# SOME REMARKS CONCERNING THE B.R.S.T. TRANSFORMATION*

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We present a new method of deriving under some mild assumptions the most general options for the B.R.S.T. transformation, without having recourse either to Lagrange or Feynman's integral-over-all-paths formalisms. It turns out that these different variants can be reduced eventually to two cases, from which one encompasses the conventional B.R.S.T. transformation.

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I am happy to be the first speaker at the meeting. One hour of emotions and stress and I am a free man. To quote Wilhelm Busch from *Fliegende Blätter:*

*Ist der Ruf erst ruiniert*
*Lebt es sich ganz ungeniert.*

Nevertheless – I hope – the subject I chose for my talk will be of some interest to this audience. It is related to some work done few years ago by Dr Zumino, who is participating in this School and is present here at this talk. As a matter of fact his clearly written lectures on gauge theories and anomalies, given in Les Houches in 1983 [1], as well as of Wess, given in Dubrovnik in 1986 [2], helped me to understand the subject better and stimulated my own activities in this field.

My talk will be devoted to some speculations concerning so called B.R.S.T. transformations. Some people use the catchword B.R.S., short for Becchi, Rouet, Stora [3]; I added also the letter T standing for Tyutin

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[4] whose work was accomplished approximately at the same time but was originally overlooked by most of the physicists. Usually the B.R.S.T. symmetry transformations arise as a substitute of gauge transformations as soon as one introduces so called ghost fields of Faddeev and Popov into the original framework of non-abelian gauge field theories. The standard procedure is to apply the functional formalism of Feynman, the integral-over-all-paths procedure, which in turn makes use of the Lagrange formalism as a tool [5]. The subintegral functional is gauge covariant, an unwanted feature under these circumstances. So this symmetry gets broken by introducing a gauge fixing term. It turns out, however, that after introducing the fictious fields, the Faddeev–Popov ghost field mentioned above arising from the Jacobi determinant of the integral, a new symmetry of the Lagrangian is born. This is just the B.R.S.T. symmetry.

In this approach the ghost fields appear rather as a technical artifact void of deeper physical meaning. The significance of this technique for recent developments in gauge field theories seems, however, to indicate that the appearance of the ghost fields can be a physically deeper rooted phenomenon. In this note we try to refrain from the approach which uses the integral-over-all-paths method and from the Lagrange formalism. We are aware of the fact that there exist a vast literature concerning the problem of ghost fields as well as B.R.S.T. transformation in which no use of Lagrange formalism is made [6]. The approach presented here differs, however, from that displayed there [7].

Incidentally, I shall underscore my conviction, based maybe upon emotions, that the Grassmannian variables are only a technical device and one could do very well without using them. We are going to use differential forms instead.

We start with the standard derivation of the B.R.S.T. transformation, which does not have recourse either to Feynman or Lagrange formalism (see [1] and [2]). One introduces a hermitian 1-form gauge field located in the 4-dimensional Minkowski space and belonging to Lie algebra of some compact semisimple gauge group, *viz.*

$$a(x) \equiv \tau_r a^r_\mu(x) dx^\mu \equiv a_\mu dx^\mu ,$$

where $a^r_\mu(x) = \overline{a^r_\mu(x)}$ is a classical gauge field, $x = (x_0, x_1, x_2, x_3)$, $\mu = 0,1,2,3$, $r = 1,\ldots,n$ and $\tau_r$ are finite dimensional hermitian matrices representing the generators of the compact semisimple gauge group G.
[ **Note:**

$$\begin{aligned} i[\tau_p, \tau_r] &= -c_{prs}\tau_s , \\ \mathrm{tr}(\tau_r\tau_s) &= \delta_{rs}, \end{aligned}$$

the structure constants $c_{prs}$ are antisymmetric in all three indices.]

These generators are, of course, $x$ independent. The gauge group itself is presented by unitary 0-form matrices $g(x,t)$, where $t$ stands for the set of $n$ group parameters $t=(t_1,\dots,t_n)$. Although $t=t(x)$ one does not go wrong by considering $x$ and $t$ as independent variables. For instance

$$dt^r\frac{\partial}{\partial t^r} \quad \text{stands just for} \quad \int dx\, dt^r\,(x)\frac{\partial}{\partial t^r(t)}\,.$$

Using $a$ and $g$ one may consider a new hermitian field, a 1-form with respect to the variables $x$ and belonging to the Lie algebra of the group G, *viz.*

$$A(x,t)=\tau_r A^r_\mu(x,t)dx^\mu \equiv g^{-1}ag+ig^{-1}dg\,, \tag{1}$$

where the 1-form in $x$

$$dg=\frac{\partial g}{\partial x^\mu}dx^\mu\,.$$

In case $a=0$ one gets the pure gauge field

$$A^0(x,t)=ig^{-1}dg\,. \tag{2}$$

One may vary $g$ or $A$ with respect to the group parameters. With the notation

$$\delta g\equiv\frac{\partial g}{\partial t^r}\,dt^r$$

(it is a 1-form with respect to the variables $t$) one gets from (1)

[ **Note:** Since $g^{-1}g=1$ we have $\delta(g^{-1})=-g^{-1}\delta g\,g^{-1}$ as well as $d(g^{-1})=-g^{-1}dg\,g^{-1}$. Taking into account that $a$ does not depend on $t$ and is a 1-form we get

$$\begin{aligned}\delta A &= -g^{-1}\delta g\,g^{-1}ag-g^{-1}a\delta g-ig^{-1}\delta g\,g^{-1}dg-ig^{-1}d\delta g\\ &= -g^{-1}\delta g\,A-Ag^{-1}\delta g+ig^{-1}dg\,g^{-1}\delta g-ig^{-1}d\delta g\\ &= -g^{-1}\delta g\,A-Ag^{-1}\delta g-id(g^{-1})\delta g-ig^{-1}d\delta g\\ &= -g^{-1}\delta g\,A-Ag^{-1}\delta g-id(g^{-1}\delta g)\end{aligned}$$

and we end up with (3).]

$$\delta A=-dv^0+iv^0A+iAv^0\,, \tag{3}$$

where

$$v^0=\tau_r(v^0)^r_s\,dt^s\equiv ig^{-1}\delta g \tag{4}$$

is a 1-form in $t$. The latter is just the Faddeev–Popov ghost field. $\delta A$ is a mixed 2-form in Minkowski as well as group parameters spaces. Notice that

$(v^0)^r_s$ is a classical field, commuting with itself as well as with other classical fields. We should keep in mind that

$$dt^r\, dx^\mu = -dx^\mu\, dt^r\,,$$
$$dx^\mu dx^\nu = -dx^\nu\, dx^\mu\,,$$
$$dt^r\, dt^s = -dt^s\, dt^r\,.$$

From (4) follows

$$\delta v^0 = i(v^0)^2\,. \tag{5}$$

One may now define the field strength as follows

[ **Note:**

$$(A^2)^+ = -A^2 \text{ when } A^a_\mu = \overline{A^a_\mu} \text{ in } A = \tau^a A^a_\mu dx^\mu.]$$

$$F(x,t) \equiv g^{-1} f g = dA - iA^2 \tag{6}$$

with

[**Note:** $da = \partial_\mu a_\nu dx^\mu dx^\nu = \frac{1}{2}(\partial_\mu a_\nu - \partial_\nu a_\mu) dx^\mu\, dx^\nu$ ]

$$f \equiv \tfrac{1}{2}\tau_r f^r_{\mu\nu} dx^\mu\, dx^\nu \equiv \tfrac{1}{2} f_{\mu\nu} dx^\mu\, dx^\nu = da - ia^2\,.$$

The field $F$ can be viewed as curvature in the space with connection $A$. For a pure gauge field $A^0$ this curvature vanishes, *viz.*

$$dA^0 = i(A^0)^2\,. \tag{7}$$

Relations (2) and (7) have similar structure to (4) and (5) *mutatis mutandis.* Indeed, (5) expresses the fact that the Lie group is flat with respect to its own geometry, the content of the Cartan–Maurer theorem [8]. This becomes clear if we write (5) in the form

$$\partial_r (v^0)^k_s - \partial_s (v^0)^k_r = i c_{lmk} (v^0)^l_r (v^0)^m_s$$

with

$$[\tau_l, \tau_m] = i c_{lmp} \tau_p\,.$$

To get the B.R.S.T. transformation one proceeds as follows. One forgets about the definitions (1) and (4) as well as about the derivation of the formulae (3) and (5) and just defines the transformation

$$sA = -dv^0 + i(v^0 A + A v^0)\,, \tag{8}$$

$$sv^0 = i(v^0)^2\,, \tag{9}$$

which we are going to call the B.R.S.T. transformation. Here $s$ is a 1-from in the parameter space, *viz.*

$$s = s_r \, dt^r .$$

Notice that we have

$$d^2 = s^2 = ds = sd = 0 . \tag{10}$$

Incidentally, relations (8) and (9) supplemented by (6) and

$$sF = i(v^0 F - F v^0)$$

are invariant under the Stora transformations [9]

$$\begin{aligned} d &\to d + s \equiv \tilde{d} , \\ s &\to s , \\ A &\to A + v^0 \equiv \tilde{A}^0 , \\ v^0 &\to v^0 , \\ F &\to F . \end{aligned}$$

Then (8), (9) and (6) can be written concisely

$$F = \tilde{d}\tilde{A}^0 - i(\tilde{A}^0)^2 . \tag{11}$$

To conform to the standard procedure we need still two more fields, the so called anti-ghost field and the gauge fixing field.

To justify the appearance of the anti-ghost field let us call attention of the reader that transformation (9) of $v^0$ was adopted because of formulae (4) and (5). Notice that it would be quite justified to call $v^0$ a pure Faddeev–Popov ghost field, in analogy to $A^0$. It is quite reasonable to ask the question: why do not we extend $v^0$ to $v$ in a similar way as we used to extend $A^0$ to $A$[10]? If we do so we get

$$v \equiv \tau_r v^r_s \, dt^s$$

which satisfies the relation

$$C = sv - iv^2 \tag{12}$$

instead of (9). Here $C$ is an additional field,

$$C \equiv \tfrac{1}{2} \tau_p C^p_{rs} dt^r \, dt^s ,$$

a 2-form in $t$ and a curvature implemented by $v$. Both fields $v^r_s$ and $C^p_{rs}$ are classical fields. Unfortunately, the relation

$$d^2 C = id(AC - CA) = 0$$

derived from (12) and (8), where $v^0$ has to be replaced by $v$, imposes a stringent condition, *viz.*

$$FC = CF\,. \tag{13}$$

The same constraint follows from

$$s^2 F = is(vF - Fv)\,.$$

To get rid of (13) we are forced to introduce, in addition to $C$, a further new field

$$\bar{v} = \tau_r \bar{v}^r_{\mu s} dx^\mu\, dt^s$$

and replace (8) by

$$sA + dv = i(vA + Av) + \bar{v}\,. \tag{14}$$

Relations (6), (12) and (14) are consistent as far as the action of $s$ and $d$ operations are concerned and display a complete reflection symmetry

$$\begin{aligned} d &\leftrightarrow s\,,\\ F &\leftrightarrow C\,,\\ A &\leftrightarrow v\,,\\ \bar{v} &\leftrightarrow \bar{v}\,. \end{aligned}$$

The equation (11) has to be replaced by

$$\tilde{F} = \tilde{d}\tilde{A} - i\tilde{A}^2\,,$$

where

$$\begin{aligned} \tilde{F} &\equiv F + C + \bar{v}\,,\\ \tilde{A} &\equiv A + v\,. \end{aligned}$$

The last relations show clearly that, in principle, we are dealing here with a gauge field theory in a $(4+n)$-dimensional space. The Minkowski and parameter spaces are treated on the same footing and are to some extent akin to a Kaluza–Klein theory. This is not our goal as far as this presentation is concerned. We want to cling to the original qualitative asymmetry between the Minkowski and parameter spaces in our present approach. To achieve that it is enough to put either $C$ or $F$ equal to zero. For physical reasons we put

$$C = 0\,.$$

As far as $\bar{v}$ is concerned we may keep it and we are going to do so.

We introduce still one more field

$$Q = \tau_p Q^p_{\mu r s}\, dx^\mu\, dt^r\, dt^s\,, \tag{15}$$

which we are going to call the gauge fixing field. The motivation for introducing such a field is only partly of conventional nature. Another reason for having such a field is that from a rotation of a vector $A$, *viz.*

$$dA$$

no conclusion whatsoever can be drawn as far as the divergence of it is concerned, *viz.*

$$d^*A\,.$$

We believe that $Q$ is somehow linked to $d^*A$ [10].

Before we enter into further considerations concerning the probing of the most general variants of the B.R.S.T. transformations we have to settle the fundamental problem, which fields can be viewed as variables independent from each other. So far we dealt with the fields $A, v, F, \bar{v}$ and $Q$ as well as $dA$ and $dv$. Further candidates are $dF, d\bar{v}$ and $dQ$ . Two of these fields can be easily excluded. Because of (6) either $F$ can be given in terms of $dA$ and $A$ or $dA$ in terms of $F$ and $A$. So one of them is redundant. We are going to keep $F$ as a variable. Notice that $F$ does not vanish identically by assumption. Notice also that in case $\bar{v}$ *versus* $dv$ in relation (14), the situation differs from the one in (6) since in (14) enters the B.R.S.T. transform of $A, sA$ and $\bar{v}$ cannot be defined in terms of $dv$ and other fields mentioned before using this relation only. Therefore we shall keep $\bar{v}$ and $dv$ as independent variables. As far as $dF$ is considered we have the Bianchi identity

$$dF = \; - AF + FA$$

which follows directly from (6).

We make an additional assumption, hiding in it an element of arbitrariness, namely we assume that the fields $d\bar{v}$ and $dQ$ depend on the remaining fields treated as independent from each other.

So we are left with the independent variables $A, v, F, \bar{v}, Q$ and $dv$, all belonging to the Lie algebra of $G$. The first step we take is to evaluate $d\bar{v}$ and $dQ$. The most general *ansatz* for the differential forms, belonging to the Lie algebra, reads

$$\begin{aligned} d\bar{v} &= \sigma_1(\bar{v}A - A\bar{v}) + \sigma_2(Fv - vF) \\ &\quad + \sigma_3(Adv - dv\,A) + \sigma_4(vA^2 - A^2v)\,, \end{aligned} \tag{16}$$

$$\begin{aligned} dQ &= \phi_1(dv\,\bar{v} - \bar{v}dv) + \phi_2(Fv^2 - v^2F) \\ &\quad + \phi_3(A^2v^2 - v^2A^2) + \phi_4(AQ - QA) \\ &\quad + \phi_5(\bar{v}Av - vA\bar{v} + \bar{v}vA - Av\bar{v}) \\ &\quad + \phi_6(vA\bar{v} - \bar{v}Av - v\bar{v}A + A\bar{v}v) \\ &\quad + \phi_7(dv\,Av - vAdv + dv\,vA - Avdv) \\ &\quad + \phi_8(vAdv - dv\,Av - vdv\,A + Adv\,v). \end{aligned} \tag{17}$$

Here $\sigma$'s and $\phi$'s are numerical coefficients to be evaluated. Notice that the term $\bar{v}^2$ does not enter on the r.h.s. of (17) as it does not belong to the Lie algebra, *viz.*

$$\bar{v}^2 = \tfrac{1}{2}(\tau_k\tau_l + \tau_l\tau_k)\bar{v}^k_{\mu r}\bar{v}^l_{\nu s} dx^\mu dt^r dx^\nu dt^s .$$

To evaluate the numerical coefficients in (16) and (17) we have to examine $d^2\bar{v} = 0$ and $d^2Q = 0$, where we made use of (10). From the requirement

$$d^2\bar{v} = 0$$

and the independence of the variables $A, v, F, \bar{v}, Q$ and$dv$ follows the relation

$$d\bar{v} = \sigma(Fv - vF - Adv + dv\,A - ivA^2 + iA^2v).$$

$$(\sigma \equiv \sigma_2 = -\sigma_3 = \sigma_4, \quad \sigma_1 = 0).$$

Because of (6) we have

$$d\bar{v} = \sigma(dA\,v - vdA - Adv + dv\,A) = \sigma d(Av + vA).$$

Hence we may replace $\bar{v}$ by a new variable, say,

$$\bar{v}' \equiv \bar{v} - \sigma(Av + vA).$$

Dropping the prime sign, (16) reduces to

$$d\bar{v} = 0. \tag{18}$$

In a similar fashion the requirement $d^2Q = 0$ yields

$$dQ = \phi(dv\,\bar{v} - \bar{v}\,dv) = \phi d(v\bar{v} - \bar{v}v) \quad (\phi \equiv \phi_1,\ \phi_j = 0\ j = 2, \ldots, 8).$$

We introduce a new variable,say,

$$Q' \equiv Q - \phi(v\bar{v} - \bar{v}v).$$

Dropping again the prime we get

$$dQ = 0. \tag{19}$$

Now we are ready to investigate the most general variants of the B.R.S.T. transformations. We recapitulate once more the standard formulation which reads $(v^0 = v)$

$$\begin{aligned} sA &= -dv + i(vA + Av), \\ sv &= iv^2, \\ s\bar{c} &= B, \\ sF &= i(vF - Fv), \\ sB &= 0. \end{aligned}$$

Here $d\bar{c}$ is linked to our $\bar{v}$ and $(-dB)$ to our $Q$. With this identification (18) and(19) are trivially satisfied.

Our conjecture is as follows. We are going to explore the action of

$$s = s_\tau \, dt^\tau$$

upon the fields $A, v, \bar{v}, F$ and $Q$ given as the most general forms with respect to the before mentioned fields and $dv$, confined to the Lie algebra. We have the *ansatz*

$$sA = \alpha_1 dv + \alpha_2(vA + Av) + \alpha_3\bar{v}\,, \tag{20}$$

$$sv = \beta v^2\,, \tag{21}$$

$$s\bar{v} = \gamma_1(v\bar{v} - \bar{v}v) + \gamma_2 Q + \gamma_3(v\,dv - dv\,v) + \gamma_4(Av^2 - v^2A), \tag{22}$$

$$sF = \epsilon_1(\bar{v}A - A\bar{v}) + \epsilon_2(Fv - vF) + \epsilon_3(A\,dv - dv\,A) + \epsilon_4(vA^2 - A^2v)\,, \tag{23}$$

$$\begin{aligned} sQ = {} & \xi_1(Qv + vQ) + \xi_2(dv\,v^2 - v^2dv) \\ & + \xi_3(\bar{v}v^2 - v^2\bar{v}) + \xi_4(Av^3 - v^3A - v^2Av + vAv^2)\,. \end{aligned} \tag{24}$$

The coefficients $\alpha, \beta, \gamma, \epsilon$ and $\xi$ are numerical coefficients to be evaluated using the consistency relations (10).

Some comments are in order. To make it plain why in Eqs (20)–(24) $vA + Av$ instead of $\bar{v}A - A\bar{v}$ has to be used let us look closer to their structure. We have

$$vA + Av = v_\tau A_\mu dt^\tau dx^\mu + A_\mu v_\tau dx^\mu dt^\tau = (v_\tau A_\mu - A_\mu v_\tau) dt^\tau dx^\mu$$

and

$$\begin{aligned} \bar{v}A - A\bar{v} &= \bar{v}_{\mu\tau} A_\nu dx^\mu dt^\tau dx^\nu - A_\nu \bar{v}_{\mu\tau} dx^\nu dx^\mu dt^\tau \\ &= (\bar{v}_{\mu\tau} A_\nu - A_\nu \bar{v}_{\mu\tau}) dx^\mu dt^\tau dx^\nu\,. \end{aligned}$$

Thus the proper sign guarantees that these expressions belong to Lie algebra of the gauge group.

To explain our method, used to evaluate the numerical coefficients, let us look first to a trivial case which,as a matter of fact, does not lead to any definite conclusion but is simple enough to illustrate clearly the idea behind the procedure used by us. Let us consider

$$sv = \beta v^2\,.$$

On the one hand we have

$$s^2 v = 0\,.$$

On the other hand

$$s^2 v = \beta(sv\,v - v\,sv) = \beta(\beta v\,v - v\,\beta v) = 0\,.$$

Hence in this case we do not gain any information on $\beta$. The method becomes more efficient as soon as applied to

$$\begin{aligned} ds\,\bar{v} &= 0\,, \\ s^2\bar{v} &= 0\,, \\ ds\,Q &= 0\,, \\ s^2 Q &= 0\,, \end{aligned} \tag{25}$$

where we made use of (18) and (19), as well as of

$$ds\,A = -sd\,A \quad \text{and} \quad s^2 A = 0\,.$$

Other relations, like $d^2 A = s^2 v = d^2 F = s^2 F = (ds + sd)F = 0$ do not provide us with any new information.

So let us look more closely to the case (25) as another exemplification of our method. We get

$$\begin{aligned} \gamma_1(dv\,\bar{v} - \bar{v}\,dv) &+ \gamma_4(dA\,v^2 - Adv\,v + Av\,dv - dv\,vA + vdv\,A - v^2 dA) \\ &= \gamma_1(dv\,\bar{v} - \bar{v}dv) + \gamma_4(Fv^2 + iA^2v^2 - Adv\,v \\ &\quad + Avdv - dv\,vA + vdv\,A - v^2F - iv^2A^2) = 0 \end{aligned}$$

or

$$\gamma_1 = \gamma_4 = 0\,.$$

Proceeding in this way (tedium!) we arrive at the final result which reads as follows.

If we discard the uninteresting case $\alpha_1 = \alpha_2 = \alpha_3 = 0$, we have to distinguish between two options

(i) $\alpha_3 = 0(|\alpha_1|^2 + |\alpha_2|^2 \neq 0)$,
(ii) $\alpha_3 \neq 0$.

The case (i) corresponds to

$$\begin{aligned} sA &= \alpha dv + \beta(vA + Av)\,, \\ sv &= \beta v^2\,, \\ s\bar{v} &= \gamma Q + \gamma'(v\,dv - dv\,v)\,, \\ sF &= \beta(A\,dv - dv\,A - Fv + vF) + i\alpha(A\,dv - dv\,A)\,, \\ sQ &= 0\,. \end{aligned}$$

For $\beta \neq 0$ we may redefine $\bar{v}$ to

$$\bar{v} \equiv \bar{v} - \frac{\gamma'}{\beta}\, dv$$

which yields (we drop the prime sign)

$$s\bar{v} = \gamma Q.$$

In case (ii) we have ($\alpha_3 \equiv \alpha \neq 0$)

$$\begin{aligned} sA &= \alpha_1\, dv + \alpha\bar{v}\,, \\ sv &= \beta v^2\,, \\ s\bar{v} &= -\frac{\alpha_1}{\alpha}\beta(v\,dv - dv\,v) = -\frac{\alpha_1}{\alpha}s(dv)\,, \\ sF &= \alpha(\bar{v}A - A\bar{v}) - \alpha_1(A\,dv - dv\,A), \\ sQ &= 0. \end{aligned}$$

After redefining $\bar{v}$ to

$$\bar{v}' = \bar{v} + \frac{\alpha_1}{\alpha}\, dv$$

we get (dropping the prime)

$$\begin{aligned} sA &= \alpha\bar{v}, \\ s\bar{v} &= 0, \\ sF &= \alpha(\bar{v}A - A\bar{v}). \end{aligned}$$

In case (i) for $\beta \neq 0$ and $\gamma \neq 0$ we may introduce the notation

$$\frac{\alpha}{\beta} \to \alpha \qquad \beta v \to iv \qquad \gamma Q \to Q\,.$$

Then

$$\begin{aligned} sA &= \alpha\, dv + i(vA + Av)\,, \\ sv &= iv^2\,, \\ s\bar{v} &= Q\,, \\ sF &= i(A\,dv - dv\,A - Fv + vF) + i\alpha(A\,dv - dv\,A)\,, \\ sQ &= 0\,. \end{aligned}$$

Only one parameter $\alpha$ remains free. For $\alpha = -1$ this coincides with the standard relation, provided $Q = -dB$ and $\bar{v} = d\bar{c}$.

In case (ii) for $\alpha \neq 0$ and $\beta \neq 0$ with

$$\alpha\bar{v} \to \bar{v}\,, \qquad \beta v \to iv$$

we get

$$\begin{aligned} sA &= \bar{v}\,, \\ sv &= iv^2\,, \\ s\bar{v} &= 0\,, \\ sF &= \bar{v}A - A\bar{v}\,, \\ sQ &= 0\,. \end{aligned}$$

In this case no free parameter is left.

If we ascribe to the "physical" fields $A, Q, F$ the zero value of the Faddeev–Popov charge and assume that the $d$-operation leaves this charge unchanged, we get in case (i) that $v$ has a charge opposite to $\bar{v}$ and the $s$-operation changes the charge from zero to that of $v$. In case (ii) the fields $v$ and $\bar{v}$ acquire the same charge as that carried by $s$.

The task to construct Lagrangians invariant under the transformations (i) and (ii) seems to be not so difficult; we did not yet, however, try to find them. It seems that the case (ii) is of limited physical interest.

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