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# MESON PROPERTIES AND CHIRAL TRANSITION AT FINITE TEMPERATURE AND DENSITY IN NAMBU–JONA-LASINIO MODEL WITH DIFFERENT REGULARIZATION SCHEMES*

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Modifications of meson and baryon properties due to the presence of an external hot baryon medium are investigated in the Nambu–Jona-Lasinio model. The corresponding meson sector is solved for a quark continuum at finite density and temperature. We use two regularization schemes with 3-dimensional sharp, Pauli–Villars and proper time cutoff types. Due to the medium the constituent quark mass and the pion and sigma masses are modified. We find a first-order chiral phase transition at relatively low temperatures less than 100 MeV which changes to second order at higher temperatures. The corresponding temperature–density phase diagram is non-monotonic.

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The QCD is generally accepted as a theory of the strong interactions with three important concepts: on the one hand the colour confinement and the asymptotic freedom and on the other — the spontaneous chiral symmetry breaking. At high density and/or temperature, however, in accordance with the asymptotic freedom idea one should expect a restoration of the

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chiral symmetry and a deconfinement. Indeed the nonperturbative QCD properties seem to be verified by the lattice calculations as well as phase transitions concerning the chiral symmetry restoration and colour deconfinement are suggested at finite temperature [1] (see also [2,3] and References therein). The pure gauge lattice calculations indicate that both phase transitions are present at approximately the same critical temperature of about 250 MeV, whereas the inclusion of light quarks ("full" lattice QCD) lowers the critical temperature to about 200 MeV or less. It is amusing that the estimates of the critical temperature done in the chiral perturbation theory [4] give similar numbers.

The nature of the phase transitions, however, is not clear. Most of the pure gauge results suggest a clear first order-phase transition whereas the "full" lattice QCD calculations do not show a consensus: the phase transition is a weaker first-order or even a second-order. Even for the simplest case of finite temperature and zero chemical potential (zero density) the present lattice QCD methods are not able to give a quantitative reliable description of the phase transition characteristics and of the bulk phase properties close to the transition, point not speaking about the meson and nucleon properties. This is a motivation to apply effective models like the Nambu–Jona-Lasinio (NJL) [5] and the linear $\sigma$-model [6], as there is some hope to justify those models as a limiting case of the low-energy QCD [7, 8]. Indeed the investigations [9–12] based on these models give a chiral phase transition from Goldstone to Wigner phase at both finite temperature and density in a qualitative agreement with the Monte-Carlo lattice calculations. Similar to the lattice calculations there is no clear opinion about the order of the phase transition: in Refs [10, 9] authors find a second order transition, whereas Asakawa and Yazaki [12] suggest a first-order at low temperatures which changes to second-order at higher temperatures. Medium [13] and low temperature [18, 19] effects in the properties of the mesons and of the nucleons have been successfully studied as well. One might expect also non-trivial effects at finite both temperature and density [20].

We study some meson properties and the chiral transition in hot and dense baryon medium as similar to Refs [17, 20]; the basic assumption is that the nucleon medium can approximately be replaced by a quark uniform medium. The latter is described by the NJL model [5] with scalar and pseudoscalar couplings regularized by means of two different regularization schemes with three cutoff types, namely noncovariant sharp cutoff, the Pauli-Villars [21] and the proper time method [22]. The parameters of the model are chosen to match PCAC and to reproduce the structured vacuum.

## 1. The model

In our approach we assume a physical picture in which the nucleon, being off-shell by the presence of the medium, is simulated by an on-shell nucleon facing meson fields modified by the medium. It means that the influence of the medium is expressed in terms of modified value of the constituent quark mass and the meson masses. We use an approximation which consists of treating the baryon medium as a uniform quark matter neglecting the nucleon substructure in it. Obviously it is rather rough approximation to study the bulk properties of the nuclear matter but seems to be reasonable [17, 20] for our aim — the modifications of the meson and nucleon properties in the medium.

### *1.1. NJL model with a chemical potential and finite temperature*

The Lagrangian of the NJL model used includes a local scalar and pseudoscalar four-fermion interaction with [5]:

$$\mathcal{L} = \overline{\Psi}(i\gamma^{\mu}\partial_{\mu} - m_0)\Psi + \frac{G}{2}[(\overline{\Psi}\Psi)^2 + (\overline{\Psi}i\gamma_5\vec{\tau}\Psi)^2]. \tag{1.1}$$

The Dirac field $\Psi$ describes the quark with SU(2)-flavours (u and d quarks) and an average current mass $m_0 = (m_{\rm u} + m_{\rm d})/2$. The non-zero current mass breaks explicitly the chiral symmetry in (1.1). Introducing auxiliary boson fields $\sigma = -g\overline{\Psi}\Psi/\lambda^2$ and $\vec{\pi} = -g\overline{\Psi}i\gamma_5\vec{\tau}\Psi/\lambda^2$ with the new constants $\alpha = -\lambda^2 m_0/g$ and $G = g^2/\lambda^2$ the Lagrangian (1.1) can be also represented in an equivalent bosonized form [23]. The new parameter $\lambda$ is not included in the original Lagrangian (1.1) and in this sense it is redundant. Because of that it should be fixed in a self-consistent way.

In accordance with the PCAC hypothesis the divergence of the axial vector current in the NJL model should be related to $f_\pi m_\pi^2 \vec{\pi}$ which gives $\alpha = -f_\pi m_\pi^2$.

Using functional integral techniques the quantized theory at finite density and temperature can be written in terms of the corresponding generating functional in grand canonical form for non-zero chemical potential $\mu$ and temperature $T = 1/\beta$:

$$\mathcal{Z} = \int D\overline{\Psi}\,D\Psi\,D\sigma\,D\vec{\pi}\,\exp\Bigg(i\int d^4x\{\overline{\Psi}[i\gamma^{\mu}\partial_{\mu} - g(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau})]\Psi + \frac{\lambda^2}{2}(\sigma^2 + \vec{\pi}^2) - f_\pi m_\pi^2\sigma - \mu\gamma^0\}\Bigg) \tag{1.2}$$

Performing a Wick rotation the quarks are integrated out replacing the integration over the imaginary time by a sum $\int_0^{\beta}\frac{dk_4}{(2\pi)} \to \frac{1}{\beta}\sum_{n=-\infty}^{+\infty}$ [24] over the fermionic Matsubara frequencies $k_4 \to (2n+1)\pi/\beta$. For the summation over $n$ we follow Dolan and Jackiw [25].

Finally one obtains for the density of the thermodynamical potential

$$\Omega \equiv \frac{S_{\text{eff}}}{\beta V} = -\sum_\alpha \left\{ \frac{1}{2}(\varepsilon_\alpha - \mu) + \frac{1}{\beta} \ln\left[1 + e^{-(\varepsilon_\alpha - \mu)\beta}\right] \right\}$$

$$+\tfrac{1}{2}\lambda^2(\sigma^2 + \vec{\pi}^2) - f_\pi m_\pi^2 \sigma \,, \tag{1.3}$$

where $V$ is the volume and $\varepsilon_\alpha$ are the solutions of the Dirac equation:

$$\left[-i\hat{\vec{\alpha}} \cdot \vec{\nabla} + \hat{\beta} g(\sigma + i\gamma_5 \vec{\pi} \cdot \vec{\tau})\right] |\alpha\rangle = \varepsilon_\alpha |\alpha\rangle \,. \tag{1.4}$$

The baryon density $\varrho_{\mathrm{B}}$ is related to the thermodynamical potential by

$$\varrho_{\mathrm{B}} N_c = -\frac{\partial \Omega}{\partial \mu} = \sum_\alpha \left\{ -\frac{1}{2} + \frac{1}{1 + e^{-(\varepsilon_\alpha - \mu)\beta}} \right\} . \tag{1.5}$$

The static solution can be obtained by minimizing the thermodynamical potential $\Omega$ (or the effective action):

$$\frac{\partial \Omega}{\partial \sigma} = \frac{\partial \Omega}{\partial \vec{\pi}} = 0 \quad \text{with} \quad -\frac{\partial \Omega}{\partial \mu} = \varrho_{\mathrm{B}} \,. \tag{1.6}$$

This procedure is equivalent to the minimization of the Helmholtz free energy $F = 1 - \mu\, \partial\Omega/\partial\mu$ with a constraint and in this case $\mu$ plays a role of a Lagrangian multiplier.

The second derivative of the free energy $F$ with respect to the meson fields determines the meson masses:

$$\frac{\partial^2 F}{\partial \sigma^2} = m_\sigma^2 \quad \text{and} \quad \frac{\partial^2 F}{\partial \vec{\pi}^2} = m_\pi^2 \,. \tag{1.7}$$

In that one has to keep in mind the constraint (the second equation of (1.6)) concerning the baryon density. It means that the fluctuations of the meson fields should be along the true path of the system. Since the inverse field propagators can be obtained from the second order variation of the effective action, the meson masses defined by Eqs (1.7) correspond to the poles at $q^2 = 0$ of the meson propagators, obtained in the small amplitude approximation, if the imaginary part appearing due to the lack of the colour confinement in the NJL model is neglected [26].

In the calculations we use the plane wave basis $\varepsilon_\alpha \equiv \varepsilon_k = \pm\sqrt{k^2 + M^2}$, where $M$ is the constituent quark mass. We also substitute:

$$\sum_\alpha \sim \int \frac{d^3k}{(2\pi)^3}(\varepsilon_k < 0) + \int \frac{d^3k}{(2\pi)^3}(\varepsilon_k > 0) \,. \tag{1.8}$$

### *1.2. Regularization schemes*

The NJL model is not renormalizable because of the local four-fermion interaction included. In order to cure the divergence of thermodynamical potential (1.3) we regularize it introducing a momentum cutoff. Actually we realize this in two different ways. In each of them we use also different types of momentum cutoff.

In the first scheme we isolate the divergent part of the thermodynamical potential $\Omega(\mu = 0,\ T = 0)$ and regularize it using a non-covariant 3-dimensional cutoff

$$\Omega^{\Lambda}(0,0) = -4N_c \int\limits_{k<\Lambda} \frac{d^3k}{(2\pi)^3}\varepsilon_k\,, \tag{1.9}$$

the proper time method [22]

$$\Omega^{\Lambda}(0,0) = \frac{N_c}{4\pi^2} \int\limits_{\Lambda^{-2}} \frac{d\tau}{\tau^3} \mathrm{e}^{-M^2\tau}\,, \tag{1.10}$$

as well as the Pauli–Villars [21] method

$$\Omega^{\Lambda}(0,0) = -4N_c \int \frac{d^3k}{(2\pi)^3} \left\{ \varepsilon_k - \varepsilon_k^{\Lambda} + \frac{\Lambda^2 - M^2}{2\varepsilon_k^{\Lambda}} \right\}. \tag{1.11}$$

In the case of Pauli–Villars we cure the divergencies subtracting two additional terms in which the momentum cutoff plays a role of a heavy mass. The divergent part is coming from the negative-energy Dirac sea. The regularization procedure is schematically illustrated in Fig. 1(a). As can be seen in this case only the negative part of the quark spectrum is cut from bellow whereas the positive part is not affected. The finite part of the thermodynamical potential is

$$\begin{aligned} &\Omega(\mu,T) - \Omega(0,0) \\ &= 4N_c \int \frac{d^3k}{(2\pi)^3} \left\{ (\varepsilon_k + \mu) - \frac{1}{\beta}\ln\left[1 + \mathrm{e}^{-(\varepsilon_k - \mu)\beta}\right] - \frac{1}{\beta}\ln\left[1 + \mathrm{e}^{(\varepsilon_k + \mu)\beta}\right] \right\} \\ &\quad + \tfrac{1}{2}\lambda^2(\sigma^2 + \vec{\pi}^2) - f_\pi m_\pi^2 \sigma\,. \end{aligned} \tag{1.12}$$

The total regularized thermodynamical potential is then given by the sum of its finite and regularized parts. In this regularization scheme the baryon density defined as $-\partial\Omega/\partial\mu$ does not include the momentum cutoff $\Lambda$.

In the second regularization scheme the momentum cutoff affects the negative- as well as the positive-energy part of the quark spectrum (see

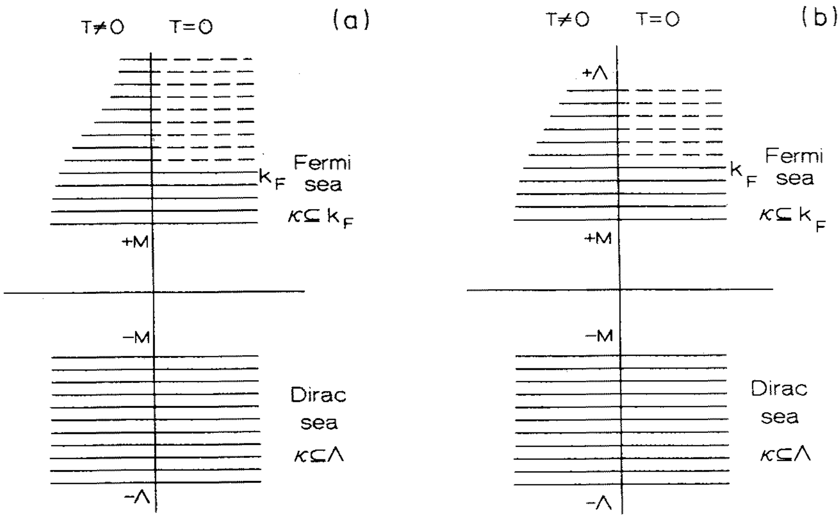


Fig. 1. The quark spectrum in NJL model regularized in two different ways: (a) a cutoff is introduced in the negative energy sea; (b) both parts of the spectrum are affected by a cutoff.

Fig. 1(b)). Because of the imaginary-time formalism only the 3-dimensional cutoff appears

$$\Omega = 4N_c \int\limits_{k<\Lambda} \frac{d^3k}{(2\pi)^3} \left\{ \mu - \frac{1}{\beta} \ln\left[1 + e^{-(\epsilon_k - \mu)\beta}\right] - \frac{1}{\beta} \ln\left[1 + e^{(\epsilon_k + \mu)\beta}\right] \right\}$$

$$+\tfrac{1}{2}\lambda^2(\sigma^2 + \vec{\pi}^2) - f_\pi m_\pi^2 \sigma \qquad (1.13)$$

and the Pauli–Villars method

$$\Omega = 4N_c \int \frac{d^3k}{(2\pi)^3} \left\{ -\frac{1}{\beta} \ln[1 + e^{-(\epsilon_k - \mu)\beta}] - \frac{1}{\beta} \ln[1 + e^{(\epsilon_k + \mu)\beta}] \right.$$

$$+ \frac{1}{\beta} \ln\left[1 + e^{-(\epsilon_k^\Lambda - \mu)\beta}\right] + \frac{1}{\beta} \ln\left[1 + e^{(\epsilon_k^\Lambda + \mu)\beta}\right]$$

$$\left. - \frac{\Lambda^2 - M^2}{2\epsilon_k^\Lambda} \left[ \frac{1}{1 + e^{(\epsilon_k^\Lambda - \mu)\beta}} - \frac{1}{1 + e^{-(\epsilon_k^\Lambda + \mu)\beta}} \right] \right\}$$

$$+\tfrac{1}{2}\lambda^2(\sigma^2 + \vec{\pi}^2) - f_\pi m_\pi^2 \sigma \qquad (1.14)$$

can be used to regularize the effective action. In contrast to the first scheme all expressions concerning the bulk quantities and in particular the baryon density include the cutoff.

### *1.3 Fixing model parameters*

Besides the redundant parameter $\lambda$ the NJL Lagrangian includes basically two parameters — the constituent mass $M$ (or coupling constant $g$) and the momentum cutoff $\Lambda$. We fix them reproducing the vacuum properties, namely the pion decay constant $f_\pi = 93$ MeV and pion mass $m_\pi = 139.6$ MeV as well as the empirical values $\langle\bar{q}q\rangle = -(283 \pm 31\ \mathrm{MeV})^3$ and $(m_u + m_d)/2 = 7 \pm 2.1$ MeV of the quark condensate and the quark bare mass [27].

The stationarity condition Eq.(1.6) at the $\mu = 0$ and $T = 0$ leads to $\vec{\pi}_0 = 0$ and to the gap equation:

$$\sigma_0\big(\lambda^2 - 4N_c g^2 J_{1/2}(M,\Lambda)\big) = f_\pi m_\pi^2\,, \tag{1.15}$$

where

$$J_\alpha(M,\Lambda) = \int^{\Lambda} \frac{d^3k}{(2\pi)^3}\frac{1}{(k^2+M^2)^\alpha}\,. \tag{1.16}$$

Due to the non-vanishing vacuum value of $\sigma_0$ the quarks acquire a constituent mass $M = g\sigma_0$. We use Eq.(1.15), however, to eliminate the parameter $\lambda$ in favour of the constituent mass $M$. Thus, the redundant parameter is determined in a self-consistent way.

Fixing the pion mass from Eq.(1.7) at its physical value one gets the Goldberger–Treiman relation at the quark level $M \equiv g\sigma_0 = g f_\pi$.

The next quantity which we use to fix our parameters is the pion decay constant. It can be calculated from the diagram shown in Fig. 2. Combining the result with the Goldberger–Treiman relation one gets the condition:

$$4N_c g^2 \int^{\Lambda} \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+M^2)^2} = 1\,. \tag{1.17}$$

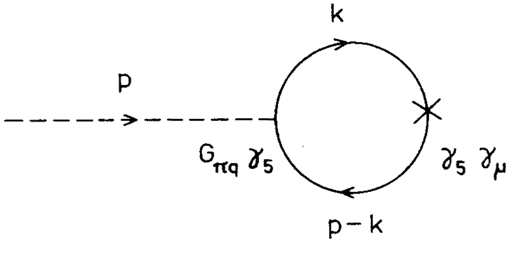


Fig. 2. One-loop diagram for the pion decay.

The integral in Eq.(1.17) is divergent and we introduce a momentum cutoff to make it finite. The particular expressions for different cutoff types are:

$$\frac{N_c g^2}{2\pi^2}\left\{\ln(\lambda/M)+\sqrt{\lambda^2/M^2+1}-\frac{\lambda/M}{\sqrt{\Lambda^2/M^2+1}}\right\}=1 \qquad (1.18)$$

in the case of 3-dimensional cutoff,

$$\frac{Nc}{4\pi^2}g^2\left\{\ln\frac{\Lambda^2}{M^2}+\frac{M^2}{\Lambda^2}-1\right\}=1 \qquad (1.19)$$

for the Pauli–Villars method, whereas in the proper time method the regularization integral cannot be calculated analytically:

$$\frac{N_c}{4\pi^2}g^2\int\limits_{1/\Lambda^2}^{\infty}\frac{d\tau}{\tau}e^{-M^2\tau}=1\,. \qquad (1.20)$$

Thus, for a given value of the constituent mass $M(g)$, the cutoff $\Lambda$ is fixed by the condition (1.17). It should be noted that the same condition is needed for a derivation of the Gell-Mann–Lévy Lagrangian as an effective Lagrangian from the NJL model using a gradient expansion [23]. The above condition leads to a cutoff independent expression for the sigma mass in the NJL model:

$$m_\sigma^2=m_\pi^2+4M^2\,. \qquad (1.21)$$

All those conditions together leave the constituent mass $M$ (or quark–meson coupling constant $g$) as the only parameter undetermined so far. To fix it one may use the values of the quark condensate and the quark current mass coming from QCD sum rules [27]. The uncertainties of the empirical values, however, leave a lot of room for the possible values of $M$.

## 2. Results and discussion

We fit simultaneously the quark condensate and the quark current mass coming from QCD sum rules [27] as well as the nucleon mass $M_N = 938$ MeV obtained [17] in the projected chiral soliton model keeping $f_\pi = 93$ MeV and $m_\pi = 139.6$ MeV. The value of the constituent mass found in that way is $M = 465$ MeV. We use this number as a common value in all regularization schemes. The momentum cutoff is determined by the conditions (1.18)–(1.20). It is assumed to be density and temperature independent. The values of the cutoff as well as the vacuum characteristics obtained using different regularization methods are presented in Table I. As can be seen the Pauli–Villars method seems to be the most preferable. It provides the best description of the vacuum because the cutoff value is larger than those of the other two methods. The sigma mass coming from the cutoff independent relation (1.21) is $m_\sigma = 940$ MeV. It should be mentioned also that apart from the sigma mass the meson properties as well as the quark condensate and current mass are rather independent [28] of the particular value of the constituent mass ranging between 360 and 500 MeV.

### *2.1. Constituent quark mass and meson masses at finite density and temperature*

In this section we present the results concerning the meson sector of the medium obtained in the first regularizations scheme with Pauli–Villars cutoff. Actually the other schemes and cutoffs show similar results.

In the medium we use unchanged vacuum values of $g = M/F_\pi$ , $\Lambda$ and $\lambda^2$. At finite density and temperature the stationarity condition (1.6) leads to the gap equation for the constituent mass value $M^*$ (marked by an asterisk in order to distinguish it from the vacuum value). The values $m_\pi^*$ and $m_\sigma^*$ of the meson masses are evaluated from Eqs (1.8). We found that the pion mass scales as

$$m_\pi^{*2} = m_\pi^2 M/M^* \tag{2.1}$$

in all regularization schemes. Thus, as can be seen from (2.1), the chiral symmetry breaking term is not modified at the medium. This implies that a relation similar to the PCAC should exist also in the hot and dense medium. For the sigma mass the cutoff independent relation (1.21) is no longer valid. The calculated constituent quark mass and the meson masses as a function of the temperature are shown in Fig. 3(a) and (b) for four different baryon densities. At finite temperature above the critical value the $m_\sigma^*$ coincides with the $m_\pi^*$ : the mesons become degenerate. The latter, together with the strongly reduced constituent mass, is a clear indication of the phase transition. The combined temperature-density effects change the behaviour

TABLE I

Vacuum properties for different cutoff types

| Cutoff | $\Lambda$ | $(-\langle\bar{q}q\rangle)^{1/3}$ | $m_0$ | $B$ |
|---|---|---|---|---|
| Sharp cutoff | 577 | 305 | 5.8 | 198 |
| Pauli–Villars | 862 | 264 | 9.0 | 199 |
| Proper time | 640 | 208 | 18.0 | 201 |
| QCD sum rules | — | $283 \pm 31$ | $7.0 \pm 2.1$ | $240 \pm 16$ |

of the constituent mass and the sigma mass to non-monotonic. In contrast, for finite temperature and density far from the critical values $m_\pi^*$ shows a rather weak medium effect. Close to the critical density all masses are affected strongly by the medium: at low temperature meson masses are very close to each other, with increasing temperature they deviate strongly and close to the critical temperature they approach again similar values. Similar to the constituent quark mass, the quark condensate shows also a non-monotonic temperature dependence at finite density. A similar increase of the quark condensate with temperature at fixed density is also found by Dey *et al.* [29].

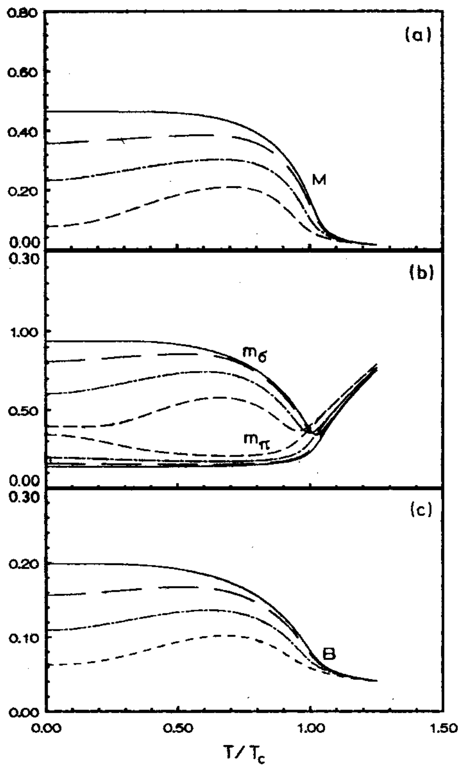


Fig. 3. Constituent quark mass $M$ (a), sigma and pion masses $m_\sigma$ and $m_\pi$ (b) and bag constant $B$ (c) as functions of temperature at different medium densities. $T_c = 200$ MeV, $\varrho_c = 2.5\varrho_{nm}$. Medium baryon density is 0 (solid line), $\varrho_{nm}$ (long-dashed line), $2\varrho_{nm}$ (short-dashed line) and $3\varrho_{nm}$ (dotted line).

### *2.2. Chiral phase transition*

Because of the finite pion mass, however, the observed chiral transition is smooth — the constituent mass is strongly reduced but does not

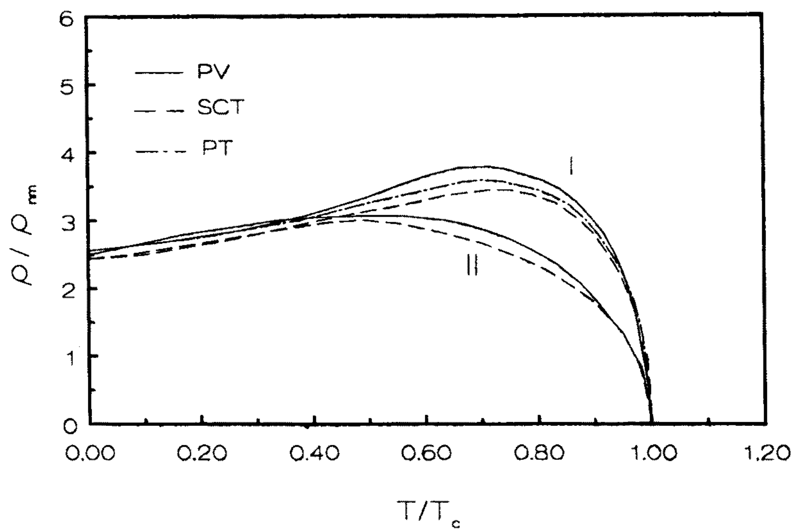


Fig. 4. $T-\varrho$ chiral phase diagram. The curves labelled by I correspond to the first regularization scheme and those with II to the second scheme.

vanish. In order to further identify the transition, besides the reduction of the constituent mass, we make use of the fact that because of the chiral symmetry restoration the pion and the sigma fields become degenerate. Operationally, we take the point at which the sigma mass changes its behaviour to the one similar to the pion mass (see Fig. 3(a) and (b)). Looking at the high density and vanishing temperature one realizes that, before the chiral transition occurs at high temperature, the chiral transition from the Wigner to Goldstone phase with a "restoration" of the chiral symmetry breaking takes place. The calculated phase diagrams on the $\varrho$–$T$-plane for the different regularization schemes used are shown in Fig. 4. As can be seen at zero temperature the critical density (about 2.5 $\varrho_{nm}$), is almost independent of the particular scheme which is not the case at the critical temperature, $T_c$, at zero density. The latter depends strongly on the way in which the momentum cutoff is introduced. In the first scheme the critical value is about 200 MeV whereas in the other case it is larger — about 260 MeV. The first number seems to be more reasonable: it agrees with the estimates of the lattice QCD calculations (see for instance Ref. [3]). All chiral transition critical curves presented in Fig. 4 show a non-monotonic behaviour of the critical density with the temperature. Quantitatively the effect, however, is much more pronounced in the first regularization scheme than in the second one. In the first scheme (see Fig. 1(a)) the cutoff is introduced only in the negative energy part of the quark spectrum. Thus at finite temperature the quarks are free to occupy all positive levels up to infinity. Critical curves for nuclear matter close to ours in the first scheme are obtained by Wakamatsu and Hayashi [30] and Kawati and Miyata [31] employing a simplified $\sigma$ model and a model with a scalar four-fermion interaction with nucleons, respectively. Similarly to our approach in both those approaches the baryon density is not affected by the regularization. In contrast to the first scheme in the second one the cutoff is introduced in both the negative and the positive part of the quark spectrum (see Fig. 1(b)). The quarks are constrained to occupy only the levels up to the cutoff. Apparently in this scheme the finite temperature effects are suppressed in some sense artificially. It is illustrated in Fig. 3 — the restoration of the Goldstone phase at intermediate temperatures is less pronounced and the critical temperature value at zero baryon density is larger than in the first scheme. The second type regularization scheme with a 3-dimensional cutoff is used by Hatsuda and Kunihiro [9, 18] as well as by Bernard *et al.* [10] and the latter show a critical curve [10] identical to ours in the case of the second scheme.

As it was outlined in the Introduction both the lattice QCD and the studies based on the NJL model do not provide an unambiguous result about the order of the phase transition. Hatsuda and Kunihiro [9, 18] as well as Bernard *et al.* [10] find the second-order phase transition at finite temperature and density. The latter use as a criterion the positivity of the energy density difference throughout the critical line. In contrast to them Asakawa and Yazaki [12] have studied carefully the behaviour of the constituent quark and meson masses as functions of the chemical potential at vanishing as well as a finite temperature. They found that the masses have a discontinuity at common critical values of the chemical potential at low temperature values which suggest a first-order phase transition. At relatively higher temperature (for parameter values used it is about 50 MeV) the picture is smeared out and no evident critical point can be defined in $T-\mu$-plane. Barducci *et al.* [32] have studied the chiral symmetry breaking for a QCD-like gauge theory at finite density and temperature. Similar to Asakawa and Yazaki [12] they have found a first-order phase transition at low temperatures which changes to second order as the temperature crosses a given critical value.

This situation motivates us to investigate more carefully the nature of the chiral transition. To that end we evaluated the equation of state (EOS) of the quark matter and apply the Ehrenfest classification of the phase transitions. In accordance with it a first-order transition is characterized by a continuous Gibbs free energy density but its derivatives are discontinuous. The calculated pressure as a function of the baryon density for different temperatures is shown in Fig. 5. The pressure of the Dirac sea ($\mu = 0$) is subtracted. The calculations are done in the first regularization scheme with the Pauli–Villars cutoff. The other scheme and cutoffs give similar

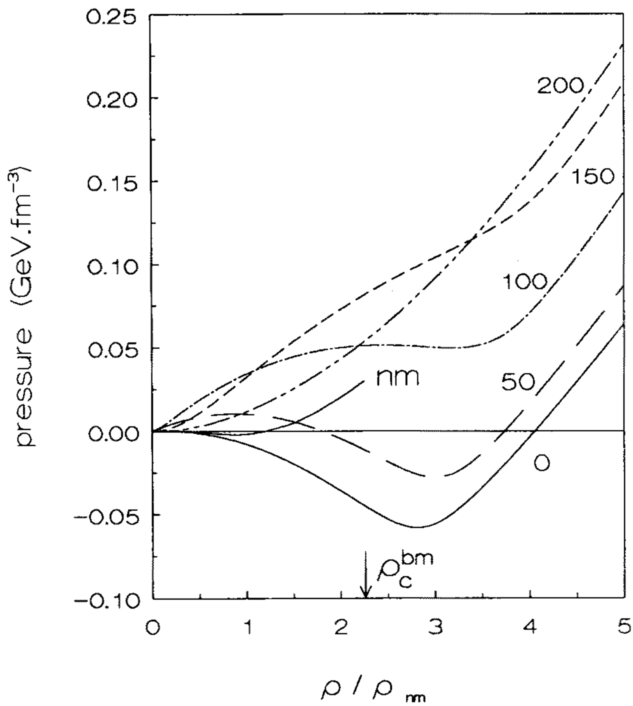


Fig. 5. Equation of state (EOS) of the quark matter. The EOS of the nuclear matter at zero temperature taken from [33] is depicted by the dotted line.

results. At lower temperatures (< 100 MeV) the isotherms show a clear minimum at the critical density and a maximum at low density. We note that for the density values lying between the maximum and the minimum the stability condition $(\partial P/\partial \varrho)T > 0$ is not satisfied and the EOS is not well defined. At these densities there is a coexistence of the two phases with equal temperature, chemical potential and pressure. At zero temperature the unstable region covers the densities from 0.15 $\varrho_{nm}$ up to 2.5 $\varrho_{nm}$. It is, however, not surprising since one should expect there the nucleon matter to be more preferable configuration than the quark one. In contrast to the quark matter the EOS of the nuclear matter [33] (shown also in Fig. 5) satisfies the stability criterion in this region. At temperatures higher than 100 MeV the isotherms change their behaviour to increasing functions of the density with an inflection. At the critical values $T_c$ it is a simple, monotonically increasing function. Using the isotherms we reconstruct the density of the Gibbs free energy. For the low-temperature cases ($T < 100$ MeV) this quantity is a continuous function but its derivative with respect to the pressure shows a discontinuity. The latter disappears at higher temperatures.

Thus we found a first-order transition at low temperature values < 100 MeV (including the zero temperature case) which changes to second-order one at higher temperatures. It means that our analysis supports the conclusions of Asakawa and Yazaki [12] and Barducci *et al.* [32] and not those of Hatsuda and Kunihiro [9, 18] and Bernard *et al.* [10]. The nature of the phase transition is important [2] for the analyses of the processes in the early universe as well as in the ultrarelativistic heavy-ion reactions. Because of the phase coexistence and possible superheating and supercooling phenomena one expects different effects in the case of the first-order transition from those in the second-order. Furthermore, one should expect signals from quark-gluon plasma formation at low as well as at high temperatures but not at intermediate temperatures and high densities.

### *2.3. Bag constant at finite density and temperature*

Following the intuitive understanding of the bag constant as an energy cost for creating an interaction-free space volume we identify it with the free energy density needed to restore the chiral symmetry $B = F(\sigma = 0, \vec{\pi} = 0) - F(\sigma_0, \vec{\pi} = 0)$. The vacuum values of the bag constant calculated using different cutoffs is presented in the last column of Table I. The numbers are practically independent on the particular cutoff and are close to the values frequently used in the bag model calculations. Fig. 3(c) shows the bag constant as a function of temperature at different density numbers. Similar to the other results shown in this figure, the first scheme with Pauli–Villars cutoff is used. As can be seen at finite temperature and density, the bag constant shows a behaviour similar to those of the constituent mass. In contrast to the latter, however, it is much less reduced at critical density and temperature values.

## 3. Summary

The Nambu–Jona-Lasinio model has been considered to study the meson properties and the chiral transition in a hot and dense baryon medium. In order to cure the divergence of the NJL model two regularization schemes with three different types of the cutoff, namely 3-dimensional sharp cutoff, Pauli–Villars and proper time method, are used. The corresponding meson sector of the model is solved for a quark continuum at finite density and temperature. This results in modified values of the constituent quark mass and the pion and sigma masses in the medium. At critical values of the density and temperature we find the chiral symmetry phase transition with a non-monotonic teperature- density phase diagram.We conclude that a first-order phase transition occurrs at finite density and relatively low temperatures (less than 100 MeV) which changes to a second-order one at high temperatures. The quark constituent mass, meson masses and the bag constant show a non-monotonic temperature dependence at finite density.

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