RENORMALIZATION GROUP IMPROVED YFS THEORY IN Z^0 PHYSICS*,**

S. JADACH

Theory Division
CERN, Geneva 23, Switzerland
and
Institute of Physics, Jagellonian University
Reymonta 4, 30-059 Cracow, Poland

AND

B.F.L. WARD

Department of Physics and Astronomy, University of Tennesee
Knoxville, TN 37996-1200, USA
and
SLAC, Stanford, CA 94309, USA

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We discuss the role of the renormalization group improved YFS theory in Z^0 physics at SLC and LEP. The general theory is reviewed. Some results of this application recently at SLC and LEP are presented. Future directions for the theory's further development and application are outlined.

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1. Introduction

The need for high precision radiative corrections has been amply described by many authors in the context of Z^0 physics. And, indeed, a substantial amount of progress in achieving such corrections in a useful way has

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been made. Here, we describe the theory, implementation and application of our approach to these corrections, namely the renormalization group improved YFS Monte Carlo approach to the higher order radiative corrections for SU(2L) × U(1) theory.

Namely, as we have described in Ref.[1], in order to provide a systematic, rigorous framework for the calculation and simulation if high precision radiative corrections at SLC and LEP, we have combined the method of Yennie, Frautschi and Suura [2] and the renormalization group of Weinberg and 't Hooft [3] to develop a Monte Carlo based approach to SU(2L) × U(1) higher order corrections in which large infrared effects are summed to all orders in α in a rigorous way. This development was motivated by the current ongoing experiments at SLC and LEP on Z^0 physics.

Specifically, the basic strategy [1] is as follows. In order to discover any deviations from Standard Model predictions which are at level of 1 %, one should know the higher order corrections to these predictions at $\lesssim$0.3 % precision; for more precise tests, the requirement on the theoretical calculations of known physics increases accordingly.

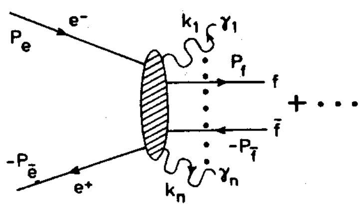

Fig. 1. The process $e^+e^- \to f\bar{f}+n(\gamma)$. P_A is the four momentum of A in the initial e^+e^- c.m. system, $A =$ e, ē, f, f̄.

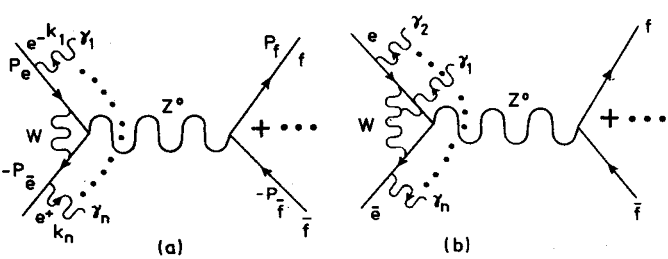

Fig. 2. Pure weak corrections to $e^+e^- \to f\bar{f} + n(\gamma)$. In (a), the $n(\gamma)$ system does not interact directly with the W; in (b), it does.

Two distinct classes of corrections may be identified at order α: the QED corrections illustrated in Fig. 1; and, the pure weak corrections which are illustrated in Fig. 2. Concerning both classes of corrections, substantial efforts have been effected for SLC and LEP physics. These efforts have led to two types of exponentiation methods for dealing with the large infrared effects in graphs such as those illustrated in Fig. 1: the improved naive exponentiation methods of Kuraev and Fadin, Berends *et al.*, Nicrosini and Trentadue, Greco, etc. [4], and, the rigorous implementation of the original theory of Yennie, Frautschi and Suura by Monte Carlo methods by us in Refs.[1]. In the former approach to the big QED effects, one arrives at an inclusive cross section in which the large infrared effects are summed to all orders by using an *ansatz* for the respective summation which was first proposed in its unimproved form in Ref.[5]. In the YFS approach, all large infrared effects are summed to all orders rigorously at the level of the Feynman amplitude so that our Monte Carlo procedure yields an event-by-event simulation of exponentiation in the actual respective *exclusive* distributions. The YFS Monte Carlo methods, because they act at the level of Feynman diagrams, admit the application of the 't Hooft–Weinberg renormalization group so that it has been systematically improved for large ultra-violet effects. This YFS Monte Carlo approach is realized for $e^+e^- \to f\bar{f} + n(\gamma)$, $f \neq e$, by the program YFS2 Fortran [1] and for $e^+e^- \to e^+e^- + n(\gamma)$ at low angles by the BHLUMI Fortran [1]. In this way we have arrived at 0.1 % accuracy on the QED effects in $e^+e^- \to f\bar{f} + n(\gamma)$, $f \neq e$, and 0.7 % accuracy on the process $e^+e^- \to e^+e^- + n(\gamma)$ in the SLC/LEP luminosity regime.

The regime of 0.1 % control over the QED effects for $e^+e^- \to f\bar{f} + n(\gamma)$, $f \neq e$, takes us naturally to the level of the pure weak effects at $O(\alpha)$, since $\alpha/\pi \simeq 0.2\,\%$. The situation is as we have illustrated in Fig. 2. One may obtain a partial simplification by observing that radiation from the internal heavy weak lines, like the W in Fig. 2(b), is suppressed by $\ln(s/m_e^2)$ from that of incoming e^+, e^- lines. This means that the dominant pure weak effects can be taken into account by improving the Born amplitudes to include one-loop pure weak effects and proceeding with the exponentiation of the respective big QED effects. It is, however, clear from Fig. 2(b), that, for higher orders in α, this factorization of the pure weak and QED effects is not strictly valid. This method of including the pure weak corrections has been used in Ref.[6] to combine YFS2 and the pure weak libraries of Hollik and of Stuart into the program KORALZ3 Fortran. In what follows, we shall present some of the results which we have been obtained in this way in the context of Z^0 physics in collaboration with Dr. Z. Wąs.

Thus our work is organized as follows. In the next Secton, we review the elements of the YFS Monte Carlo approach to $SU(2L) \times U(1)$ radiative corrections. In Sect. 3 we present some of the recent applications of this approach to Z^0 physics. In Sect. 4, we present an overview of the various future directions of investigations which exist for our methods. Sect. 5

contains some summary remarks.

2. Review of the YFS Theory

In this Section, we present the basic elements of the YFS theory, including its renormalization group improvement. We begin with the basic elements of the theory.

Specifically, the $n(\gamma)$ emission amplitude illustrated in Fig. 1, $M^{(n)}$, may be represented as

$$M^{(n)} = \exp(\alpha B) \sum_{n'=0}^{\infty} \mathcal{M}_{n'}^{(n)}, \tag{1}$$

where n' is the number of γ-loops in the respective Feynman diagrams corresponding to $\mathcal{M}_{n'}^{(n)}$ and the superscript (n) denotes the number of real emitted photons in these diagrams. $\mathcal{M}_{n'}^{(n)}$ is free of all virtual infrared divergencies; for, these are all contained in the YFS virtual infrared function B, where

$$\begin{aligned} B = &-\frac{i}{8\pi^3} \int \frac{d^4k}{k^2 - m_\gamma^2 + i\epsilon} \\ &\times \left\{ -\left(\frac{-2P_e - k}{k^2 + 2kP_e + i\epsilon} - \frac{-2P_e + k}{k^2 - 2kP_e + i\epsilon} \right)^2 + \cdots \right\}. \end{aligned} \tag{2}$$

The rate corresponding to $M^{(n)}$ is determined by

$$\begin{aligned} |M^{(n)}|^2 &= \exp(2\alpha \operatorname{Re} B) \left| \sum_{n'=0}^{\infty} \mathcal{M}_{n'}^{(n)} \right|^2 \\ &= \exp(2\alpha \operatorname{Re} B) \left\{ \widetilde{S}(k_1) \ldots \widetilde{S}(k_n) \bar{\beta}_0 + \cdots + \bar{\beta}_n(k_1, \ldots, k_n) \right\}, \end{aligned} \tag{3}$$

where the infrared emission factor $\widetilde{S}(k)$ is well-known and is given by

$$\widetilde{S}(k) = -\frac{\alpha}{4\pi^2} \left(\frac{P_e}{P_e k} - \frac{P_{\bar{e}}}{P_{\bar{e}} k} \right)^2 + \cdots \tag{4}$$

and $\bar{\beta}_j$ are the famous YFS infrared divergence free hard photon residuals. The corresponding cross section for $e^+e^- \to X$ is then

$$
d\sigma = \exp\{2\alpha(\mathrm{Re}\,B + \widetilde{B})\} \int \frac{d^4y}{(2\pi)^4} \exp\left\{i\big(y(P_e + P_{\bar{e}} - P_{X'}) + D\big)\right\}
$$
$$
\times \left\{\bar{\beta}_0 + \sum_{n=1}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3k_j}{k_j} e^{-iyk_j} \bar{\beta}_n \right\} dE_{X'}\, d^3P_{X'}, \tag{5}
$$

where

$$
D = \int \frac{d^3k}{k} \widetilde{S}(k) \left(e^{-iyk} - \theta(K_{max} - k)\right), \tag{6}
$$

for some dummy parameter K_{max}, which may actually be a dummy functional of the experimental scenario under study. The point is that the real infrared function $\widetilde{B}$ is given by

$$
2\alpha\widetilde{B} = \int^{k \le K_{max}} \frac{d^3k}{(\vec{k}^2 + m_\gamma^2)^{1/2}} \widetilde{S}(k), \tag{7}
$$

so that (5) does not depend on K_{max}. The formula (5) is the formula which we have realized *via* Monte Carlo methods in Refs [1] in the programs YFS2 Fortran and BHLUMI Fortran.

We emphasize that, because the starting point for (5) is the actual Feynman amplitude $M^{(n)}$, we may apply the renormalization group to this amplitude, and, hence, to the $\bar{\beta}_n$. The result is that, under the renormalization group of 't Hooft and Weinberg [7], we have the replacements

$$
\bar{\beta}_n\big(\lambda\{P_j\}\big) \to \lambda^{d_n} \bar{\bar{\beta}}_n\big(\{P_j\}, \alpha(\lambda), m_{iR}(\lambda), \mu\big), \tag{8}
$$

etc., where we show the running of α and m_{iR} explicitly only (the SU(2L) coupling constant g_R also would run) for reasons of pedagogy, d_n is the respective engineering dimension for $\bar{\beta}_n$ and μ is the normalization point. In this way, we have arrived at our renormalization group improvement of (5) and its Monte Carlo realizations.

This completes our review of the methods. We turn now to some recent applications.

3. Recent Applications in Z^0 Physics

In this Section, we shall present some of the recent applications of our YFS methods in Z^0 physics. We begin with the total cross section in $e^+e^- \to f\bar{f} + n(\gamma)$, $f \neq e$.

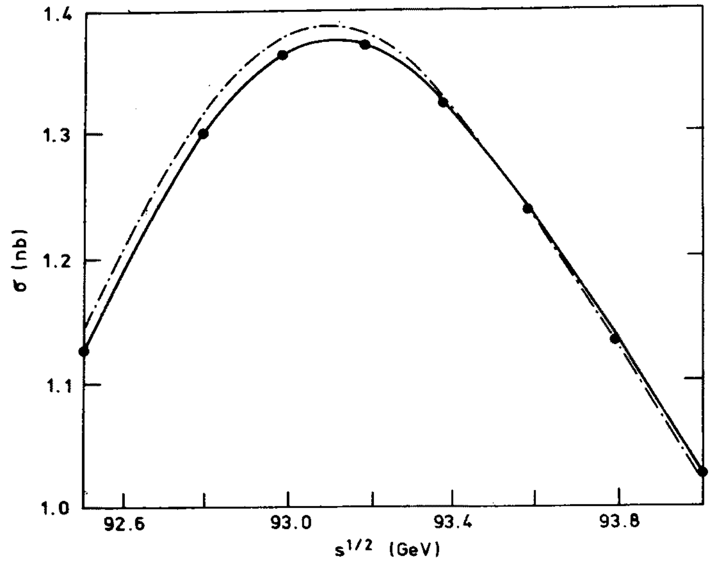

Fig. 3. Numerical realization of the YFS Monte Carlo methods. The dash–dotted and solid curves represent the $O(\alpha^2)$ and naive exponentiated $O(\alpha^2)$ curves of Ref.[8] for $\mu\bar{\mu}$ production; the solid dots come from our Monte Carlo. The MC numerical accuracy is better than 0.1 %. Here, $s'/s = 1 - v$.

Specifically, the era of $\gtrsim 10^6$ Z^0's at LEP (or the SLC) would require that the radiative corrections to $e^+e^- \rightarrow f\bar{f} + n(\gamma)$, $f \neq e$ should be known to $\lesssim 0.1$ %. Accordingly, we have studied in Ref.[1] the comparison of our YFS2 Fortran Monte Carlo simulation of the cross section $e^+e^- \rightarrow \mu\bar{\mu} + n(\gamma)$ with the exact second order exponentiated results of Berends *et al.*[4.8]. Our findings are illustrated in Fig. 3. In this figure, the large dots are the YFS2 MC points, the dash–dotted curve is the exact second order result of this same reference. The statistical error of the large dots is below their size. Thus, in this way, we have indeed verified that our YFS2 MC simulation of the total cross section in $e^+e^- \rightarrow \mu\bar{\mu} + n(\gamma)$ is accurate to 0.1 % insofar as the pure QED initial state radiation is concerned. This YFS2 MC has been recently incorporated into the MC program KORALZ Fortran [6] and, indeed, for example multiple photon effects in $e^+e^- \rightarrow \tau\bar{\tau} + n(\gamma)$ have been properly realized in the respective KORALZ simulations by several Collaborations at LEP in determining ther various efficiences for such events. We re-emphasize that, from Fig. 3, we know that these effects are properly simulated at the $\lesssim 0.1$ % level; for, in KORALZ, the final state radiation of one photon is also allowed.

Turning next to the subject of cross secton asymmetries, we shall consider the following standardly defined quantities of this type: in $e^+e^- \rightarrow$

$\mathrm{f}\bar{\mathrm{f}} + n(\gamma)$,

$$A_{\mathrm{FB}} = \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}}, \tag{9}$$

$$A_{\mathrm{LR}} = \frac{\sigma_{\mathrm{L}} - \sigma_{\mathrm{R}}}{\sigma_{\mathrm{L}} + \sigma_{\mathrm{R}}}, \tag{10}$$

$$A_{\mathrm{FB,pol}} = \frac{(\sigma_{\mathrm{LF}} - \sigma_{\mathrm{LB}}) - (\sigma_{\mathrm{RF}} - \sigma_{\mathrm{RB}})}{\sigma_{\mathrm{LF}} + \sigma_{\mathrm{LB}} + \sigma_{\mathrm{RF}} + \sigma_{\mathrm{RB}}}, \tag{11}$$

where we agree that $\sigma_{\mathrm{F(B)}}$ is the respective cross section when $\cos\theta_{\mathrm{f}} > 0$ ($<$ 0) and $\sigma_{\mathrm{L(R)}}$ is the respective cross section when e^- is left-(right-) handedly polarized. Here, θ_{f} is the $\mathrm{e}^+\mathrm{e}^-$ c.m.s. f production angle relative to the incoming e^- direction. σ_{HD} then has both of the restrictions of σ_{D} and σ_{H}, D = F, B and H = L, R for example. This completely defines the asymmetries which we shall consider.

What we wish to explore is the flavor dependence of the interplay of realistic detector cuts on the one hand and multiple initial state radiative effects on the other hand. This we shall do in two steps as follows. In the first step, we study the interplay of initial state multiple photon effects and cuts for $\mu\bar{\mu} + n(\gamma)$ final states. In the second step we study the flavor dependence by comparing our $\mu\bar{\mu} + n(\gamma)$ results with analogous results for $\mathrm{b}\bar{\mathrm{b}} + n(\gamma)$. In all of our work, the cuts are the MkII SLC/LEP type cuts [9]:

$$|\cos\theta_\mu| < 0.8, \qquad |cos\theta_\gamma| < 0.95,$$
$$E_\mu > 2\ \mathrm{GeV}, \qquad E_\gamma > 0.2\ \mathrm{GeV}, \qquad E_{\mathrm{vis}} > 0.1\sqrt{s}, \tag{12}$$

where E_a is the $\mathrm{e}^+\mathrm{e}^-$ c.m.s. energy of a, $a = \mu, \gamma$, and E_{vis} is the total visible energy of the respective event in the detector. The effect of the multiple photons may be isolated by comparing the results of our YFS2 Fortran simulation with those of a typical 1γ MC of the type in Ref.[10] (B-K-J). In this 1γ MC, the value of the infamous k_0 parameter which separates real unsimulated and real simulated photons is set at 3×10^{-3} to get agreement between the normalizations of the YFS2 and B-K-J simulations. In this way, we arrive at the result in Table I, where we have imposed the further cut on acollinearity in the A_{FB} simulations. The statistics is $\sim 10^5$ events per entry.

We see in Table I that the polarized asymmetries are only moderately affected (a few % effect at most) by the radiative phenomena. This is true for the YFS MC and the 1γ MC. For $A_{\mathrm{FB}}(\mu)$, however, the size of the radiative effects is comparable to $A_{\mathrm{FB}}(\mu)$ itself. For both the 1γ MC and YFS results, the size of the radiative effects on $A_{\mathrm{FB}}(\mu)$ decreases as

TABLE I

$\mu\bar{\mu}$ Asymmetries

No radiative corrections:	
	$A_{LR} = 0.1164$
	$A_{FB,pol} = 0.0767$
	$A_{FB} = 0.00893$
Radiative corrections:	
$1\gamma = $ (B-K-J, $k_0 = 3 \times 10^{-3}$)	$A_{LR} = 0.1139 \pm 0.0012$
YFS	$A_{LR} = 0.1137 \pm 0.0012$
$1\gamma = $ (B-K-J, $k_0 = 3 \times 10^{-3}$)	$A_{FB,pol} = 0.0753 \pm 0.0012$
YFS	$A_{FB,pol} = 0.0755 \pm 0.0012$
1γ (no acol. cut)	$A_{FB} = -0.00779 \pm 0.00166$
YFS (no acol. cut)	$A_{FB} = -0.00463 \pm 0.00166$
1γ (10° acol. cut)	$A_{FB} = -0.00685 \pm 0.00166$
YFS (10° acol. cut)	$A_{FB} = -0.00398 \pm 0.00169$
1γ (3° acol. cut)	$A_{FB} = -0.00373 \pm 0.00167$
YFS (3° acol. cut)	$A_{FB} = -0.00130 \pm 0.00171$
1γ (1° acol. cut)	$A_{FB} = 0.00179 \pm 0.00172$
YFS (1° acol. cut)	$A_{FB} = 0.00331 \pm 0.00170$

TABLE II

$b\bar{b}$ Asymmetries

No radiative corrections:	
	$A_{FB} = 0.0708$
	$A_{LR} = 0.116$
	$A_{FB,pol} = 0.609$
Radiative corrections:	
5×10^5 events (YFS2)	
	$A_{LR} = 0.1126 \pm 0.0012$
	$A_{FB,pol} = 0.6143 \pm 0.0012$
(no acol. cut)	$A_{FB} = 0.0672 \pm 0.0012$
(1° acol. cut)	$A_{FB} = 0.0689 \pm 0.0012$

we tighten the acollinearity cut. Regarding the comparison of 1γ MC and YFS MC results, the two sets of results are generally consistent with one another if one allows for the errors; however, for $A_{FB}(\mu)$, the two results at no acollinearity cut are almost 2σ apart. This suggests, but does not prove, that they may be different. A higher statistics sample is under investigation in this regard. Further, we should note that the closeness of the 1γ and YFS MC results for the polarized asymmetries does not mean that the respective values of $\sigma_{\rm HD}$ are just as close. Indeed, the $\sigma_{\rm HD}$ differ at the 3–4 % level, so that the YFS methods are indeed necessary for the highest precision work. One conclusion from Table I is manifest: to measure $A_{FB}(\mu)$, one must observe a large enough sample of events to unravel the large radiative effect. From Table I, 10^5 $\mu\bar{\mu}$ pairs may not be a large enough sample; the specific required sample will of course depend on the level of accuracy one is seeking.

Turning now to the second step of our asymmetries study, we consider A_{FB}, A_{LR} and $A_{FB,pol}$ for the $b\bar{b} + n(\gamma)$ final states. The cuts are the μ-like cuts in (12), where we now consider A_{FB} for the acollinearity angle cut of $1°$ and for no such cut at all. Using again our YFS2 Fortran MC program, we arrive at the results in Table II. Similarly to the $\mu\bar{\mu} + n(\gamma)$, the effect of the radiation is small on A_{LR} and $A_{FB,pol}$. For A_{FB}, we see a dramatic change in the nature of the radiative effect: the percentage changes due to the radiation are $-5.1\,\%$ for no acollinearity cut and $-2.75\,\%$ for a $1°$ acollinearity cut. This leaves a relatively large asymmetry which appears to be measurable with high precision, in view of the statistics in Table II, in high luminosity unpolarized scenarios such as LEP and its planned upgrade [11]. We should note that our results in Table II are consistent with the semi-analytic results in Ref.[12], where a similar conclusion regarding $A_{FB}(b)$ and its potential for an unpolarized LEP has been reached. Our analysis then verifies this conclusion in the presence of realistic detector cuts and rigorous $n(\gamma)$ radiation.

We wish to stress that, in the $b\bar{b} + n(\gamma)$ case, we have not discussed the effects of tagging, hadronization (QCD), etc. Matters such as these are discussed in the semi-analytic work in Ref.[12] to some extent. What remains to be done is to combine our YFS2 MC with a state of the art QCD generator such as the Lund MC [13], augmented with a realistic detector simulation scenario. This would afford a complete assessment of $A_{FB}(b)$ in Z^0 physics with an unpolarized LEP, for example. We hope to participate in such a study in the not-too-distant future. Our work here illustrates the first phase of this complete assessment.

Turning next to our work on the luminosity regime for SLC and LEP, w call attention to our YFS Monte Carlo program BHLUMI in Ref.[1]. In BHLUMI Fortran we have realized the luminosity cross section $e^+e^- \to e^+e^- + n(\gamma)$ with θ_e, $\theta_{\bar{e}}$ restricted to the region $2m_e/\sqrt{s} \ll \theta \lesssim 250$ mrad, $\theta = \theta_e$, $\theta_{\bar{e}}$, on an event-by-event basis, when the energies of the outgoing leptons are required to stay above some value $x_{\rm cut}$ EBEAM, for $x_{\rm cut} \sim 0.5$ and EBEAM $= \sqrt{s}/2$ at SLC and LEP. We illustrate this in Fig. 4,

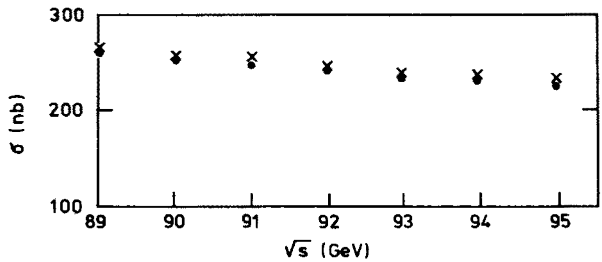

Fig. 4. Luminosity monitor results for $\sqrt{s} \sim M_{Z^0}$; the dots represent our multiple photon Monte Carlo result for $\bar{\beta}_0 + \bar{\beta}_1$; the crosses represent the 1 photon Monte Carlo result of the type of Berends and Kleiss [14]. The statistical error is the size of the dot. The monitor configuration is that of the MINISAM at the MkII at the SLC: 16.2 mrad $\leq \theta_e \leq$ 24.5 mrad, 15.2 mrad $\leq \theta_{\bar{e}} \leq$ 25 mrad, where θ_f is the respective c.m. scattering angle of f, f = e^-, e^+; $E'_e + E'_{\bar{e}} \geq 0.6\sqrt{s}$ in the c.m. system, where E'_f is the final state energy of f, f = e^-, e^+.

where, for the MkII MINISAM cross section, which we define by the cuts 16.2 mrad $\leq \theta_e \leq$ 24.5 mrad, 15.2 mrad $\leq \theta_{\bar{e}} \leq$ 25 mrad, $E'_e + E'_{\bar{e}} \geq 0.6\sqrt{s}$ for $x_{\rm cut}$ = 0.5, where E'_{rmf} is the final state of f in the c.m. system, we show the comparison of the YFS multiple photon results from BHLUMI with the corresponding results from a 1γ Monte Carlo program which is similar to that of Berends and Kleiss in Ref.[14]; this latter program is obtained as a switch in BHLUMI for the user's convenience and we use it in Fig. 4 at the 1γ k_0 parameter of 1×10^{-2} so that its normalization is close to that of our YFS multiple photon result from BHLUMI. We see in Fig. 4 that the 1γ and YFS results are close throughout the luminosity regime. Indeed, at $\sqrt{s}$ = 92 GeV, we get the comparison, for 6×10^5 events, $\sigma(1\gamma) = 246.8 \pm 0.3$ nb and $\sigma(\text{YFS}) = 264.4 \pm 0.7$ nb; these numbers are fortuitously close, in view of the errors, and, if one studies this comparison over the 92 GeV region as it is done in Ref.[15], one may conclude that the 1γ and YFS results are indeed within ~ 1 % of one another for that region. This knowledge that the higher order corrections to the SLC/LEP luminosity are $\lesssim$1 % has already contributed to the discovery [16] by the MkII Collaboration, which has been confirmed by the ALEPH Collaboration and subsequently by the remaining LEP Collaboration, that the number of massless neutrino generations, N_ν, is 3, since the uncertainty on the effect of the higher order corrections gives an uncertainty to the absolute luminosity measurement which generates an uncertainty on the absolute normalization of the observed cross sections at SLC and LEP and it is from the latter that the shape and peak parameters of the Z^0 line shapes are determined. These parameters then determine the respective visible and invisible widths, the latter of which is directly related to the value of N_ν.

What remains to be done in this area of our investigations is to establish the absolute normalization of our BHLUMI simulations to the 0.2 % and below 0.2 % regimes respectively, as the statistical errors at LEP are now (September, 1990) $\sim$ 0.1 % on luminosity and the systematic errors are $\sim$ 0.7 % currently and are expected to be improved to the 0.2–0.3 % regime eventually [17] and the error on the radiative corrections should be at or below 1/3 of this systematic error in order that it does not play an unacceptable role in the respective physics analysis. Such accuracies on the absolute normalization are indeed possible and, recently, we have made progress in achieving them [18].

The high precision work on the luminosity cross section simulation (and measurements) then naturally lead to the question of a corresponding accuracy of the non-luminosity cross section, such as $e^+e^- \to \mu\bar{\mu} + n(\gamma)$, for example. In our YFS2 Fortran program, we already have an event generator which simulates the respective initial state radiation effects at the 0.1 % level. In practice, it is oftentimes efficient, for certain inclusive distributions, to use semi-analytic one-dimensional integral representations of a given cross section in which the higher order corrections are represented using the naive exponentiation procedure of Jackson, Scharre and Tsai [5]; this can be particularly useful if one is studying the μ-pair line shape for the Z^0, for example. Indeed, in Ref.[4], Berends *et al.* have considered five different improvements of the Kuraev–Fadin type of the methods in Ref.[5], one of which is based on our rigorous YFS results in Ref.[1]. The conclusion of Berends *et al.* is that the five different forms of improved naive exponentiation are with 0.2–0.3 % of one another, with no definitive way of distinguishing between them. Clearly, in the regime of 0.1 % physics at LEP (and, perhaps, at SLC), such a conclusion needs to be sharpened. Recently [19], we have been able to argue that the naive improved exponentiation procedure based on our YFS Monte Carlo methods is in fact the best such procedure among the five procedures in Ref.[4].

Specifically, we have analyzed the naive exponentiation techniques in a "toy model" which was developed by one of us (S. J.) in collaboration with Skrzypek in Ref.[20]. This toy model consists of an exact solution for the all orders leading-log results for the simultaneous fragmentation of two incoming beams (e^+e^-); this solution is constructed *via* a Monte Carlo method using the representation

$$\varrho^{(\infty)}(v) \doteq \int_0^1 dv_1 \int_0^1 dv_2\, \delta\big(1 - v - (1 - v_1)(1 - v_2)\big)\, D(v_1)D(v_2), \qquad (13)$$

where the exact solution for the one beam energy loss is given by the following integral

$$D(v) = \exp\left[\tfrac{1}{2}\gamma \ln\left(\epsilon/(1-v)\right) + \tfrac{3}{4}\gamma\right]$$
$$\times \sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{\gamma}{2}\right)^n \prod_{i=1}^{n} \int_{\epsilon}^{v} \frac{dx_i}{x_i} \delta\left(v - \sum_{i=1}^{n} x_i\right)$$
$$\times \prod_{i=1}^{n} \chi\left(x_i\left(1 - \sum_{j=1}^{i-1} x_j\right)^{-1}\right) \theta\left(x_i - \frac{\epsilon}{1-v}\left(1 - \sum_{j=1}^{i} x_j\right)\right), \quad (14)$$

where $\chi(x) = \frac{1}{2}(1 + (1-x)^2)$ and $\gamma = (2\alpha/\pi)\left(\ln(s/m_e^2) - 1\right)$. Hence, we can compare any given improved naive exponentiation procedure A with our exact solution (13) by comparing (13) to the respective prediction of the procedure for the simultaneous fragmentation of two incoming beams, $\varrho_A(v)$. For example, the formula of Kuraev and Fadin (as it would be realized by use of the result of Berends *et al.* [8]) would give

$$\varrho^{(2)}_{\rm KF} = F(\gamma)\,\gamma v^{\gamma-1}\left\{1 + \tfrac{3}{4}\gamma + \tfrac{1}{2}(\tfrac{3}{4}\gamma)^2\right\}$$
$$+ \gamma(\tfrac{1}{2}v - 1)(1 + \gamma \ln v)$$
$$+ \gamma^2\left(-\frac{3}{4} + \frac{v}{8} - \frac{1 + 3(1-v)^2}{8v}\ln(1-v)\right), \quad (15)$$

whereas, in Ref.[1], we have found that our YFS Monte Carlo procedure in the program YFS2 Fortran is, for $\varrho(v)$, very well approximated by

$$\varrho^{(2)}_{\rm YFS} = F(\gamma)\,\gamma v^{\gamma-1} e^{\gamma/4}\left\{1 + \tfrac{1}{2}\gamma + \tfrac{1}{2}(\tfrac{1}{2}\gamma)^2 + v(\tfrac{1}{2}v - 1)\right.$$
$$\left. + \gamma\left[-\frac{v}{2} - \frac{1 + 3(1-v)^2}{8}\ln(1-v)\right]\right\}, \quad (16)$$

where $F(\gamma) \equiv \exp(-C\gamma)/\Gamma(1+\gamma) \simeq 1 - \pi^2\gamma^2/12$, and $C = 0.5772156\ldots$ is Euler's constant. These formulas (15) and (16) are two of the *ansatz's* considered in Ref.[4]. In Fig. 5, we show comparison of our $\varrho^{(\infty)}$ result with (15) and (16), as well as with the "so-called" most complete *ansatz*, Eq.(3.31), in Ref.[4]. What we see is that, indeed, in the 0.1% regime, the YFS-based formula is preferred. We conclude then, that for the high precision Z^0 physics, either our YFS Monte Carlo procedures [1] or, for inclusive analysis, the YFS-based formula (16) should indeed be used.

Perhaps, one of our more important applications is in the area of the interplay of the YFS multiple photon radiation and the pure weak corrections to $e^+e^- \to \gamma$, $Z^0 \to X$, as we illustrated in Fig. 6. Indeed, in Ref.[6],

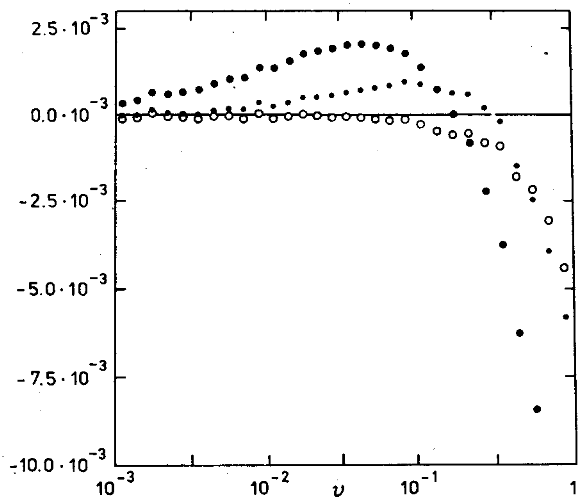

Fig. 5. The ratio of the second order exponentiated distribution $\varrho^{(2)}(v)$ and the infinite order solution $\varrho^{(\infty)}$ calculated by means of the Monte Carlo ($10^6 - 4 \cdot 10^{-6}$ MC events per point). Numerical input was set for e^+e^- beams at $\sqrt{s} = 92\,\text{GeV}$. Three curves represent exponentiation of the type (a) Kuraev–Fadin, see Eq.(15), (b) YFS, see Eq.(16) and (c) LEP workshop [4].

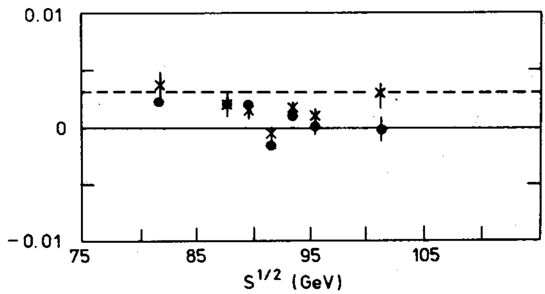

Fig. 6. Comparison of Stuart and Hollik *et al.* pure electroweak libraries in $e^+e^- \to \mu^+\mu^-$. Here, $M_{Z^0} = 92\,\text{GeV}$, $m_t = 60\,\text{GeV}$, and $m_H = 100\,\text{GeV}$. In both cases, the libraries were obtained from its principal author *via* private communication. Note that vvmax is the maximum value of $1 - s'/s = v$, where $s = (p_e + p_{\bar{e}})^2$ and $s' = (p_\mu + p_{\bar{\mu}})^2$. (This set of values for M_{Z^0} and m_t is of mainly pedagogical interest now.)

YFS2 has been interfaced to two different pure weak correction libraries, one by R. Stuart and one by W. Hollik in the program KORALZ3. (In collaboration with Z. Wąs, we are in the process of interfacing YFS2 to the weak corrections library of Bardine *et al.* [21].) Hence, KORALZ3 has now available for use the state-of-the-art multiple photon and pure weak corrections to $e^+e^- \to f\bar{f}$ where for the latter corrections, we have implemented them as provided by the respective authors. Accordingly, we may show in Fig. 6 the comparison of the total cross section, as an illustration, for $e^+e^- \to \mu^+\mu^- + n(\gamma)$ for the case that one uses the Stuart library or the Hollik library [22,23]. What we see is that the two weak libraries produce the same cross section at the 0.3 % level. If we look at the *available* literature [21–24] from the various authors of weak corrections libraries, we see that, for $A_{FB}(\mu)$ at the Z^0 pole, the authors differ significantly as we show in Table III (July, 1989). These discrepancies are being worked-on by the authors [23] and by us. Such discrepancies as those in Fig. 6 and Table III are unacceptable for the truly < 0.1 % physics regime which LEP will want to probe in the not-too-distant future. Currently, however, the systematic errors on total cross sections at LEP/SLC are $\sim$ 0.7 %, so that the pure weak corrections libraries in KORALZ3 are currently accurate enough for use at LEP and SLC. And, indeed, it is in use at LEP and SLC for physics analysis.

TABLE III

Comparison of A_{FB} for $e^+e^- \to \mu^+\mu^-$; $M_{Z^0} = 90\,\text{GeV}$, $m_H = 100\,\text{GeV}$.

Authors	m_t [GeV]				
	50	100	150	200	
Bardin *et al.*	0.0036	0.0044	0.0050	0.0054	
Hollik *et al.*	0.0037	0.0043	0.0054	0.0071	
Lynn *et al.*	0.0038*	0.0041*	0.0035*	0.0012*	
	m_t [GeV]				
	60	90	130	180	230
Lynn *et al.*	0.0037	0.0041	0.0040	0.0028	0.0012

* Obtained by linear interpolation from the published results at m_t = 30, 60, 90, 130, 180 and 230 GeV.

Thus, the YFS MC approach to 1 % precision SU(2L) $\times$ U(1) radiative corrections is in active use at SLC and LEP. We look forward to future applications with even higher precision and/or at higher energies.

4. Future Directions

The natural question to pose at this point is what should be the next areas of investigation for our YFS MC approach to SU(2L) $\times$ U(1) radiative corrections. We discuss the answer to this question in this Section.

Specifically, what is not implemented in our YFS2 MC are the final state radiative effects. These are handled at $O(\alpha)$ by KORALZ3 but a complete treatment of the $n(\gamma)$ final state radiation is not implemented in currently available software. We have recently completed the construction of such a final state $n(\gamma)$ version of our YFS Fortran Monte Carlo's; it is version YFS3 and we shall present it in detail in Ref.[25]. These final state effects, which enter at the level of $\alpha/\pi \simeq 0.2$ %, are clearly necessary to probe the below 0.1 % regime in Z^0 physics.

Perhaps, one of our most immediate issues is to resolve the 0.3 % cross section discrepancies between the pure weak libraries in KORALZ3 Fortran. Clearly, the high precision ($\simeq 0.1$ %) regime which our YFS $n(\gamma)$ methods would otherwise avail to us will not be accessible if we do not resolve this pure weak discrepancy.

The most pressing issue is, of course, the normalization process $e^+e^- \rightarrow e^+e^- + n(\gamma)$ at LEP and SLC. As we have noted above, the near term regime of 0.2 % for the absolute normalization uncertainty is a near term goal on which we have made recent progress [18]. We should follow-up on the work in Ref.[18] by extending it to the below 0.1 % regime in the not-too- distant future.

Concommitent with our normalization research in connection with BHLUMI Fortran is our effort to extend BHLUMI to wide angles. The corresponding version of BHLUMI, version 3.0, is in its initial stages of testing. We hope to make it available in the near term.

We should also note the various applications of our work to other electroweak physics scenarios. In particular, we have in mind the NLC/LEP II type scenarios. Recently, we have been able to initiate the application of our YFS methods to the process $e^+e^- \rightarrow W^+W^- + n(\gamma)$. Preliminary MC results have been obtained and we hope to arrive at a complete application of our YFS methods to the NLC/LEP II scenario in the not-too-distant-future.

5. Conclusions

In conclusion, it may be said that our YFS methods are indeed of significant use in the precision tests of the electroweak theory in Z^0 physics. We are encouraged by their utilization at LEP and SLC to date.

However, several outstanding issues, each interesting in its own right, must be addressed before we can talk about a complete application of our methods to the Z^0 physics programs at SLC/LEP; below 0.1 % absolute normalization uncertainty for BHLUMI Fortran, wide angles extension of BHLUMI Fortran, resolution of the pure weak libraries discrepancy in KORALZ3, and inclusion of $n(\gamma)$ final state radiation in our YFS Monte Carlo procedures. In all cases, we are making progress on the respective issue and we look forward to its resolution in the near term.

Further, we have initiated the application of our methods to the higher energy scenarios at NLC/LEP II. We look forward to a complete application of our methods to these higher energy scenarios in the near term also.

In summary, the YFS Monte Carlo approach to higher radiative corrections is in place at SLC and LEP and, more importantly, it is in use at both scenarios. We anticipate with excitement its many further applications.

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