QUARKS IN THE INSTANTON MEDIUM

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A systematic theory of quarks in the instanton medium is developed beyond the framework of the zero-mode approximation. At large number of colours a closed equation for the quark propagator is derived. The chiral symmetry is shown to be spontaneously broken. The effective quark mass and the quark condensate are calculated.

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1. Introduction

Since the discovery of the instanton solution [1] of the Yang-Mills equations, the instantons in QCD continue to attract theorists. In particular, one should mention the work of Diakonov and Petrov who, starting from the variational approach to the instanton vacuum in the pure gluon theory [2], suggested a theory of light quarks in the instanton vacuum [3,4], explained the mechanism of the spontaneous breakdown of chiral symmetry, derived the effective chiral Lagrangian and proposed a chiral theory of nucleons [5].

In these papers the field of the instanton medium was represented as the sum of the fields of separate pseudoparticles (instantons and anti-instantons)

$$A_\mu(x) = \sum_I A_{I\mu}(x). \tag{1.1}$$

The fundamental parameters of the instanton medium are the average size of pseudoparticles $\bar{\varrho}$ and their density N/V, where N is the total number of pseudoparticles ($N/2$ instantons and $N/2$ anti-instantons) in the Euclidean

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four dimensional volume V. These parameters have been calculated in QCD without quarks by means of the variational method [2]:

$$N/V = (200\,\mathrm{MeV})^4, \tag{1.2}$$

$$\bar{\varrho} = (600\,\mathrm{MeV})^{-1}, \tag{1.3}$$

and agree with the phenomenological analysis.

Let us now place quarks into the instanton medium (1.1). It is well known that the influence of quarks on the gluon sector of QCD vanishes at large number of colours N_c. Hence, one can use "quenched approximation", treating instanton medium as a given classical background field. In this approximation the quark propagator $\bar{S}$ is

$$\bar{S} = \left\langle (i\not{\partial} + im + \sum_I \not{A}_I)^{-1} \right\rangle. \tag{1.4}$$

Here the brackets denote averaging over positions, sizes and orientations of all pseudoparticles. Naturally, such an averaging is a difficult problem. Taking into account the diluteness of the instanton medium with the parameters (1.2), (1.3) one can neglect the correlation of different pseudoparticles and can average over all pseudoparticles in Eq. (1.4) independently. Another approximation used in papers [3,4] was the limit of large number of colours N_c.

It is well known that the massless Dirac operator in the background field of one instanton A_I has a zero mode ψ_I:

$$(i\not{\partial} + \not{A}_I)\psi_I = 0. \tag{1.5}$$

In the instanton medium (1.1) zero modes of separate pseudoparticles mix up and delocalize. The mechanism of spontaneous breakdown of chiral symmetry suggested by Diakonov and Petrov [3,4] is based on this phenomenon. Taking into account the importance of zero modes these authors approximated the quark propagator in the background field of a single instanton by the sum of the free propagator and of zero-mode contribution

$$\langle x|(i\not{\partial} + \not{A}_I + im)^{-1}|y\rangle \cong \langle x|(i\not{\partial} + im)^{-1}|y\rangle + \frac{\psi_I(x)\psi_I^\dagger(y)}{im}. \tag{1.6}$$

This approximation can be regarded as an interpolation between the high momentum regime where the free propagator dominates, and the low-momentum domain where the zero-mode contribution is the most essential for the light quarks due to the singular factor m^{-1}.

The zero-mode approximation (1.6) simplifies considerably the calculation of the quark propagator (1.4) and of the correlation functions of quark currents in the instanton vacuum. The results are in good agreement with the experiment. However, this approximation faces certain problems. One of them is the breakdown of the current conservation by the zero-mode approximation.

This list of the approximations used by Diakonov and Petrov to construct the theory of light quarks in the instanton vacuum [3, 4] is rather large:

1) quenched approximation (1.4),
2) neglection of the instanton correlations,
3) large N_c limit,
4) zero-mode approximation (1.6).

A natural desire arises to construct a more precise theory. As for direct computer calculations beyond the assumptions 1)–3) one should mention paper [6].

The aim of the present work is to exceed the limits of the zero-mode approximation preserving, however, assumptions 1)–3). As mentioned above, the zero mode approximation breaks the conservation of vector and axial currents. As a consequence one faces an ambiguity in extracting the pion axial constant F_π from the correlation functions of quark currents. The approach we developed restores the current conservation. It also helps to understand the status of the zero-mode approximation. A small parameter is found and the zero-mode approximation will be shown to reproduce correctly the first order of the perturbation theory in this small parameter.

2. Equation for the quark propagator in the instanton medium

Let us construct a diagram technique for the quark propagator in the instanton medium. In the Euclidean QCD the quark propagator can be represented as the functional integral

$$\left\langle \psi(x)\psi^\dagger(y)\right\rangle = Z^{-1}\int DA\int D\psi\int D\psi^\dagger$$
$$\times \exp\left[\int d^4z\psi^\dagger(i\partial\!\!\!/ + A\!\!\!/ + \ im)\psi + S(A)\right], \tag{2.1}$$

where $S(A)$ involves both the Yang–Mills action and the gauge fixing terms. Integrating over the quark fields $\psi, \psi^\dagger$ one obtains

$$\left\langle \psi(x)\psi^\dagger(y)\right\rangle = Z^{-1}\int DA\ \exp\ S(A)\ \det\,(i\partial\!\!\!/ + A\!\!\!/_I + im)$$

$$\times \left\langle x|(-i\partial\!\!\!/ - A\!\!\!/ - im)^{-1}|y\right\rangle . \tag{2.2}$$

It is well known that at large number of colours N_c the planar graphs without quark loops are dominating. Hence, in the limit $N_c \to \infty$ one can neglect the determinant of the Dirac operator in Eq. (2.2)

$$\left\langle\psi(x)\psi^\dagger(y)\right\rangle \overset{N_c\to\infty}{=} Z^{-1} \int DA \exp S(A)$$

$$\times \left\langle x|(-i\partial\!\!\!/ - A\!\!\!/ - im)^{-1}|y\right\rangle . \tag{2.3}$$

In the instanton vacuum model an assumption is made that instead of performing the full functional integral over gluon fields in Eq. (2.3) one can use averaging over collective coordinates of instantons. The quark propagators $\bar{S}$ in the instanton vacuum (1.1) can be written then in the form (1.4). The technique of calculating averages of this type was developed by Diakonov and Petrov [3,4]. One expands the inverse Dirac operator as a power series in $A\!\!\!/_I$

$$-(i\partial\!\!\!/ + \sum_I A\!\!\!/_I + im)^{-1} = \sum_{n=0}^{\infty} \sum_{I_1 \ldots I_n} S_0 A\!\!\!/_{I_1} S_0 \ldots A\!\!\!/_{I_n} S_0 , \tag{2.4}$$

where S_0 is the free propagator

$$S_0 = -(i\partial\!\!\!/ + im)^{-1} . \tag{2.5}$$

Generally, the neighbour pseudoparticles are different in Eq. (2.4): $I_1 \neq I_2, I_2 \neq I_3, \ldots$ but in some terms there may appear insertions of a certain repeated pseudoparticle $I : \ldots S_0 A\!\!\!/_I S_0 A\!\!\!/_I S_0 \ldots$. Such insertions can be summed up

$$S_0 A\!\!\!/_I S_0 + S_0 A\!\!\!/_I S_0 A\!\!\!/_I S_0 + \cdots = S_I - S_0 , \tag{2.6}$$

where

$$S_I = -(i\partial\!\!\!/ + A\!\!\!/_I + im)^{-1} \tag{2.7}$$

is the quark propagator in the background field of a single pseudoparticle.

The partial summation (2.6) allows one to rewrite the expansion (2.4) in the form

$$-(i\partial\!\!\!/ + \sum_I A\!\!\!/_I + im)^{-1} = S_0 + \sum_I (S_I - S_0)$$

$$+ \sum_{I\neq J} (S_I - S_0) S_0^{-1} (S_J - S_0)$$

$$+ \sum_{I\neq J,\, J\neq K} (S_I - S_0) S_0^{-1} (S_J - S_0) S_0^{-1} (S_K - S_0) + \cdots . \tag{2.8}$$

Now we have to average Eq. (2.8) over collective coordinates of the instanton medium. This problem can be simplified drastically if we neglect the correlation of different pseudoparticles. It can be justified by the diluteness of the instanton medium (see Eqs (1.2), (1.3)). We get from Eq. (2.8)

$$
\begin{aligned}
-\left\langle \left(i\partial\!\!\!/ + \sum_I A\!\!\!/_I + im\right)^{-1} \right\rangle &= S_0 + \sum_I \langle S_I - S_0 \rangle \\
&+ \sum_{I \neq J} \langle S_I - S_0 \rangle \, S_0^{-1} \, \langle S_J - S_0 \rangle \\
&+ \sum_{I \neq J \neq K \neq I} \langle S_I - S_0 \rangle \, S_0^{-1} \, \langle S_J - S_0 \rangle \, S_0^{-1} \, \langle S_K - S_0 \rangle \\
&+ \sum_{I \neq J} \left\langle (S_I - S_0) \, S_0^{-1} \, \langle S_J - S_0 \rangle_J \, S_0^{-1} (S_I - S_0) \right\rangle_I + \cdots . \qquad (2.9)
\end{aligned}
$$

Here it is taken into account that two cases are possible for three pseudoparticles $I, J, K : I \neq K$ or $I = K$. Only the neighbour pseudoparticles in the rhs of (2.8) fall under restriction $I \neq J$, $J \neq K, \ldots$. Therefore, in some terms of the expansion (2.8) certain pseudoparticles need special care. Though we neglect the correlation of different pseudoparticles one has to take into account the correlation induced by recurrence of some pseudoparticles.

A diagram technique can be developed for the expansion (2.9) as shown in Fig. 1. A circle with I inside denotes $S_I - S_0$ and a solid line represents S_0^{-1}. The circles of the same pseudoparticle are connected with a dashed line (or with a bunch of dashed lines if the pseudoparticle appears more than twice).

Let us now study the diagram expansion of Fig. 1 at large number of colours N_c. The situation is quite similar to that in the perturbative multicoloured QCD. The diagrams surviving at large N_c make possible a simple description. A graph for the quark propagator in the instanton medium will be called planar if the dashed lines corresponding to different pseudoparticles do not intersect. Among the graphs of Fig. 1 only the last one is non-planar.

Just as in the perturbative QCD, in the instanton vacuum at large N_c the planar graphs dominate. To check this statement one has to take into account the following sources of N_c. First, the density of the instanton medium N/V is of order N_c. Therefore, if a graph contains k different pseudoparticles the summation over them gives a factor N_c^k.

Besides that, certain powers of N_c arise after averaging over SU(N_c) orientation of instantons.

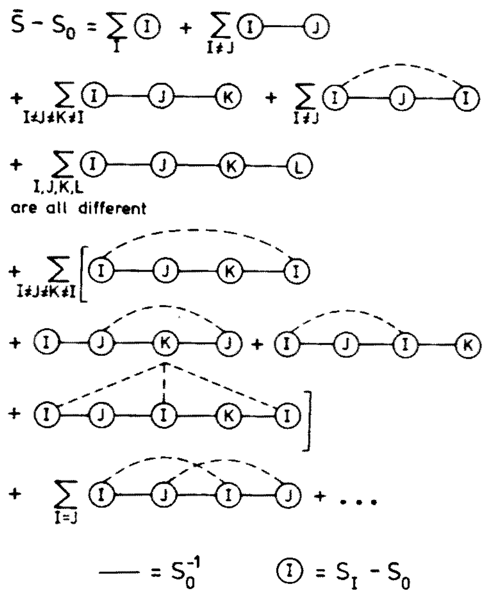

Fig. 1. Graphs for the quark propagator.

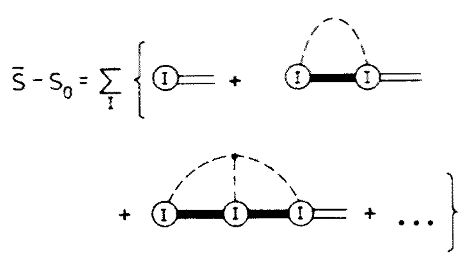

Fig. 2. Skeleton expansion for the quark propagator.

Now our task is to sum up all the planar graphs for the quark propagator. Although this cannot be done directly, it is possible to write down a closed equation for the quark propagator with all planar graphs taken into account. First of all, let us construct a skeleton planar equation. To this end we fix the first left pseudoparticle in each planar graph. Generally speaking, this pseudoparticle may appear in the same graph more than once. If one sums over all other pseudoparticles of the graph one will obtain a skeleton graph with bold lines between the circles of the fixed pseudoparticle (Fig. 2). A diagram expansion can be written for the bold lines (Fig. 3). If one compares it with the diagram expansion for the quark propagator $\bar{S}$ (Fig. 1) one can see that the bold line is equal to $S_0^{-1}\bar{S}S_0^{-1}$. Here we have subtracted S_0^{-1}. because of the summation condition $I \neq J \neq K \neq \cdots$ for the neighbour pseudoparticles in Eq. (2.8).

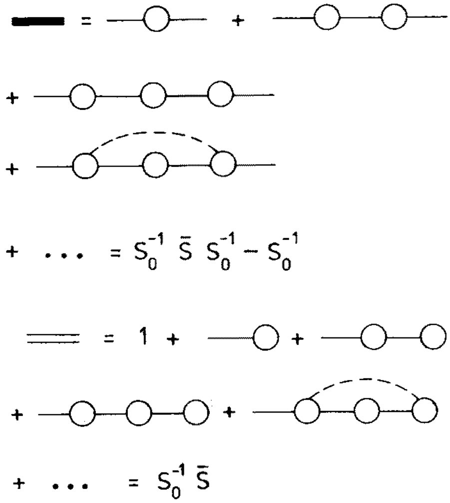

Fig. 3. Diagram expansion of skeleton lines.

Next, we insert expressions of the skeleton lines in terms of $\bar{S}$ into the skeleton expansion for $\bar{S}$ (Fig. 3). We get

$$\begin{aligned}\bar{S} - S_0 =& \sum_I \langle\{(S_I - S_0) \\ &+ (S_I - S_0)(S_0^{-1}\bar{S}S_0^{-1} - S_0^{-1})(S_I - S_0) + \cdots\} S_0^{-1}\bar{S}\rangle \\ =& \sum_I \langle\{\bar{S}^{-1}(S_0^{-1} - S_I^{-1})^{-1}S_I^{-1} - (S_0^{-1} - \bar{S}^{-1})\}^{-1}\rangle . \qquad (2.10)\end{aligned}$$

Here according to Eqs (2.5), (2.7)

$$S_I^{-1} = S_0^{-1} - \not{A}_I. \qquad (2.11)$$

Therefore, we can rewrite Eq. (2.10) in the form

$$\bar{S}^{-1} - S_0^{-1} = \sum_I \left\langle(\bar{S} - \not{A}_I^{-1})^{-1}\right\rangle . \qquad (2.12)$$

Let us suppose that there are N pseudoparticles ($N/2$ instantons and $N/2$ anti-instantons) in the four-dimensional Euclidean volume V. It is clear that all the $N/2$ instantons give the same contribution to the sum in the rhs of (2.12). Hence

$$\bar{S}^{-1} - S_0^{-1} = N/2\left\langle(\bar{S} - \not{A}_I^{-1})^{-1}\right\rangle + N/2\left\langle(\bar{S} - \not{A}_{\bar{I}}^{-1})^{-1}\right\rangle . \qquad (2.13)$$

Hereafter we denote the instanton field A_I, and $A_{\bar{I}}$ is an anti-instanton. It is essential that in both terms of the rhs of (2.13) only one pseudoparticle is involved in averaging, but not the whole instanton medium as in the original expression (1.4). In Eq. (2.13) the operator $(\bar{S} - \not{A}_I^{-1})^{-1}$ should be averaged over the instanton position z_I, its orientation matrix U_I and its size ϱ_I. Averaging over z_I implies

$$\langle \cdots \rangle z_I = V^{-1} \int d^4 z_I \cdots . \tag{2.14}$$

The instanton orientation in the colour space is characterized by a $\mathrm{SU}(N_c)$ matrix U_I

$$A_I \to U_I A_I U_I^+ , \tag{2.15}$$

and averaging over the orientation reduces to the integration with Haar measure

$$\langle U_I(\cdots)U_I^\dagger \rangle_{U_I} = \int dU_I U_I(\cdots)U_I^\dagger = \frac{1}{N_c}\mathrm{Tr}_{\mathrm{colour}}(\cdots), \tag{2.16}$$

where $\mathrm{Tr}_{\mathrm{colour}}$ is the trace over colour indices.

As to the instanton size ϱ_I we shall put it equal to the average size $\bar{\varrho}$ (1.3). According to Ref. [2] the size distribution tends to $\delta(\varrho - \bar{\varrho})$ at large N_c). Taking into account Eqs (2.14), (2.16) we obtain from Eq. (2.13) the equation for the quark propagator

$$\bar{S}^{-1} - S_0^{-1} = \frac{N}{2VN_c} \cdot \mathrm{Tr}_{\mathrm{colour}}\{ \int \mathrm{d}^4 \mathrm{z_I} (\bar{\mathrm{S}} - \not{A}_{\mathrm{I}}^{-1})^{-1} + \int d^4 z_{\bar{I}} (\bar{S} - \not{A}_{\bar{I}}^{-1})^{-1} \}. \tag{2.17}$$

3. The quark propagator at small N/VN_c

Eq. (2.17) for the quark propagator is a complicated nonlinear operator equation. It can not be solved analytically. But this equation contains the parameter N/VN_c. It should be reminded that in the instanton vacuum at large N_c $N/V = O(N_c)$, so N/VN_c is stable in the limit of large N_c. In the real world N/VN_c is numerically small. Indeed, in the chiral limit $m = 0$ we can use only the size of instantons $\bar{\varrho}$ to build up a dimensionless ratio out of N/VN_c. Using the values (1.2), (1.3) for N/V and $\bar{\varrho}$ one obtains

$$\bar{\varrho}^4 N/VN_c \simeq 0.004 \tag{3.1}$$

At first sight Eq. (2.17) can be easily solved at small N/VN_c by iterations. In the zeroth order one obtains the free propagator $\bar{S} = S_0 + O(N/VN_c)$ and the first iteration gives

$$\bar{S}^{-1} = S_0^{-1} + \frac{N}{2VN_c}\mathrm{Tr}_{\mathrm{colour}}\{\int \mathrm{d}^4 \mathrm{z_I}(\mathrm{S_0} - \not{A}_{\mathrm{I}}^{-1})^{-1} + (\mathrm{I} \longrightarrow \bar{\mathrm{I}})\}$$

$$+O[(N/VN_c)^2]. \qquad (3.2)$$

Here we face the problem of inverting the operator

$$S_0 - \not{A}_I^{-1} = S_0(i\not{\partial} + \not{A}_I + im)\not{A}_I^{-1}. \qquad (3.3)$$

At zero quark masses this operator cannot be inverted because the Dirac operator $i\not{\partial} + \not{A}_I$ has the zero mode (1.5). Therefore, the iteration procedure fails at m=0. The reason of this failure is obvious. As we shall see below, in the instanton medium model the spontaneous breaking of the chiral symmetry occurs at whatever small N/VN_c. Therefore, Eq. (2.17) has not a single solution but a family of solutions with different γ_5 phases of the quark condensate. It is clear that the iteration procedure which starts from the free propagator cannot generate a family of solutions with broken chiral symmetry.

It is well known that the spontaneous breakdown of any symmetry usually leads to a nonanalytical behaviour of various observables. It is our case too. The propagator $\bar{S}$ should be expanded not in integer but in half-integer powers of the small parameter N/VN_c.

Let us denote

$$\alpha = \sqrt{N/2VN_c}. \qquad (3.4)$$

We shall search the solution of Eq. (3.2)

$$\bar{S}^{-1} - S_0^{-1} = \alpha^2\,\mathrm{Tr}_{\mathrm{colour}}\left\{\int d^4 z_I(\bar{S} - \not{A}_I^{-1})^{-1} + (I \longrightarrow \bar{I})\right\} \qquad (3.5)$$

in the form

$$\bar{S}^{-1} = S_0^{-1} + \alpha\sigma_1 + \alpha^2\sigma_2 + \cdots. \qquad (3.6)$$

Then the lhs of (3.5) is of order α. Let us show that the rhs of (3.5) is also of the order of α in spite of the explicit factor α^2. Indeed, in the rhs of (3.5) one deals with the operator

$$(\bar{S} - \not{A}_I^{-1})^{-1} = \not{A}_I(\not{A}_I - \bar{S}^{-1})\bar{S}^{-1} = \not{A}_I(i\not{\partial} + \not{A}_I - \alpha\sigma_1$$

$$+O(\alpha^2))^{-1}(-i\not{\partial} + O(\alpha)). \qquad (3.7)$$

As mentioned above, the Dirac operator in the background instanton field has a zero mode ψ_I (1.5). Due to the perturbation $-\alpha\sigma_1$ in the rhs of (3.7) this zero mode becomes quasizero. At $\alpha \to 0$ the quasizero mode dominates in the inverse operator

$$\left[i\partial\!\!\!/ + A\!\!\!/_I - \alpha\sigma_1 + O(\alpha^2)\right]^{-1} = \frac{|\psi_I\rangle\langle\psi_I|}{\alpha\langle\psi_I|\sigma_1|\psi_I\rangle} + O(\alpha^0) . \tag{3.8}$$

Let us insert (3.7), (3.8) into (3.5). Taking into account Eq. (1.5) one obtains

$$\sigma_1 = -\operatorname{Tr}_{\text{colour}}\left\{\int d^4 z_I \frac{i\partial\!\!\!/|\psi_I\rangle\langle\psi_I|i\partial\!\!\!/}{\langle\psi_I|\sigma_1|\psi_I\rangle} + (I \to \bar{I})\right\} . \tag{3.9}$$

It is a closed equation for σ_1. Thus the assumption about the expansion of $\bar{S}$ in half-integer powers of N/VN_c is selfconsistent.

Equation (3.9) is easy to solve. Since σ_1 is translation-invariant, its matrix element $\langle\psi_I|\sigma_1|\psi_I\rangle$ does not depend on z_I. Due to the P-reflection symmetry,

$$\langle\psi_I|\sigma_1|\psi_I\rangle = \langle\psi_I^-|\sigma_1|\psi_I^-\rangle. \tag{3.10}$$

Inserting Eq. (3.10) into Eq. (3.9) and passing to the momentum space one obtains

$$\sigma_1(k) = -\frac{1}{< \psi_I|\sigma_1|\psi_I >} \operatorname{Tr}_{\text{colour}} k\!\!\!/[\psi_I(k) \times \psi_I^+(k) + \psi_I(k) \times \psi_I^\pm(k)]k\!\!\!/ . \tag{3.11}$$

The explicit expressions for the projectors onto the zero modes ψ_I, ψ_I^- (see Refs [3,4]) are

$$\psi_I(k) \times \psi_I^+(k) = \frac{[\varphi'(k)]^2}{8k^2}\tau_\mu^-\tau_\nu^+ k\!\!\!/\gamma_\mu\gamma_\nu k\!\!\!/ \frac{1-\gamma_5}{2} ,$$

$$\psi_I^-(k) \times \psi_I^\pm(k) = \frac{[\varphi'(k)]^2}{8k^2}\tau_\mu^+\tau_\nu^- k\!\!\!/\gamma_\mu\gamma_\nu k\!\!\!/ \frac{1+\gamma_5}{2} . \tag{3.12}$$

Here $\tau_\mu^\pm = (\vec{\tau}, \mp i)$, $\vec{\tau}$ are the usual Pauli matrices,

$$\varphi'(k) = \pi\varrho^2\, d/dz[I_0(z)K_0(z) - I_1(z)K_1(z)]|_{z=k\varrho/2} , \tag{3.13}$$

where I_n, K_n are the modified Bessel functions and ϱ is the instanton size.

Putting (3.12) into (3.11) one gets

$$\sigma_1(k) = -\frac{k^2[\varphi'(k)]^2}{\langle\psi_I|\sigma_1|\psi_I\rangle} . \tag{3.14}$$

According to Eq. (3.12)

$$\langle\psi_I|\hat{\sigma}_1|\psi_I\rangle = 2\int \frac{d^4q}{(2\pi)^4}\sigma_1(q)[\varphi'(q)]^2\,. \tag{3.15}$$

Inserting Eq. (3.15) into Eq. (3.14) one has

$$\sigma_1(k) = -\frac{k^2[\varphi'(k)]^2}{2\int d^4q/(2\pi)^4\sigma_1(q)[\varphi'(q)]^2}\,. \tag{3.16}$$

The obvious solution of this equation is

$$\sigma_1(k) = -i\frac{k^2[\varphi'(k)]^2}{\sqrt{2}\|q\varphi'^2\|}\,, \tag{3.17}$$

where

$$\|q\varphi'^2\|^2 = \int \frac{d^4q}{(2\pi)^4}\, q^2[\varphi'(q)]^4\,. \tag{3.18}$$

These two solutions correspond to different realizations of the spontaneous breakdown of chiral symmetry. Substituting Eq. (3.17) into Eq. (3.6) we see that, due to the spontaneous breakdown of chiral symmetry, the quark has acquired a dynamical mass

$$M(k) = i\alpha\sigma_1(k) + O(\alpha^3) = \frac{\alpha k^2[\varphi'(k)]^2}{\sqrt{2}\|q\varphi'^2\|} + O(\alpha^3)\,. \tag{3.19}$$

Using the values (1.2), (1.3) for the parameters of the instanton medium we get

$$M(0) = 300\text{ MeV}\,. \tag{3.20}$$

The spontaneous breakdown of chiral symmetry generates the quark condensate

$$\langle\bar{\psi}\psi\rangle = iN_c\int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\,\bar{S}(k)\,. \tag{3.21}$$

In the first order in α we use Eqs (3.6), (3.19)

$$\langle\bar{\psi}\psi\rangle = -4\alpha N_c\int \frac{d^4k}{(2\pi)^4}\,\frac{\sigma_1(k)}{k^2}$$

$$= -2\sqrt{2}\alpha N_c\frac{\|\varphi'\|^2}{\|q\varphi'^2\|^2} \cong -(250\text{ MeV})^3\,. \tag{3.22}$$

The quark condensate is not renormalization invariant. Here it was calculated in a classical instanton medium without quantum corrections. Therefore, the value (3.22) corresponds to the scale which is determined by the instanton size $\bar{\varrho}^{-1}= 600$ MeV. At this scale the phenomenological value of the quark condensate is (240–250 MeV) [3].

We have calculated the effective quark mass and the quark condensate at leading order in N/VN_c. Comparing these results with the values obtained by Diakonov and Petrov [3,4], one can see that they are numerically close. This fact has a simple explanation. It can be shown [8] that the zero-mode approximation of Diakonov and Petrov gives correct results at the leading order in N/VN_c with some incomplete admixture of high orders.

The technique described here can also be applied to correlators of quark currents [7]. In this case the zero-mode approximation also reproduces the correct leading-order contribution of the N/VN_c expansion but involves incomplete addition of higher orders which is responsible for the currents non-conservative in the zero mode approximation. The present approach allows us, in principle, to construct a systematic N/VN_c expansion with the axial and vector currents conserved at each order.

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