WARMING UP A NUCLEON*

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(Received September 18, 1990)

The problem of temperature dependence of nucleon mass is addressed by considering a retarded correlator of two currents with quantum numbers of a nucleon at finite temperature $T \ll F_\pi$ in the chiral limit. It is shown that at Euclidean momenta the leading one-loop corrections arise from direct interaction of thermal pions with the currents. A dispersive representation for the correlator shows that this interaction smears the nucleon pole over frequency interval with width $\sim T$. This interaction does not change the exponential fall-off of the correlator in Euclidean space but gives an $O(T^2/F_\pi^2)$ contribution to the pre-exponential factor.

PACS numbers: 12.40.-y

The recent interest in finite temperature theories is connected with the expected phase transition from hadrons to quark–gluon plasma at a critical temperature which ranges from 100 to 300 MeV according to different estimates. It is a common belief that at higher temperatures in spite of additional infra-red divergences coming from gluonic zero-modes the interaction between quarks and gluons becomes weak due to asymptotic freedom and perturbation theory can be used at least in low enough orders [1]. At low temperatures hadronic phase was investigated by Leutwyler and his co-workers by using effective chiral Lagrangians [2]. They obtained low temperature expansions for thermodynamic properties of hadronic gas such as pressure and energy density and also for the quark condensate which is given by a derivative of the energy density with respect to the quark mass.

It is also interesting to ask what happens to a particle mass at finite temperature. However, at $T \neq 0$ there is no unique definition of mass. *E.g.* one can define the mass either as the position of pole in the propagator or as the inverse correlation length at large space-like distances. These

* Presented at the XXX Cracow School of Theoretical Physics, Zakopane, Poland, June 2-12, 1990.

definitions do not coincide even for a free Fermi particle due to non-zero lowest Matsubara frequency. For a discussion of different definitions of mass at $T \neq 0$ see Ref. [3].

In this paper we are interested in a finite temperature nucleon. Intuitively one would expect that the nucleon melts with rising T and its mass decreases. Indeed, as is seen, *e.g.* from the zero-temperature QCD sum rules for the nucleon [4], its mass is due to spontaneous chiral symmetry breaking, $m_{\rm N} \sim \langle\bar{\psi}\psi\rangle^{1/3}$, and according to Gasser and Leutwyler [2] temperature effects reduce the quark condensate, $\langle\bar{\psi}\psi\rangle_T = \langle\bar{\psi}\psi\rangle_0(1-T^2/8F_\pi^2)$. This is supported by calculations in the chiral soliton model [5], which give nucleon mass decreasing with rising temperature. Here we shall approach the problem of temperature dependence of nucleon mass by considering the following finite temperature correlator[1]

$$C_{\rm R}(\omega,\vec{p}) = -i\int d^4x e^{ipx}\vartheta(x^0)\,\langle\{\eta(x),\bar{\eta}(0)\}\rangle_T\,. \tag{1}$$

We use the retarded correlator since it is just retarded (or advanced) Green functions that are analytic at $T \neq 0$ and have spectral representations [8]. Here $\eta(x)$ is an external current with quantum numbers of a nucleon whose explicit form is unessential. For definiteness we can take a protonic current [4] $\eta = \varepsilon^{abc}(u^aC\gamma_\mu u^b)\gamma_5\gamma_\mu d^c$, where C is the charge conjugation matrix. In Eq. (1) $\langle\ldots\rangle_T$ denotes the Gibbs average $\langle A\rangle_T = {\rm Tr}(Ae^{-H/T})/\,{\rm Tr}\,e^{-H/T}$, where the trace is over the full set of eigenstates of the Hamiltonian. Due to Lorentz invariance breaking at $T \neq 0$, $C_{\rm R}$ depends on two variables ω and $\vec{p}$. The function $C_{\rm R}(\omega,\vec{p})$ is analytic in the upper half-plane of complex ω and has a spectral representation

$$C_{\rm R}(\omega,\vec{p}) = \frac{1}{\pi}\int_{-\infty}^{\infty} d\omega'\frac{{\rm Im}\,C_{\rm R}(\omega',\vec{p})}{\omega'-\omega-i\varepsilon}\,. \tag{2}$$

At points $\omega = \omega_n = (2n+1)\pi iT$, $C_{\rm R}$ coincides with the correlator calculated in imaginary time approach. We are interested in Euclidean region $\omega^2 - \vec{p}^{\,2} < 0$. For simplicity we shall put $\vec{p} = 0$ and $\omega = ip_0$.

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[1] Such a correlator was used in Ref. [6] to write hot QCD sum rules for the nucleon. It was also investigated recently in Ref. [7] within the chiral Lagrangian approach with an emphasis on its behavior in the vicinity of the pole $p^2 = m^2$. It was shown that in the chiral limit temperature corrections do not shift the pole, but just renormalize the residue at this pole. On the contrary, here we are interested in behavior of the correlator at Euclidean momenta. Below we discuss relation of our results with those of Ref. [7] in some detail.

At low temperatures only interaction with thermal pions must be taken into account since the effects of massive particles are exponentially suppressed. To this end one can use the chiral pion–nucleon Lagrangian (see, *e.g.* [9])

$$L^{\pi N} = -g_A \bar{\psi}\gamma_\mu\gamma_5 \vec{\tau}\vec{\varphi}_\mu \psi - \frac{1}{4F_\pi^2}\bar{\psi}\gamma_\mu \vec{\tau}\,[\vec{\varphi}, \vec{\varphi}_\mu]\psi \,, \tag{3}$$

where ψ and φ are nucleon and pion fields, $\vec{\varphi}_\mu \equiv \partial_\mu \vec{\varphi}/(1+\vec{\varphi}^2/4F_\pi^2)$, $F_\pi \cong 93$ MeV is the pion decay constant, $g_A \cong 1.27$. We shall consider pion as massless. The correlator $C_{\rm R}$ at large Euclidean times is saturated by the lowest state, the proton, see Fig. 1. At the one-loop level interaction with the thermal pion gas is given by the diagrams of Fig. 2. Diagrams 2a and 2b correspond to direct interaction of pions with the current. Dashes on the pion lines denote thermal pions. Since $T \ll m$, the diagrams with protonic temperature insertions are suppressed by a factor $\exp(-m/T)$ and are unessential. The tree graph gives at $\omega = ip_0$

$$C_{\rm R}^{(1)} = -\frac{\lambda^2(ip_0\gamma_0 + m)}{p_0^2 + m^2} \,. \tag{4}$$

Here λ is the residue defined by the matrix element $\langle 0|\,\eta\,|{\rm N}\rangle = \lambda v_{\rm N}$, where $v_{\rm N}$ is the nucleon spinor and m is its mass.

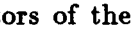

Fig. 1. Tree contribution to the correlator.

In calculating Diagrams 2a, 2b and 2d one must know matrix elements $\langle 0|\,\eta\,|{\rm N}\pi\rangle$ and $\langle 0|\,\eta\,|{\rm N}\pi\pi\rangle$. They are estimated using PCAC and are expressed in terms of commutators of the current η with the axial charge $Q_5^a = (i/\sqrt{2})\int \bar{q}\gamma_0\gamma_5\tau^a q\; d^3x$

$$\langle 0|\eta|{\rm N}\pi^a\rangle = \frac{i}{F_\pi}\langle 0|[\eta, Q_5^a]|{\rm N}\rangle = \frac{i\lambda}{2F_\pi}\gamma_5\tau^a v_{\rm N}\,,$$
$$\langle 0|\eta|{\rm N}\pi^a\pi^b\rangle = -\frac{1}{F_\pi^2}\langle 0|\,\big[[\eta, Q_5^a], Q_5^b\big]\,|{\rm N}\rangle = -\frac{\lambda\delta^{ab}}{4F_\pi^2}v_{\rm N}\,. \tag{5}$$

A rigorous formulation of real time perturbation theory at $T \neq 0$ is given by the two-component technique. We are not going to describe it here (for details see, *e.g.* Refs [10]), but just note that in this technique propagators are 2×2 matrices with off-diagonal components being non-zero

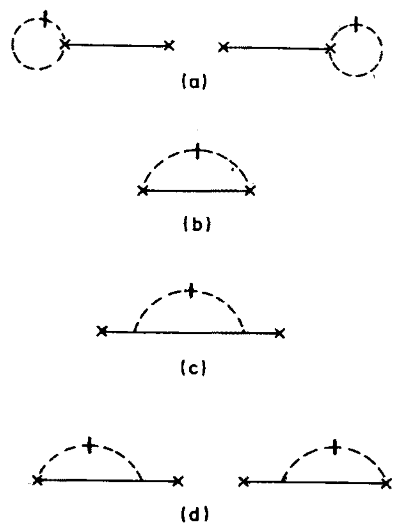

Fig. 2. One-loop corrections to the correlator. Dashed lines correspond to thermal pions.

only on mass-shell and proportional to statistical Bose or Fermi factors. Of four propagator components only two combinations are independent which can be chosen to be the retarded (or advanced) Green function having the same form as at $T = 0$, and the distribution of the particle in the heat bath. The retarded correlator $C_{\rm R}$ is expressed through these functions.

The contribution of Diagram 2a is obtained from the tree graph multiplying it by the factor $\int \frac{d^4k}{(2\pi)^3} \frac{1}{\exp(|k_0|/T)-1}\delta(k^2) = \frac{1}{12}T^2$ corresponding to the closed thermal pion loop and using the second matrix element from Eqs (5)

$$C_{\rm R}^{(2a)} = \frac{T^2}{16F_\pi^2} \frac{\lambda^2(ip_0\gamma_0 + m)}{p_0^2 + m^2} . \tag{6}$$

For Diagram 2b using Eqs (5) we have

$$C_{\rm R}^{(2b)} = \frac{3\lambda^2}{4F_\pi^2} \int \frac{d^4k}{(2\pi)^4} \gamma_5 G_{\rm R}(p+k)\gamma_5 \,{\rm Im}\, D_{\rm R}(k)\big(\mathrm{cth}(k_0/2T) - \mathrm{sgn}(k_0)\big) . \tag{7}$$

Substituting here ${\rm Im}\, D_{\rm R}(k) = -\pi\delta(k^2)\,\mathrm{sgn}(k_0)$ and $G_{\rm R}(k) = (\hat{k} + m) \times \big(k^2 - m^2 + i\varepsilon\mathrm{sgn}(k_0)\big)^{-1}$ we obtain at $\omega = ip_0$, $\vec{p} = 0$

$$C_{\rm R}^{(2b)} = \frac{3\lambda^2}{2F_\pi^2}\int \frac{d^3k}{(2\pi)^3 2k}\,\frac{1}{\exp(k/T)-1}$$
$$\times\,\frac{(m-ip_0\gamma_0)(p_0^2+m^2)-2ip_0\gamma_0 k^2}{(p_0^2+m^2)^2+4p_0^2k^2}$$
$$= -\frac{T^2}{16F_\pi^2}\,\frac{i\lambda^2(ip_0\gamma_0-m)}{p_0^2+m^2}+O(T^4)\,. \tag{8}$$

Similarly for Diagram 2c we have

$$C_{\rm R}^{(2c)} = \frac{3\lambda^2 g_A^2}{2F_\pi^2}\,\frac{p_0^2}{p_0^2+m^2}\,ip_0\gamma_0\int \frac{d^3k}{(2\pi)^3 2k}\,\frac{1}{\exp(k/T)-1}$$
$$\times\,\frac{k^2}{(p_0^2+m^2)^2+4p_0^2k^2}\,. \tag{9}$$

Due to an extra factor of k^2 in the numerator $C_{\rm R}^{(2c)} = O(T^4)$. It is easy to show that Diagram 2d is also of order T^4.

Let us compare our results with those of Ref. [7]. In that paper it was shown that interaction of a nucleon with thermal pion gas does not shift the physical pole in Minkowski space, but contributes to the residue at this pole. In this region contrary to Euclidean point which we consider here only diagrams 2a and 2c are relevant. Diagram 2b was not considered in Ref. [7] since it has no pole at $\omega^2 = m^2$. Indeed, it is easy to see that its real part vanishes at the pole as $(\omega^2-m^2)\ln|\omega^2-m^2|$.

On the other hand as shown above at Euclidean point $\omega = ip_0$, $\vec{p}=0$ the leading $O(T^2)$ correction comes from Diagram 2b as well as from Diagram 2a. Diagrams 2c and 2d contribute to order $O(T^4)$. Besides, from Eq. (8) it is seen that the contribution of Diagram 2b is of pole type (with the pole at $p_0 = \pm im$). This seeming paradox in fact disappears if one looks at the corresponding imaginary part $\mathrm{Im}\,C_{\rm R}^{(2b)}$. Indeed, from Eq. (7) one easily gets the following expression which has transparent physical meaning

$$\mathrm{Im}\,C_{\rm R}^{(2b)} = \frac{6\lambda^2\pi^4}{F_\pi^2}\int \frac{d^3k}{(2\pi)^3 2k}\,\frac{d^3k'}{(2\pi)^3 2E'}\,\delta^{(3)}(\vec{k}-\vec{k}')\,\frac{1}{\exp(k/T)-1}$$
$$\times\,\Big\{(m-\gamma_0(\omega-k))\left[\delta(\omega-k-E')-\delta(\omega-k+E')\right]$$
$$+\,(m-\gamma_0(\omega+k))\left[\delta(\omega+k-E')-\delta(\omega+k+E')\right]\Big\}\,, \tag{10}$$

where $E' = (m^2 + k'^2)^{1/2}$. Integrating over k', we obtain

$$\mathrm{Im}\, C_{\mathrm{R}}^{(2b)} = \frac{3\lambda^2}{16\pi F_\pi^2}\left(m - \gamma_0 \frac{\omega^2 + m^2}{2\omega}\right)$$
$$\times \left\{ \frac{(\omega^2 - m^2)/2\omega^2}{\exp((\omega^2 - m^2)/2|\omega|T) - 1} (\vartheta(\omega - m) - \vartheta(-\omega - m)) \right.$$
$$\left. + \frac{(m^2 - \omega^2)/2\omega^2}{\exp((m^2 - \omega^2)/2|\omega|T) - 1} (\vartheta(m - \omega)\vartheta(\omega) - \vartheta(m + \omega)\vartheta(-\omega)) \right\}. \quad (11)$$

It is seen that $\mathrm{Im}\, C_{\mathrm{R}}^{(2b)}(\omega, 0)$ at $T \ll m$ is exponentially small everywhere except a narrow region of width $\sim T$ around the points $\omega = \pm m$. At these points $\mathrm{Im}\, C_{\mathrm{R}}^{(2b)} = (3\lambda^2 T/16\pi F^2)(\pm 1 - \gamma_0)$. $\mathrm{Im}\, C_{\mathrm{R}}^{(2b)}(\omega, 0)$ is depicted in Fig. 3. Substituting Eq. (11) into the dispersive integral of Eq. (2) it is easy at $T \ll m$ to reproduce Eq. (8).

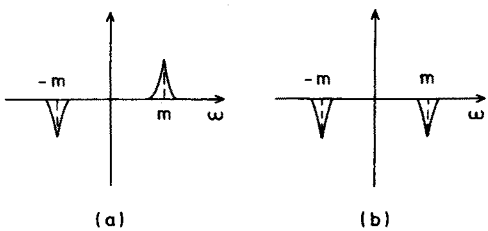

Fig. 3. Imaginary part of diagram 2b: (a) structure ~ 1 and (b) structure $\sim \gamma_0$.

Thus, we see that at $T \neq 0$ there is a contribution to $\mathrm{Im}\, C_{\mathrm{R}}$ due to Diagram 2b. At $\omega^2 > m^2$ it corresponds to a process in which production of a nucleon by the current is accompanied by induced production of a pion in the presence of thermal pion gas. At $\omega^2 < m^2$ it describes inelastic scattering of a thermal pion from the heat bath on the current with production of a nucleon. This last process is analogous to the mechanism of Landau damping in ordinary plasma. The two processes result in smearing of the nucleon pole at $\omega = \pm m$ over the region of width $\sim T \ll m$. This explains why the contribution of Diagram 2b to $C_{\mathrm{R}}(ip_0, 0)$ has a pole form.

In the same way considering the self-energy insertion in the Diagram 2c one sees that due to an extra factor of k^2 in the numerator (see Eq. (8)) its spectral density though being smeared over the region of width $\sim T$ is now of order $\sim T^3$. So, from Eq. (2) its contribution to Diagram 2c in Euclidean region is $\sim T^4$. The other part of $\mathrm{Im}\, C_{\mathrm{R}}^{(2c)}$ which corresponds to one of the nucleons being on-shell is proportional to the real part of the self-energy insertion which was calculated in Ref. [7] and is of order T^2. However, the corresponding integrand in Eq. (2) is odd in ω in this case and there is no contribution to $C_R(ip_0, 0)$.

Vanishing of $O(T^2)$ terms from Diagrams 2c and 2d in Euclidean region can also be seen if one considers their behavior at large Euclidean time. To this end it is necessary to take into account the nearest singularities in the Fourier integral over p_0. Consider for example Diagram 2c. In the upper half-plane of complex p_0 the corresponding expression in Eq. (9) has three poles $\omega = ip_0$, $\omega = i(\sqrt{m^2 + k^2} \pm k)$. Evaluating their contributions and integrating over k afterwards it is easy to see that though each pole contributes to order T^2 in the integral $\int dp_0 \exp(ip_0 t) C_R(\omega = ip_0)$, however the contribution of the first pole is canceled by the sum of contributions from the second and the third ones. It is worth mentioning that only Euclidean times $t \ll 1/T$ make sense here, because Euclidean correlator is (anti)periodic in t with a period of $2\pi T$. Since $T \ll m$, this means that one can really go as far as $t \sim 1/m$.

Thus, we have calculated $O(T^2)$ corrections to the retarded correlator of nucleonic currents at $T \ll m$ in Euclidean region. These corrections have a pole form though they are contributed by a non-pole diagram which vanishes at the physical pole $\omega^2 = m^2$. This is due to the fact that at $T \neq 0$ there is a cut along the whole real axis in ω-plane. At this cut the correlator C_R has an imaginary part which at $T \ll m$ is effectively non-zero only in the vicinity of the physical poles $\omega = \pm m$. The obtained corrections do not change the factor $\exp(-mt)$ describing the correlator fall-off at large Euclidean time and contribute only to pre-exponential factors.

In conclusion an important note is in order. Due to the current algebra relations (5) the thermal corrections to the correlator (1) at $T^2 \ll F_\pi^2$ are expressed through the zero-temperature correlators $\langle 0| \eta\bar{\eta} |0\rangle$ and $\langle 0| \gamma_5\eta\bar{\eta}\gamma_5 |0\rangle$ multiplied by factors of order T^2/F_π^2. In saturating these correlators the parity partner of the nucleon must also come into play. The problem will thus become two-channel. The same happens in the case of correlators of vector and axial currents. Presumably, the temperature-induced mixing of parity partners will result in chiral symmetry restoration with formation of parity doublets as suggested in Refs [11] on the basis of lattice calculations. This possibility is now being under consideration and the results will be published elsewhere.

Useful discussions with A.A. Abrikosov (Jr.), M. Dey, B.L. Ioffe, H. Leutwyler and A.V. Smilga are gratefully acknowledged. I am indebted to H. Leutwyler and A.V. Smilga for showing me the manuscript of Ref. [7] prior to publication.

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