 parameters of the Lagrangian to the pion decay constant and to the meson masses. The Lagrangian has remarkably few parameters and we shall see that the model has the appealing feature of clearly displaying its own limitations. They are due to its lack of confinement. Mesons and baryons, for example, will appear as bound (but not confined) $\bar{q}q$ or qqq states but not all such states turn up to be bound.

3. Meson propagators

The action (2.4) has the stationary point:

$$S_0 = M_0\,, \quad \vec{S} = 0\,, \quad P_a = 0\,, \quad V_{\mu a} = 0\,, A_{\mu a} = 0\,, \tag{3.1}$$

which is translationally invariant and which can represent the physical vacuum. In this vacuum only the (σ meson) field S_0 acquires a non-vanishing value M_0, a mass which we can determine from the condition that the action is stationary at that point. This is expressed by the so-called gap equation:

$$a^2 = \frac{2\nu}{(2\pi)^4} \int d^4k \frac{1}{k^2 + M_0^2}\,, \tag{3.2}$$

where ν is the spin-flavor-color degeneracy of the quark orbits. When u and d quarks are included, $\nu = 12$. In the physical vacuum, the quarks acquire a mass M_0 which we call the constituent quark mass.

The gap equation can be used to eliminate the constant a^2 in favor of the constituent quark mass M_0. By subtracting its value at the stationary point (3.1), the action (2.4) can be written in the form:

$$I = -\mathrm{Tr}\ln\left(1 + \frac{1}{-i\partial + M_0}(\tilde{S} + iP\gamma_5 + V + A\gamma_5)\right)$$

$$+\frac{1}{2}\mathrm{Tr}\frac{1}{i\partial_\mu^2 + M_0^2}(S_0^2 + \vec{P}^2 - M_0^2) + \frac{b^2}{2}\int d^4x\,\left(V_{\mu a}^2 + A_{\mu a}^2\right)\,, \tag{3.3}$$

where we defined:

$$S \equiv \tilde{S} + M_0 \ , \quad \tilde{S} = \tilde{S}_0 + \vec{S} \cdot \vec{\tau} \, . \tag{3.4}$$

If we assume that the mesons are low amplitude qq excitations of the vacuum, the meson propagators can be calculated by expanding the action to second order in the fluctuating parts of the fields around the stationary point [15, 16, 17]. The calculation is somewhat tedious but straightforward. It is found that the scalar fields give contributions of the form:

$$I = \frac{1}{2(2\pi)^4} \int d^4q \ \phi(q)\phi(-q)z(q)(q^2 + m^2(q)) \, , \tag{3.5}$$

where ϕ stands for the scalar or pseudo-scalar fields S or P and where:

$$\phi(q) \equiv \int d^4x \ e^{iqx} \phi(x) \, . \tag{3.6}$$

The functions $z(q)$ and $m(q)$ are given below in Table (3.10a).

The propagator of the field ϕ is diagonal in momentum space. It is equal to:

$$K(q) = \frac{1}{z(q)(q^2 + m^2(q))} \, , \tag{3.7}$$

and $m(q)$ is the meson mass operator or self-energy [15].

The vector fields give contributions of the form:

$$\begin{aligned} I &= \frac{1}{2(2\pi)^4} \int d^4q \ V_\mu(q)V_\alpha(-q)z(q) \left((\delta_{\mu\alpha}q^2 - q_\mu q_\alpha) + \delta_{\mu\alpha}m^2(q) \right) \\ &= \frac{1}{2(2\pi)^4} \int d^4q \ z(q)(\tfrac{1}{2}F_{\mu\alpha}(q)F_{\mu\alpha}(-q) + m^2(q)V_\mu(q)V_\mu(-q)) \, , \end{aligned} \tag{3.8}$$

where V stands for the vector or axial fields V or A and $F_{\mu\alpha} \equiv (\partial_\mu V_\alpha) - (\partial_\alpha V_\mu)$. The propagators $K_{\mu\nu}(q)$ of the vector fields are diagonal in momentum space and equal to:

$$K_{\mu\alpha}(q) = \frac{1}{[z(q)(q^2 + m^2(q))]} \left(\delta_{\mu\alpha} + \frac{q_\mu q_\alpha}{m^2(q)} \right) \, , \tag{3.9}$$

so that $m(q)$ in (3.9) is the vector meson mass operator or self-energy.

As in most models involving chiral and vector fields, the action mixes the P_a and $A_{\mu a}$ fields. The pion field turns out to be a linear combination of the two. It is made explicit in Ref.[17].

The meson mass operators $m(q)$ and the field strength renormalization constants $z(q)$ are given in the table below:

Meson	Field	$z(q)$	$m^2(q)$
σ	$\tilde{S}_0$	$\nu f(q)$	$4M_0^2$
$\vec{a}_0$	$\vec{S}$	$\nu f(q)$	$4M_0^2$
$\eta^\dagger$	P_0	$\nu f(q)\dfrac{b^2+\nu T(q)}{b^2+\nu T(q)+4M_0^2\nu f(q)}$	0
$\vec{\pi}$	$\vec{P}$	$\nu f(q)\dfrac{b^2+\nu T(q)}{b^2+\nu T(q)+4M_0^2\nu f(q)}$	0

(3.10a)

Meson	Field	$z(q)$	$m^2(q)$
ω	V_μ^0	$\nu S(q)$	$\dfrac{\nu T(q)+b^2}{\nu S(q)}$
$\vec{\rho}$	$\vec{V}_\mu$	$\nu S(q)$	$\dfrac{\nu T(q)+b^2}{\nu S(q)}$
f_1	B_μ^0	$\nu S(q)$	$\dfrac{b^2+\nu T(q)+4M_0^2\nu f(q)}{\nu S(q)}$
$\vec{a}_1$	$\vec{B}_\mu$	$\nu S(q)$	$\dfrac{b^2+\nu T(q)+4M_0^2\nu f(q)}{\nu S(q)}$

(3.10b)

More complete tables are listed in Refs [16, 17]. The functions $f(q)$, $S(q)$ and $T(q)$ are given in the Appendix.

The function $T(q)$ appearing in the vector meson mass operators is a gauge breaking contribution of the quark loop which is due to the gauge breaking regularization. Alone, it would yield a mass term (of the wrong sign) even in the absence of the explicit mass term proportional to b^2. When a gauge invariant proper-time regularization is used, the function $T(q)$ vanishes. With a sharp 4-momentum cutoff, it acquires the form (A3) given in the Appendix. It is negative and weakly (less than 10%) dependent on q in the allowed range of q values.

Tables (3.10) show that $T(q)$ can always be grouped together with the constant b^2 of the Lagrangian. This way, the gauge breaking terms of the quark loop can be lumped together with the explicit gauge breaking mass terms of the Lagrangian. The function $T(q)$ depends quadratically on the cutoff. However, when $b^2 + T(q)$ at $q^2 \simeq -m_\varrho^2$ is fitted to the ϱ meson mass m_ϱ as in Eq.(4.5) for example, the action becomes only logarithmically dependent on the cutoff. Thus we can replace the 4-fermion coupling constants $1/a^2$ and $1/b^2$ of the Lagrangian (2.1), respectively, by the σ and ϱ masses. The action is then only logarithmically dependent of the cutoff.

4. The pion decay constant and the constituent quark mass

The axial current $j_\mu^a(x)$ is the Noether current associated to a chiral rotation in which the fields undergo the following transformation:

$$
\begin{aligned}
S^0 \Rightarrow S^0 - \vec{\alpha}\cdot\vec{P}\,, \quad \vec{P} \Rightarrow \vec{P} + \vec{\alpha}S^0\,,\\
\vec{S} \Rightarrow \vec{S} - \vec{\alpha}P^0\,, \quad P^0 \Rightarrow P^0 + \vec{\alpha}\cdot\vec{S}\,,\\
V_\mu^0 \Rightarrow V_\mu^0\,, \quad A_\mu^0 \Rightarrow A_\mu^0\,, \quad \vec{V}_\mu \Rightarrow \vec{V}_\mu + \vec{\alpha}\times\vec{A}_\mu\,,\\
\vec{A}_\mu \Rightarrow \vec{A}_\mu + \vec{\alpha}\times\vec{V}_\mu + \tfrac{1}{2}(\partial_\mu\vec{\alpha})\,.
\end{aligned}
\tag{4.1}
$$

Substituting into the action (3.3), we obtain the axial current:

$$
j_\mu^a(x) \equiv \frac{\partial I}{\partial(\partial_\mu \alpha(x))}
$$

$$
= \frac{1}{2(2\pi)^4}\sum_q (-iq_\mu)P_a(q)\mathrm{e}^{-iqx}M_0\nu f(q)\frac{b^2+\nu T(q)}{b^2+\nu T(q)+4M_0^2\nu f(q)}\,, \tag{4.2}
$$

where only the linear terms in the pion field are retained. The matrix element of the axial current between the vacuum and a one-pion state normalized in a volume Ω is:

$$
\langle 0|j_\mu^a(x)|\vec{q},a\rangle = -iq_\mu\frac{\mathrm{e}^{iqx}}{\sqrt{2\omega_q\Omega}}\frac{\nu f(0)M_0}{\sqrt{Z_\pi}}\frac{b^2+\nu T(0)}{b^2+\nu T(0)+4M_0^2\nu f(0)}
$$

$$
\equiv -iq_\mu\frac{\mathrm{e}^{iqx}}{\sqrt{2\omega_q\Omega}}f_\pi\,, \tag{4.3}
$$

where f_π is the pion decay constant, measured to be $f_\pi = 93$ MeV and where $Z_\pi \equiv z(q=0)$ can be read off Table (3.10a). Since we have zero mass pions, the pion decay constant turns out to be:

$$
f_\pi = M_0\sqrt{\nu f(0)}\sqrt{\frac{b^2+\nu T(0)}{b^2\nu T(0)+4M_0^2\nu f(0)}}\,. \tag{4.4}
$$

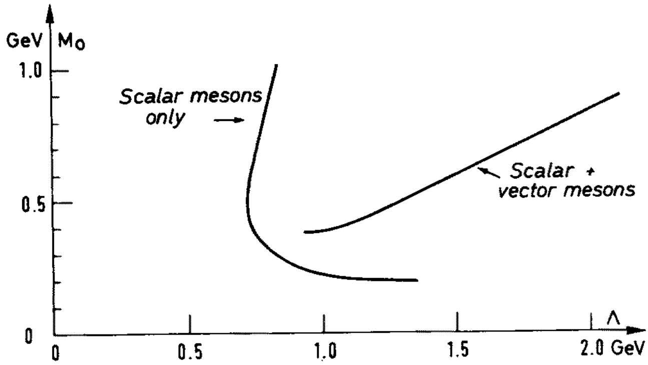

Fig. 1. The relation between the constituent quark mass M_0 and the cutoff Λ obtained by fitting the pion decay constant ($f_\pi = 93$ MeV) and the ϱ mass ($m_\varrho = 770$ MeV). The calculation was made using a sharp 4-momentum cutoff. The two curves show the results obtained with and without vector mesons. When vector mesons are not included, only f_π is fitted.

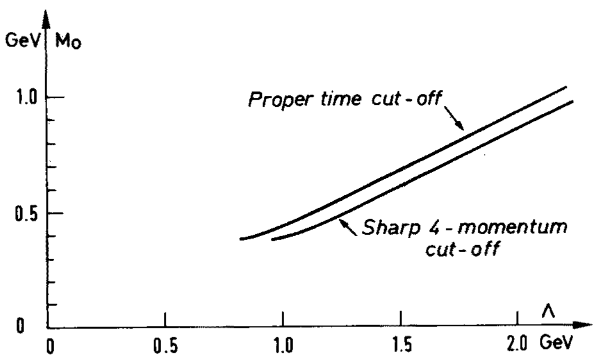

Fig. 2. The relation between the constituent quark mass M_0 and the cutoff Λ obtained with proper time and 4-momentum cutoff regularizations.

As shown in the next Section, the ϱ-meson mass is given by the expression:

$$m_\varrho^2 = \frac{b^2 + \nu T(q^2 = -m_\varrho^2)}{\nu S(q^2 = -m_\varrho^2)} . \tag{4.5}$$

We can solve Eq. (4.5) for b^2 and substitute its value into Eq. (4.4). When the experimental values $f_\pi = 93$ MeV and $m_\varrho = 770$ MeV are used, the expression (4.4) becomes a relation between the cutoff Λ (used to evaluate the functions f, S and T) and the constituent quark mass M_0. This relation is depicted in Fig. 1. We see that vector mesons completely modify the relation between Λ and M_0. This is why the model predicts very different results when vector mesons are included. The effect is due to the mixing of the $A_{\mu a}$ and $(\partial_\mu P_a)$ fields. Fig. 2 compares the relations between Λ and M_0 obtained with proper -time and 4-momentum cutoff regularizations. To obtain a constituent quark mass $M_0 = 0.5$ GeV for example, a 10 % larger cutoff is required with the 4-momentum regularization. This can be understood by the fact that the proper time regularization cuts off momenta larger than Λ in a smooth way so that it includes some higher momenta.

5. Bound and unbound q̄q excitations. Critique of Skyrme-like local Lagrangians

On-shell mesons are poles of the propagators (3.7) and (3.9). They occur for negative values of q^2 which are the solutions of the equation:

$$q^2 + m^2(q) = 0\,. \tag{5.1}$$

The squared mass operators $m^2(q)$ are listed in Tables (3.10). The mass of an on-shell meson is $m(q)$ where q is a solution of (5.1).

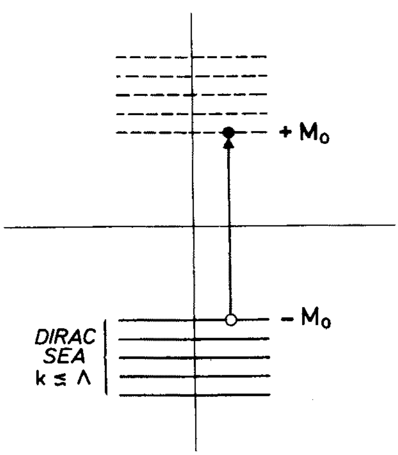

Fig. 3. The physical vacuum is a Dirac sea composed of quarks filling the negative energy plane wave states with momentum $k \leq \Lambda$. The positive energy excited states are empty. Free q̄q excitations of the vacuum occur at energies greater than the mass gap $2M_0$.

In the Nambu–Jona-Lasinio model, the vacuum is a Dirac sea made up of quarks which have mass M_0 (Fig. 3). If, for a given meson, Eq.(5.1) has a solution such that $-q^2 > 4M_0^2$ then the meson will have a mass larger than $2M_0$ and it can decay into free $\bar{q}q$ excitations shown in Fig. 3. The pole of the propagator represents then an **unbound** $\bar{q}q$ excitation and not a meson particle. Table (3.10a) shows that the σ meson has an on-shell mass exactly equal to $2M_0$ so that it occurs just at the $\bar{q}q$ threshold. Table (3.10b) shows that:

$$m_{a_1}^2(q) = m_\varrho^2(q) + 4M_0^2 \frac{f(q)}{S(q)} . \tag{5.2}$$

The expressions in the Appendix show that $f(q) > S(q)$ so that:

$$m_{a_1}^2 > 4M_0^2 . \tag{5.3}$$

Thus, in the model described by the action (2.4), it is not possible to form a bound $\bar{q}q$ excitation with the quantum numbers of the a_1 meson. The same holds for the f_1.

Fig. 4 shows the ϱ and a_1 mass operators plotted against $-q^2$. The curves cannot be continued to the region where $-q^2 > 4M_0^2$ because the mesons become then unbound. Although no bound state is obtained for a_1, we see that it is **almost** bound. It is tempting to extrapolate (dotted line) the a_1 mass operator to the point where it intersects the $-q^2$ line and where Eq.(5.1) would be satisfied. We see that for a constituent quark mass $M_0 = 0.55$ GeV the a_1 meson is almost bound at its observed mass of 1260 MeV.

The lack of a bound states for the a_1 is due to the lack of confinement of the Nambu–Jona-Lasinio model. Free quarks of finite mass M_0 do not exist, of course, no more than free $\bar{q}q$ excitations of mass $2M_0$. The model is clearly showing its limitations. Whether the model will be successful in binding baryons is not yet known.

When gradient expansions are used to generate Skyrme-like effective actions, only off-shell $q = 0$ meson masses are obtained. Figure 4 shows that this would overestimate the squared ϱ and a_1 masses by 40–50 %.

This is yet another example of the inadequacy of gradient expansions applied to low-energy hadronic physics. Such was the impact of Skyrmion physics during the past decade that even the Nambu–Jona-Lasinio model was often used to generate local meson Lagrangians using gradient expansions. The effective Lagrangians obtained this way were compared to the Skyrme Lagrangian and observables such as meson masses and decay rates were calculated from them. This way of proceeding kills one of the main virtues of the Nambu–Jona-Lasinio model, namely its ability to treat correctly quark dynamics. Furthermore, gradient expansions become ambiguous when axial-vector mesons mix with gradients of the pion field. When

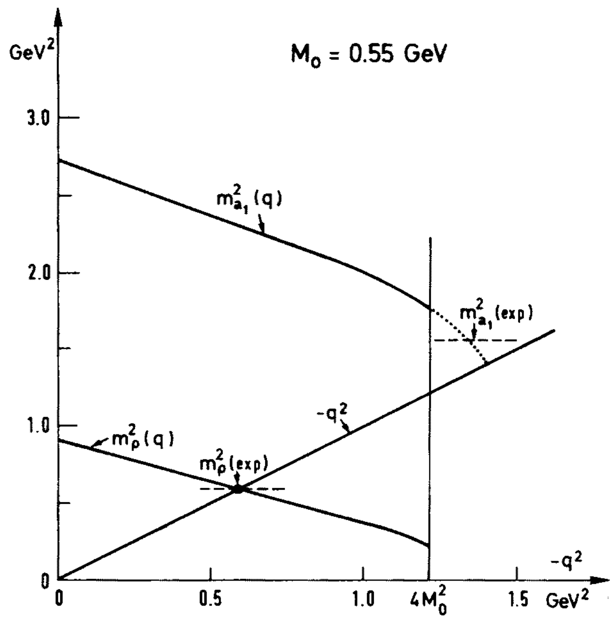

Fig. 4. The a_1 and ϱ mass operators are plotted against $-q^2$. The point at which they cross the $-q^2$ line gives the poles (5.1) corresponding to on-shell mesons. The curves stop when $-q^2 > 4M_0^2$ beyond which the mesons become unbound. The dotted line is a qualitative extrapolation of the a_1 mass operator. Horizontal dashed lines are drawn at the position of the observed a_1 and ϱ meson masses. The calculation was performed with a sharp 4-momentum cutoff.

an effective local meson Lagrangian is derived from a quark Lagrangian one tends to forget the finite constituent quark mass M_0 which can cause certain hadrons to be unbound. Local effective meson Lagrangians need to be corrected by Wess–Zumino terms which mimic the quark dynamics in an unnecessarily complicated way. The spin and statistics of solitons are obvious when quark degrees of freedom are taken explicitly into account.

Quark models are richer, not in their mathematical properties, but in the physics they describe. Low energy hadron dynamics is probably not the most useful place to apply the rich topological properties of Skyrme-like actions.

6. Partial chiral symmetry restoration in dense baryonic matter

For a translationally invariant system, the energy per unit volume is given by the expression:

$$\frac{E}{\Omega} = \frac{I}{\beta\Omega}, \tag{6.1}$$

where I is the Euclidean action, Ω the volume of the system and β the (imaginary) time interval: $\int d^4x = \beta\Omega$. Let us use the action (3.3) to study the energy as a function of the scalar field $S_0 \equiv M$, keeping the other fields zero.

When the action (3.3) is regularized with a sharp 4-momentum cutoff Λ, the energy per unit volume of the vacuum (or the Dirac sea) is found to be:

$$\frac{E_D(M))}{\Omega} = \frac{\nu\pi^2}{(2\pi)^4} \int_0^{\Lambda^2} y\,dy \left(\ln\left(\frac{y+M^2}{y+M_0^2}\right) + (M^2 - M_0^2)\frac{1}{y+M_0^2} \right). \tag{6.2}$$

When the proper-time regularization is used, the energy per unit volume is:

$$\frac{E_D(M)}{\Omega} = \frac{\nu\pi^2}{(2\pi)^4} \int_{1/\Lambda^2}^{\infty} \frac{ds}{s^3} \left(e^{-sM^2} - e^{-sM_0^2} + s(M^2 - M_0^2)e^{-sM_0^2} \right). \tag{6.3}$$

In the expressions above, we have assumed that all fields except the scalar field $S_0 \equiv M$ are zero.

Figure 5 compares expressions (6.2) and (6.3) for a typical constituent quark mass $M_0 = 0.5$ GeV. The corresponding cutoffs can be read off Fig. 2. Much to our satisfaction, the two curves are very similar, thus illustrating our contention that observables which depend only logarithmically on the cutoff do not depend significantly on the method used to cutoff the high momenta.

The vacuum energy per unit volume is a very sensitive function of the constituent quark mass M_0. It is roughly proportional to M_0^4. This is illustrated in Fig. 6. As the constituent quark mass M_0 increases, the chirally broken phase of the vacuum becomes more rigid: it costs more and more energy to deviate from the vacuum value $M = M_0$. For example, the energy per unit volume of the chirally restored phase ($M = 0$), which is the MIT bag constant, increases from 200 MeV/fm^3 to 500 MeV/fm^3 when the constituent quark mass M_0 increases from 0.4 GeV to 0.5 GeV.

The rigidity of the chirally broken phase is an important parameter which determines the modification of the nucleon immersed in baryonic matter. We can represent baryonic matter by a Fermi sea of nucleons, filling positive energy plane wave states with momenta $k < k_F$ [6], where, at normal density, $k_F = 1.36$ fm^{-1} = 0.268 GeV. The contribution of the

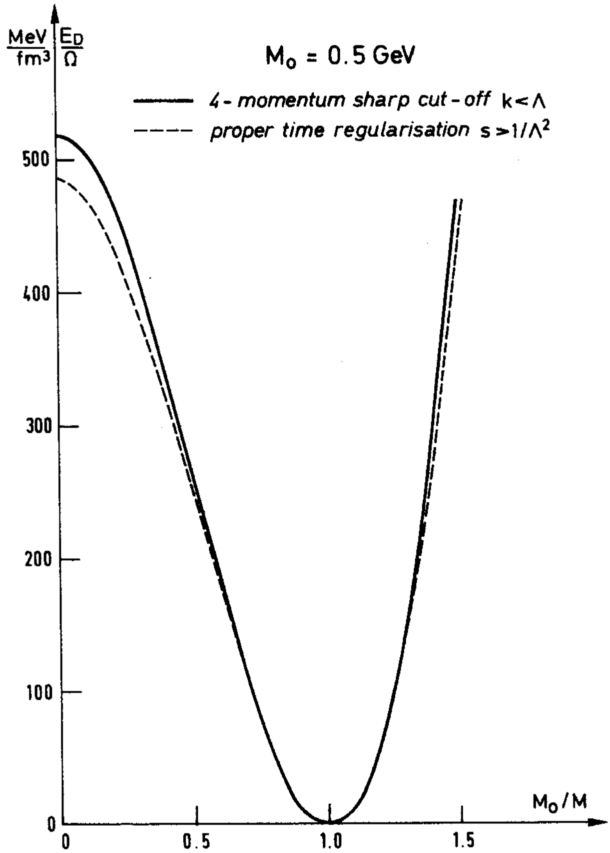

Fig. 5. The Dirac sea energy per unit volume is plotted against the scalar field strength $S_0 \equiv M_0$. The full line is obtained with a sharp 4-momentum cut off and the dashed line with a proper time regularization.

Fermi sea nucleons to the energy per unit volume is:

$$\frac{E_F}{\Omega} = \frac{\nu_N}{(2\pi)^3} 4\pi \int_0^{k_F} k^2 dk \sqrt{k^2 + g^2 M^2}\,, \qquad (6.4)$$

where $\nu_N = 4$ is the spin–isospin degeneracy of the nucleons. The nucleons have a structure so that they couple to the scalar field with a coupling constant g. In the vacuum where the scalar field M acquires the equilibrium value M_0, the nucleons acquire their observed mass $M_N = 989$ MeV so that the coupling constant is given by $g = M_N/M_0$. Using this value the Fermi

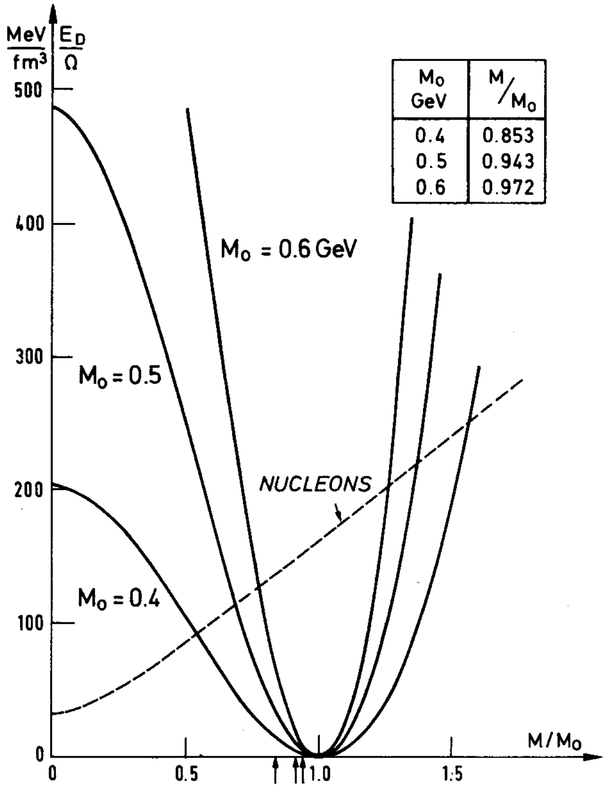

Fig. 6. The vacuum (Dirac sea) energy per unit volume is plotted against the ratio M/M_0 for constituent quark masses $M_0 = 0.4 - -0.6$ GeV. The dashed line gives the contribution (6.5) of Fermi sea nucleons (see Fig. 7). The insert gives the values of M/M_0 at which the Dirac + Fermi sea energy is minimum. These values are further indicated on the abscissa by little arrows. The calculation was done with a sharp 4-momentum cutoff.

sea energy (6.4) becomes:

$$\frac{E_F}{\Omega} = \frac{\nu_N}{(2\pi)^3} 4\pi \int_0^{k_F} k^2 dk \sqrt{k^2 + M_N^2 \frac{M^2}{M_0^2}} \,. \tag{6.5}$$

In several applications [4] nuclear matter is represented by a Fermi sea of quarks rather than of nucleons. A Fermi sea of quarks gives the following contribution to the energy per volume:

$$\frac{E_F}{\Omega} = \frac{\nu}{(2\pi)^3} 4\pi \int_0^{k_F} k^2 dk \sqrt{k^2 + M^2} \,, \tag{6.6}$$

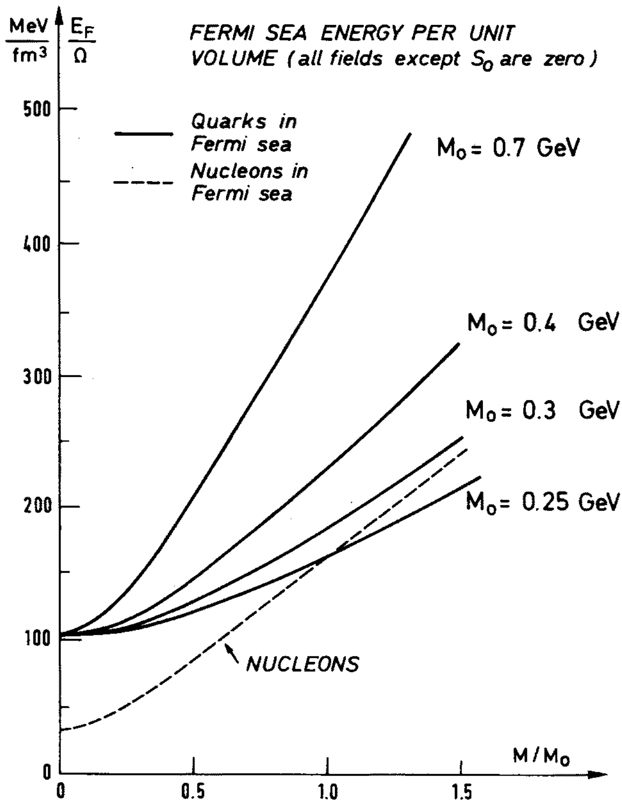

Fig. 7. The Fermi sea contribution to the energy per unit volume. The full curves are the quark Fermi sea contributions for various constituent quark masses. The dashed curve gives the nucleon Fermi sea contribution.

where $\nu = 12$ is the quark spin, flavor, color degeneracy.

The nucleon and quark Fermi sea contributions (6.5) and (6.6) are compared in Fig. 7. When the constituent quark mass is about 0.4 GeV, the quark and nucleon contributions are represented by approximately parallel curves and they produce, therefore, similar shift of M/M_0 at a given baryonic density.

A Fermi sea composed of plane wave states couples only to the scalar field $S_0 \equiv M$ and to the $\mu = 0$ component of the vector field $V_{0,\mu=0} \equiv \Phi$. In nuclear matter at normal density the fields differ little from their vacuum values and the changes in M or Φ produced by the Fermi sea are independent. The change in the scalar field M is obtained by minimizing the total (Dirac + Fermi) sea energies. This is illustrated in Fig. 8. An approximately 5 % reduction of the ratio M/M_0 is obtained for a constituent quark mass $M_0 = 0.5$ GeV. The insert in Fig. 6 gives the values of the ratio

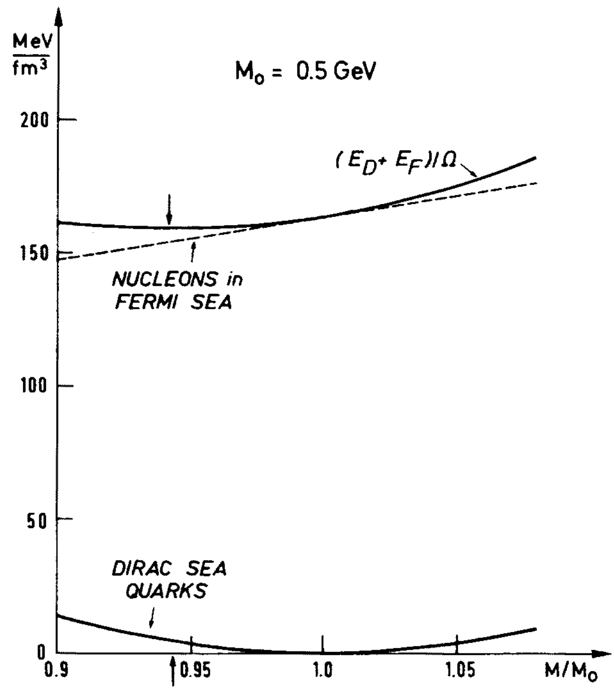

Fig. 8. This is a blow up of a part of Fig. 6 in the region close to the energy minimum. The bottom curve gives the Dirac sea contribution to the energy per unit volume. The dashed curve gives the contribution of nucleons in the Fermi sea. The upper curve is the sum of the Dirac+Fermi sea contributions. Its minimum is shifted by 5.6 little arrow. The calculation is made with a constituent quark mass $M_0 = 0.5$ GeV and a sharp 4-momentum cutoff.

M/M_0 for different constituent quark masses.

The presence of vector mesons significantly reduces the shift in M/M_0. This is because they stiffen the chirally broken phase as illustrated in Fig. 9. The Table below gives the values of M/M_0 obtained at normal density for various constituent quark masses and it compares the values obtained with nucleons and quarks in the Fermi sea:

M_0	chiral field only (nucleons)	chiral + vector fields (nucleons)	chiral field only (quarks)	chiral + vector fields (quarks)
0.4	0.734	0.853	0.701	0.831
0.5	0.850	0.943	0.768	0.912
0.6	0.900	0.972	0.810	0.947
0.8	0.945	0.991	0.860	0.976

(6.7)

Table (6.7) shows how much vector mesons reduce the shift of M/M_0.

The shifts are still strongly dependent on M_0. At low values such as 0.4 GeV the ϱ and ω mesons are barely bound so that higher values of the order of 0.5–0.6 GeV may be preferred. At such values M/M_0 is shifted by about 5 % which is close to a recent estimate [18] but smaller than the value reported last summer in Sao Paolo [19] where vector mesons were not included. In either case we are very far from the value $M/M_0 \approx 0.6$ required by Walecka's model of nuclear matter saturation [20].

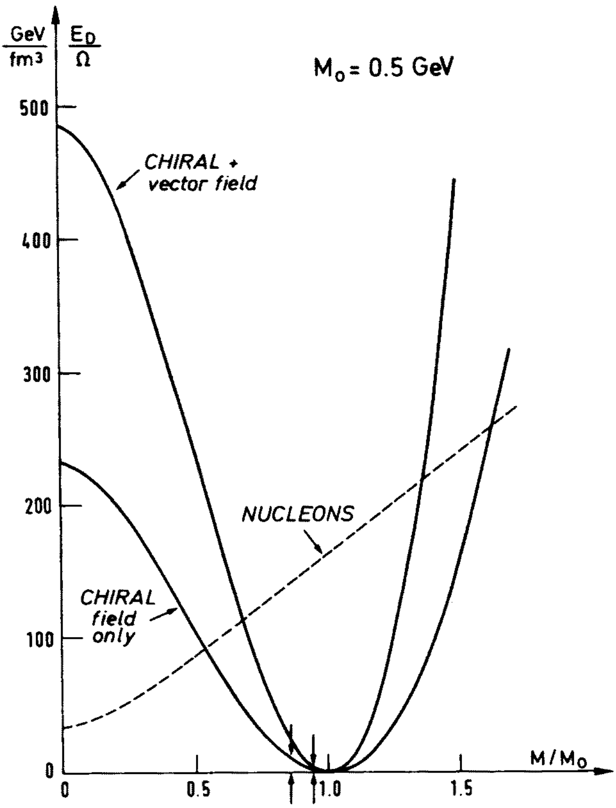

Fig. 9. The vacuum energy per unit volume calculated with and without vector mesons. The shift in the ratio M/M_0 produced by nucleons in the Fermi sea (dashed line) is reduced from 0.85 to 0.94 when vector mesons are presented. The calculation was made with a sharp 4-momentum cutoff.

The shift in M/M_0 produced by the presence of baryons in the Fermi sea can be related to the change in size of the nucleon. In chiral models such as the Skyrmion or the σ model, the size of the soliton varies as the inverse of M/M_0. In the Nambu–Jona-Lasinio model this is not strictly true because the size also depends on the ratio Λ/M_0. Recent calculations of this effect [21, 22] give contradictory results.

7. Conclusion

So far we have only considered quantities which have a logarithmic dependence on the cutoff. We have seen that they do not depend significantly on the way the high momenta are cut off so long as observables such as meson masses and the pion decay constant are used to fix the parameters of the model. This is, however, not the case for quantities which depend quadratically on the cutoff such as the quark condensate $\langle\overline{\psi}\psi\rangle$ or the inverse coupling constants a^2 and b^2 for example. The current quark mass, fitted to the observed pion mass, is another example not considered in this lecture. In lattice calculations, such quantities have abnormal dimensions [24]. The Table below compares such quantities calculated with a sharp 4-momentum cutoff and with a proper time regularization. The values quoted are obtained with a constituent quark mass $M_0 = 0.5$ GeV:

$M_0 = 0.5$ GeV	sharp 4-momentum cutoff	proper time regularization
a^2 [(GeV)^2]	0.17	0.11
b^2 [(GeV)^2]	0.165	0.109
$b^2 + \nu T(0)$ [(GeV)^2]	0.057	0.109
$(b^2 + \nu T(0))/a^2$	0.33	0.98
$\langle\bar{u}u\rangle^{1/3}$ [MeV]	−221	−304

(7.1)

If one assumes a one-gluon exchange potential between the quarks, a Fierz transformation shows that the coupling constants of the Lagrangian (2.2) are in the ratio $b^2/a^2 = 2$ which is far from the values displayed in Table (7.1).

The significant differences appearing in Table (7.1) show that quantities, which have a quadratic dependence on the cutoff, are more dependent of the high momenta than quantities which have only a logarithmic dependence. This is expected and it does not make much sense to fit both kinds of quantities indiscriminately as is often done. If one wishes to fit both kinds of quantities, an extra parameter should be added to the model which can modify the profile function used to cut off the high momenta [23, 24]. This can be done both for the gauge breaking momentum cutoff method or for the proper time method which preserves the gauge invariance of the quark loop. Two regularization methods can then be compared by fitting one quantity, for example the quark condensate, which depends quadratically on the cutoff.

We see that it is very important in phenomenology to distinguish quantities which have a logarithmic and quadratic dependences on the cutoff. Otherwise comparisons of values obtained for meson masses and quark condensates, for example, are meaningless.

APPENDIX

When a sharp 4-momentum cutoff Λ is used, the functions defined in Section 3 are:

$$f(q) \equiv \frac{1}{(2\pi)^4} \int_{k<\Lambda} d^4k \frac{1}{\left(\left(k-\frac{q}{2}\right)^2+\phi^2\right)\left(\left(k+\frac{q}{2}\right)^2+\phi^2\right)}$$

$$= \int_{-1}^{1} \frac{du\pi^2}{2(2\pi)^4} \int_0^{\Lambda^2} \frac{y\,dy}{\left(y+\phi^2+\frac{q^2}{4}(1-u^2)\right)^2}, \tag{A.1}$$

$$S(q) \equiv \int_{-1}^{1} \frac{du\pi^2}{2(2\pi)^4} \int_0^{\Lambda^2} \frac{(1-u^2)y\,dy}{\left(k^2+\phi^2+\frac{q^2}{4}(1-u^2)\right)^2}, \tag{A.2}$$

$$T(q) = \int_{-1}^{1} \frac{du}{2(2\pi)^4} \int_{-1}^{1} \frac{du}{2(2\pi)^4} \int_{k<\Lambda} d^4k \left(\frac{k^2+q^2u^2}{\left(k^2+\phi^2+\frac{q^2}{4}(1-u^2)\right)^2} - \frac{2}{k^2+\phi^2}\right). \tag{A.3}$$

When a sharp cutoff is used in the proper-time regularization, the functions are:

$$f(q) = \frac{1}{(2\pi)^4} \int d^4k \int_{1/\Lambda^2}^{\infty} s\,ds \int_0^1 dx \exp\left[-sx\left((k-\tfrac{1}{2}q)^2+\phi^2\right)\right.$$
$$\left.- s(1-x)\left((k+\tfrac{1}{2}q)^2+\phi^2\right)\right]$$
$$= \frac{\pi^2}{(2\pi)^4} \int_{-1}^{1} \frac{du}{2} \int_{1/\Lambda^2}^{\infty} \frac{ds}{s} \exp\left[-s\left(\phi^2+\frac{q^2}{4}(1-u^2)\right)\right], \tag{A.4}$$

$$S(q) = \frac{\pi^2}{(2\pi)^4} \int_{-1}^{1} \frac{du}{2} \int_{1/\Lambda^2}^{\infty} \frac{ds}{s} \exp\left[-s\left(\phi^2 + \frac{q^2}{4}(1-u^2)\right)\right](1-u^2), \quad (A.5)$$

$$T(q) = 0. \quad (A.6)$$

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