

# SUPERSYMMETRY BREAKING FROM SUPERSTRINGS AND THE GAUGE HIERARCHY\*

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The gauge hierarchy problem is reviewed and a class of effective field theories obtained from superstrings is described. These are characterized by a classical symmetry, related to the space-time duality of string theory, that is responsible for the suppression of observable supersymmetry breaking effects. At the quantum level, the symmetry is broken by anomalies that provide the seed of observable supersymmetry breaking, and an acceptably large gauge hierarchy may be generated.

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## 1. Introduction

In these talks I will report on some recent work [1] with Pierre Binétruy on effective field theories obtained from superstrings. The physics motivation is the gauge hierarchy problem, which I will first review. I will then review the theoretical framework in which we are working, namely effective supergravity theories obtained from  $E_8 \times E_8$  heterotic string.

A certain class of these theories is characterized by an invariance, at the classical level, under a group of global, nonlinear transformations among the fields of the effective theory. We have shown [1] that this symmetry can protect the scalars and gauginos of the observed gauge group from acquiring masses when supersymmetry (SUSY) is broken in a "hidden" sector of the theory, that couples to our world with interactions of gravitational strength only.

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This symmetry group includes chiral transformations on fermion fields, as well as scale transformations, and is therefore broken at the quantum level by the well known chiral and conformal anomalies. These anomalies, in collusion with nonperturbative effects in the strongly coupled gauge interactions of the hidden sector, provide the seed of SUSY breaking in the observable sector. We find [1] that a very mild hierarchy between the Planck scale and the scale (*i.e.*, the gravitino mass) of SUSY breaking in the hidden sector is sufficient to generate an acceptably large (for phenomenology) hierarchy in the observed sector. I will first give a qualitative description of these results, and then a more technical explanation of the construction of the effective low energy field theory using the underlying classical symmetries and their anomaly structure. Finally, I will comment on more recent developments and their implications for our analysis.

## 2. The standard model and beyond

The aim of theoretical physics is to provide an understanding of observed phenomena; in the context of particle physics, what is observed is the Standard Model, namely the  $SU(3)_c \times SU(2)_L \times U(1)$  gauge theory of the strong and electroweak interactions. The electroweak  $SU(2)_L \times U(1)$  theory is characterized by a spontaneous breakdown to the  $U(1)$  of QED *via* an as yet unknown Higgs mechanism, giving rise to weak vector boson masses of the order of 100 GeV. The strong  $SU(3)_c$  gauge theory is characterized by asymptotic freedom and infrared enslavement, entailing confinement of particles that carry the strong color charge, as well as chiral symmetry breaking *via* a nonperturbatively induced quark condensate

$$\langle \bar{q}_R q_L \rangle + \text{h.c.} \neq 0 \quad (1)$$

that breaks the symmetry under chiral transformations:

$$q_L \rightarrow e^{i\alpha} q_L, \quad q_R \rightarrow e^{-i\alpha} q_R. \quad (2)$$

A mechanism similar to (1) plays a central role in the scenarios for SUSY breaking that I will describe.

The Standard Model is further characterized by the spectrum of matter fermions that couple to one another *via* the gauge forces. These are three “families” or “generations” of quarks and leptons, with identical properties from one generation to the next, except for widely different masses and flavor changing weak couplings *via* which the heavier fermions cascade decay to the lightest ones.

The Standard Model describes observed physics well — in fact so well that we are left with no clue as to how to proceed from here. Expected to

lie beyond the Standard Model are answers to the many questions that the theory leaves unresolved. I will briefly enumerate these.

**What is the origin of electroweak symmetry breaking?** This is the most immediate question facing us, because we know [2] that some indication of the answer, that is, some manifestation of the (elementary or composite) “Higgs sector” must show up at hard collision energies of a few TeV or less, within reach of currently planned, if not existing, collider facilities. A closely related issue is the infamous gauge hierarchy problem, which will be a central theme of these talks.

**What is the origin of CP violation, and what determines fermion mass hierarchies and weak flavor mixing?** These questions are connected to the overall issue of electroweak symmetry breaking; in the Standard Model the associated parameters are all determined by the Higgs Yukawa couplings to fermions — that is, by a large number of arbitrary constants. The underlying physics relevant to these questions may be manifest only at energies considerably higher than a TeV, possibly out of reach of any foreseeable accelerator facility. B-physics will play an important role in addressing these issues, at the very least in pinning down accurately the parameters of the Kobayashi–Maskawa matrix. Continued searches for neutrino masses and/or neutrino (and charged lepton) flavor mixing, and for a nonvanishing neutron dipole moment may either turn up clues or severely constrain the viable possibilities.

**What is the origin of the particle spectrum itself, and, for that matter, of the gauge group?** LEP has now provided a convincing case for the most standard of standard models, namely the three-generation one. New physics that might shed light on these questions surely lies well beyond a TeV. Rare decay searches that provide limits on lepton flavor-changing couplings (relevant to a gauged family symmetry) and on flavor-changing axion emission (relevant to a global family symmetry) can probe such ideas up to scales of  $10^3$  to 100 TeV.

**Is the observed gauge group unified by a larger, simple group, i.e., GUT?** If so, the measured couplings of the observed group tell us that the scale of the relevant physics is  $10^{15}$  GeV or more, so we must rely only on indirect probes such as proton decay and neutrino masses and oscillations. A very important low energy indicator is the precise value of the weak mixing angle,  $\sin^2 \theta_w$ .

**Are the observed gauge interactions unified with gravity?** If so, the relevant physics lies at the Planck energy scale of about  $2 \times 10^{18}$  GeV, and we do not even know what we might look for as a low energy probe.

**“Is there a Theory of Everything?”** is a more fashionable way to phrase the last question. If the answer is positive, the T.O.E. will of course answer all the above. In spite of meager theoretical progress in

making contact with observed physics, superstring theory [3] is still the prime candidate for a T.O.E.. I will describe one possibility as to how the gauge hierarchy may emerge in this context.

### 3. The gauge hierarchy problem

The gauge hierarchy problem may be simply expressed in the context of the standard Model by writing the renormalized Higgs mass  $m_H$  as

$$m_H^2 = \frac{1}{8}\lambda(\text{TeV})^2 = m_H^2(\text{tree}) + \frac{g^2}{16\pi^2}\Lambda^2 + \dots \quad (3)$$

Here  $g$  is the weak gauge coupling constant, and  $\lambda$  is the renormalized coupling constant for scalar self-couplings. The right hand side of (3) represents the classical value plus the sum of quantum corrections, which are quadratically divergent, as indicated by the appearance of the cut-off  $\Lambda$ . If perturbation theory makes sense,  $\lambda$  can be no larger than 1 (or at least  $4\pi$ ). Then the first equality suggests  $m_H < (0.35 - 1.2) \text{ TeV}$ , and so we need  $\Lambda < (8 - 30) \text{ TeV}$ . Of course, purely within the context of the renormalizable standard model, there is not really a gauge hierarchy problem. The infinite quadratic divergences can be absorbed into a redefinition of the Higgs mass, whose value is simply fixed by measurement. However if the underlying theory includes Higgs couplings to heavier particles, such as GUT vector bosons, quantum corrections will include terms with  $\Lambda$  in (3) replaced by the masses of these particles. Gravitational couplings of matter imply the presence of at least one large mass scale: the Planck scale.

There are three standard "solutions" to the gauge hierarchy problem, which I briefly recall. I will list them in what I view as increasing order of plausibility; many people would disagree with my ordering.

**Compositeness.** In this scenario, the standard model is an effective theory, some or all of whose "elementary" particles are bound states of yet more elementary objects. The theory makes sense up to momentum scales of order of the inverse radius of compositeness  $r_c$ , so

$$\Lambda \rightarrow \Lambda_c \sim r_c^{-1} \quad (4)$$

in (3). If quarks and leptons are composite, those with common constituents should couple to one another *via* four-fermion interactions with an effective Fermi constant  $G \sim 4\pi r_c^2$ . Existing experiments suggest  $r_c < (\text{TeV})^{-1}$ ; recent results from Tristan [4] give more stringent limits, with  $\Lambda_c > 5 \text{ TeV}$  in one channel.

**Technicolor.** In this case only the Higgs sector is composite. The theory [5] mimics the observed properties of QCD. New asymptotically free

gauge interactions are assumed, which break the electroweak symmetry *via* a technifermion condensate

$$\langle \bar{f}^T f^T \rangle \simeq (f_\pi^T)^3 \equiv \left(\frac{1}{4}\text{TeV}\right)^3. \quad (5)$$

Here  $f_\pi^T$  is the strength of the coupling to the axial current of the technipion  $\pi^T$ , analogous to the pion decay constant,  $f_\pi$ . This number is fixed at 250 GeV, so as to correctly reproduce the observed W, Z masses. The scale at which the effective “low energy” theory ceases to be valid is determined by the scale  $\Lambda_{\text{TQCD}}$  at which the technigauge interactions become strong:

$$\Lambda \rightarrow \Lambda_{\text{TQCD}} \sim f_\pi^T \simeq 250 \text{ GeV}. \quad (6)$$

As yet, no one has succeeded in constructing an experimentally viable, nor a grand unifiable, model that incorporates this idea.

**Supersymmetry.** In this case [6] the quantum corrections on the right hand side of (3) are damped by cancellations between boson and fermion loops, which are complete if SUSY is unbroken. Since observation tells us that SUSY is certainly broken, the effective cut-off is provided by the fermion–boson mass splitting:

$$\Lambda \rightarrow \Lambda_{\text{SUSY}} = |m_{\text{fermion}} - m_{\text{boson}}|. \quad (7)$$

It is possible to construct viable SUSY extensions of the standard model, but the scale parameter (7) is simply put in by hand, so we have not really solved the gauge hierarchy problem in this way.

Before proceeding to a T.O.E., I wish to emphasize that one cannot evade the gauge hierarchy problem by a strongly interacting scalar sector, *i.e.*, by letting  $\lambda \gg 1$  in (3). In this case the scalar sector, described classically by the Standard Model Higgs potential

$$\mathcal{L}_{\text{Higgs}} = \frac{1}{4}\lambda [(H+v)^2 + \varphi_0^2 + 2\varphi^+\varphi^- - v^2]^2, \quad v = 250 \text{ GeV}, \quad (8)$$

becomes a system of strongly interacting Goldstone bosons [7]. At energies  $E \ll m_H$ , the physical Higgs field  $H$  is not excited, and  $\varphi^+$ ,  $\varphi^-$ ,  $\varphi^0$ , which are in fact the longitudinally polarized components  $W_L^+$ ,  $W_L^-$ ,  $Z_L$  of the weak vector bosons, interact in exactly the same way as the pions  $\pi^+$ ,  $\pi^-$ ,  $\pi^0$  of low energy QCD, with the replacement  $f_\pi \simeq 125 \text{ MeV} \rightarrow v \simeq 250 \text{ GeV}$ . These interactions should be observable [8], with sufficiently high energy and luminosity, such as planned for the SSC, as an excess of W and Z pairs with invariant masses of TeV or more. Their interactions are described by an effective Lagrangian  $\mathcal{L}_{\text{eff}}$  whose low energy form is dictated by the global symmetry of the potential (8), analogous to the chiral symmetry of QCD. Including quantum corrections,

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} \partial_\mu \varphi^i \partial^\mu \varphi^j \left( \delta_{ij} + \frac{\varphi_i \varphi_j}{v^2 - |\varphi|^2} \right) \left( 1 - \frac{\Lambda^2}{8\pi^2 v^2} + \dots \right) \\ + \text{higher derivative terms} + \text{resonance effects}. \quad (9)$$

Just as the quadratic divergence in (3) can be absorbed into the definition of the physical Higgs mass, the one in (9) can be absorbed into the definition of the physical (*i.e.*, renormalized) vacuum expectation value  $v_R \equiv 250 \text{ GeV}$ :

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} \partial_\mu \varphi_R^i \partial^\mu \varphi_R^j \left( \delta_{ij} + \frac{\varphi_R^i \varphi_R^j}{v_R^2 - |\varphi_R|^2} \right) + \dots, \\ \frac{\varphi_R}{\varphi} = \frac{v_R}{v} = \left( 1 - \frac{\Lambda^2}{8\pi^2 v^2} + \dots \right)^{\frac{1}{2}}. \quad (10)$$

However once the theory is embedded in a larger theory (as it should be, since a pure scalar field theory is apparently not self-consistent) including large mass scales, one still has to invoke a physical origin for the cut-off,  $\Lambda < 3 \text{ TeV}$ , to understand the “small” observed value of  $v_R$ . Technicolor in fact provides an explicit example of a theory with the effective Lagrangian (9), and with the cut-off (6). The resonances in (9) are in this case predictable, roughly by scaling observed resonance masses in QCD by the factor  $v/f_\pi$ .

#### 4. SUSY, GUTs and SUSY GUTs

There is no direct evidence for supersymmetry in nature. Even more stringent limits on sparticle masses are emerging from the LEP collaborations and from CDF. (The CDF collaboration has previously reported squark and gaugino mass limits as high as about 100 GeV, but these entail decay branching ratio assumptions that are apparently not valid in the relevant mass range [9].) Moreover, results from Higgs searches at LEP are closing the window of allowed parameters in the minimal SUSY extension of the Standard Model, with just two  $\text{SU}(2)_L$  doublets of scalar fields and their fermionic partners. However there is no particular reason — especially within the context of a T.O.E. — to believe that a SUSY extension of the Standard Model should be minimal. If one adds just one chiral supermultiplet (*i.e.* a complex scalar and a Weyl fermion) that is singlet under the Standard Model gauge group, the parameters are much less constrained, and one even loses the prediction of the minimal SUSY model that the lightest scalar is lighter than the  $Z$ .

There is also no direct evidence for a Grand Unified Theory. Limits on the lifetime for nucleon decay to mesons and leptons presumably rule out

the minimal [10] SU(5) GUT (with the caveat as to whether the value of the SU(3)<sub>c</sub> fine structure constant  $\alpha_s$  — or, equivalently,  $\Lambda_{\text{QCD}}$  — is sufficiently well established). On the other hand, predictions in the context of SUSY GUTs, or a T.O.E., are highly model dependent.

Do we have indirect evidence for either of these ideas? If the Standard Model gauge interactions are unified at some scale, their values, as determined by the renormalization group equations, should all become equal at a single energy scale [11]. Modulo assumptions about massive gauge nonsinglet particles that can contribute to the R.G.E.'s, coupling constant unification can be checked by comparing the measured value of  $\sin^2 \theta_w$  with the predicted one, with the fine structure constants  $\alpha$  and  $\alpha_s$  as input. Here I will quote verbatim from Sirlin's talk at Les Arcs [12]. He gave the value of  $\sin^2 \theta_w$  at Z mass scale, in the modified minimal subtraction scheme, averaged over the results of UA1, UA2, CDF and LEP, as

$$\sin^2 \theta_{\overline{\text{MS}}}(m_Z) = 0.2327 \pm 0.0012. \quad (11)$$

The comparable value, after appropriate radiative corrections [12], from the CHARM II collaboration is 0.232, with a similar (experimental + theoretical) error. These results apparently [12] differ by a few standard deviations from the Standard Model prediction, but are consistent with the minimal SUSY prediction obtained by Marciano's estimate:

$$\sin^2 \theta_{\overline{\text{MS}}}(m_Z) = 0.237^{+0.003}_{-0.004} - \frac{4}{15} \frac{\alpha}{\pi} \ln \left( \frac{\Lambda_{\text{SUSY}}}{m_W} \right). \quad (12)$$

Thus an optimist might conclude that there is indirect evidence for a SUSY GUT. Aside from modern refinements that should be included [12] in the estimate (12), this result could be modified by contributions from nonstandard massive particles, and the conclusions may be subject to the above-mentioned caveat. However, the predictions for  $\sin^2 \theta_w$  are much less sensitive to uncertainties (which are reflected in the quoted theoretical errors) in  $\Lambda_{\text{QCD}}$  than are those for the proton lifetime.

## 5. T.O.E.: the heterotic string

According to the presently most popular hope for a fully unified theory, the Standard Model is an effective theory that is a low energy limit of the heterotic string [13] theory. Starting from a string theory in 10 dimensions with an  $E_8 \times E_8$  gauge group, one ends up, at energies sufficiently below the Planck scale, with a supersymmetric field theory in 4 dimensions [14], with a generally smaller gauge group  $\mathcal{H} \times \mathcal{G}$ .  $\mathcal{H}$  describes a "hidden sector", that has interactions with observed matter of only gravitational strength,

and  $\mathcal{G} \supset SU(3)_c \times SU(2)_L \times U(1)$  is the gauge group of observed matter. Part of the gauge symmetry may be broken (or additional gauge symmetries may be generated) the  $10 \rightarrow 4$  dimensional compactification process itself, and part of it may be broken by the Hosotani mechanism [15], in which gauge flux is trapped around space-tubes in the compact manifold. There are now many more examples of effective theories from superstrings than one once thought could emerge. For illustrative purposes, I will stick to the original "conventional" scenario, in which the "observed"  $E_8$  is broken to  $E_6$ , long known to be the largest phenomenologically viable GUT, by the compactification process. Then the observed sector is a supersymmetric Yang-Mills theory, with gauge bosons and gauginos in the adjoint representation of  $\mathcal{G} \subset E_6$ , coupled to matter, i.e., to quarks, squarks, leptons, sleptons, Higgs, Higgsinos, ....

The hidden sector is assumed to be described by a pure SUSY Yang-Mills theory,  $\mathcal{H} \subset E_8$ , which is asymptotically free, and therefore infrared enslaved. At some energy scale  $\Lambda_c$ , below the compactification scale  $\Lambda_{\text{GUT}}$  at which all the gauge couplings are equal, the hidden gauge multiplets become confined and chiral symmetry is broken, as in QCD, by a fermion condensate. In this case the fermions are the gauginos of the hidden sector:

$$\langle \bar{\lambda} \lambda \rangle_{\text{hid}} \sim \Lambda_c^3 \neq 0. \quad (13)$$

The condensate (13) breaks SUSY [16], and by itself would generate a positive cosmological constant. If this were the only source of SUSY breaking, and of a cosmological constant, the condensate would be forced dynamically to vanish, due to the condition that the vacuum energy be minimized.

Another source of SUSY breaking is the (quantized) vacuum expectation value of an antisymmetric tensor field  $H_{LMN}$ , that is present in 10-dimensional supergravity:

$$H_{LMN} = \nabla_L B_{MN}, \quad L, M, N = 0, \dots, 9,$$

$$\int dV^{lmn} \langle H_{lmn} \rangle = 2\pi n \neq 0, \quad l, m, n = 4, \dots, 9. \quad (14)$$

The VEV (14) can arise if  $H$ -flux is trapped around a 3-dimensional space-hole in the compact 6-dimensional manifold, in a manner analogous to the Hosotani mechanism for breaking the gauge symmetry. When (13) and (14) are both present,  $\lambda$  and  $H_{LMN}$  couple in such a way [17] that the overall contribution to the classical cosmological constant vanishes. There are other potential sources of SUSY breaking, such as a gravitino condensate [18], that might play a similar role.



The particle spectrum of the effective four dimensional field theory includes the gauge supermultiplets  $W^a = (\lambda^a, F_{\mu\nu}^a - i\tilde{F}_{\mu\nu}^a)$  (gauginos and gauge bosons) and chiral supermultiplets  $\Phi^i = (\varphi^i, \chi^i)$  that contain the matter fields ( $\varphi^i$  = squarks, sleptons, Higgs particles, ...,  $\chi^i$  = quarks, ...). In the “conventional” scenario these are all remnants of the gauge supermultiplets in ten dimensions:

$$A_M \rightarrow A_\mu + \varphi_m, \quad \mu = 0, \dots, 3, \quad m = 4, \dots, 9. \quad (15)$$

Thus for each gauge boson  $A_M$  in ten dimensions, there are potentially one gauge boson  $A_\mu$  and six scalars  $\varphi_m$  (and their superpartners) in four dimensions. However not all of these are massless. In the “conventional” picture ( $E_8 \rightarrow E_6$  in the observed sector) the massless 4-vectors are in the adjoint of  $E_6$ , while the massless scalars are in  $(27 + \bar{27})$ 's that makes up the difference:  $(\text{adjoint})_{E_8} - (\text{adjoint})_{E_6}$ . In addition there are gauge singlet chiral supermultiplets associated with the structure of the compact manifold. Two of these,  $S = (s, \chi^S)$  and  $T = (t, \chi^T)$  are of special interest. Their scalar components are [19]

$$\begin{aligned} s &= e^{3\sigma} \phi^{-\frac{3}{4}} + 3i\sqrt{2}D, \\ t &= e^\sigma \phi^{\frac{3}{4}} - i\sqrt{2}a + \frac{1}{2} \sum_i |\varphi^i|^2. \end{aligned} \quad (16)$$

In (16)  $\phi$  is the dilation of ten-dimensional supergravity,  $D$  and  $a$  are two axions that are remnants of the antisymmetric tensor (14):

$$a \propto \epsilon^{lm} B_{lm}, \quad \partial_\mu D \propto \epsilon_{\mu\nu\rho\tau} \phi^{-\frac{3}{2}} e^{6\sigma} H^{\nu\rho\tau}, \quad (17)$$

and  $\sigma$  is the “breathing mode” or “compacton” whose VEV determines the size of the compact manifold with metric  $g_{lm} = g_{lm}^{(0)} e^\sigma$ . Thus the GUT — or compactification — scale, which is the inverse of the radius  $R$  of compactification, is determined by the VEV (in Planck mass units)

$$\Lambda_{\text{GUT}}^2 = R^{-2} = \langle e^{-4\sigma} \rangle = \langle (\text{Re } s \text{ Re } t)^{-1} \rangle. \quad (18)$$

The total number of gauge singlet chiral multiplets, as well as the number of matter generations ( $\#27$ 's -  $\#\bar{27}$ 's) is determined by the detailed topology of the compact manifold.

The Lagrangian of the effective four dimensional field theory, with nonperturbative SUSY breaking included [17] has, in a broad class of models, the following properties at the classical level. The gravitino mass  $m_{\tilde{G}}$  can be nonvanishing, so that local supersymmetry is broken. The cosmological constant vanishes, as do the observable gaugino masses  $m_{\tilde{g}}$ , the gauge nonsinglet scalar masses  $m_\varphi$ , and “A-terms”, which are trilinear gauge

nonsinglet scalar self-couplings that, if present, would also break SUSY. Thus there is no manifestation of SUSY breaking in the observable sector.

One loop corrections have been evaluated [20] in this effective (non-renormalizable) theory, which is cut off at the scale of gaugino condensation

$$\Lambda_c = e^{-b_0/2g^2} \Lambda_{\text{GUT}} = \left\langle \frac{e^{-b_0 \text{Re } s/2}}{\sqrt{\text{Re } s \text{Re } t}} \right\rangle. \quad (19)$$

The first equality in (19) is just the standard R.G.E. result, where  $b_0$  is a group theory number that determines the  $\beta$ -function of the hidden sector Yang-Mills theory. The second equality follows from (18) and the relation (there are no free parameters in the T.O.E.!) between the VEV of  $s$  and the gauge coupling constant  $g$  at the GUT scale, where all gauge couplings are equal:

$$g^2 (\Lambda_{\text{GUT}}) = \langle (\text{Re } s)^{-1} \rangle. \quad (20)$$

The result found [20] is that the classical features described above are unchanged at the one loop level.

In fact, the class of 4-d theories considered possesses [21] a classical non-linear, noncompact global symmetry. They are in fact nonlinear  $\sigma$ -models, much like the effective pion theory of low energy QCD, where chiral  $\text{SU}(2)$  symmetry is realized *via* nonlinear transformations among the pion fields. The difference here is that the global symmetry group is the noncompact group  $\text{SU}(1,1) \times \text{U}(1)_{\text{R}}$ , where  $\text{U}(1)_{\text{R}}$  is the usual R-symmetry of supersymmetric theories. The group of transformations includes [1] a subset under which

$$t \rightarrow b^2/t, \quad (21)$$

where  $b$  is a finite, continuous, real parameter. The string scale  $M_{\text{S}}$  is related to the Planck scale  $M_{\text{P}}$  by

$$M_{\text{P}} = \langle (\text{Re } s)^{\frac{1}{2}} \rangle M_{\text{S}}, \quad (22)$$

so when the theory is expressed in string mass units, (21) corresponds to an inversion of the radius of compactification (18):

$$R^2 = \Lambda_{\text{GUT}}^{-2} = \langle \text{Re } s \text{Re } t \rangle / M_{\text{P}}^2 = \langle \text{Re } t \rangle / M_{\text{S}}^2 \rightarrow b^2 / R^2. \quad (23)$$

For the special case of integer  $b$ , this is the well known "duality" transformation, which leaves the string spectrum invariant. We have recently shown [1] that this classical  $\text{SU}(1,1) \times \text{U}(1)_{\text{R}}$  symmetry is responsible for the cancellation of observable SUSY breaking effects, as found [20] by explicit calculation.

## 6. Anomalies as the seed of observable symmetry breaking

Under the classical  $SU(1, 1) \times U(1)_R$  symmetry of the effective low energy theory, the fermions undergo chiral phase transformations:

$$f_L \rightarrow e^{i\alpha} f_L, \quad f_R \rightarrow e^{-i\alpha} f_R, \quad (24)$$

so at the quantum level the symmetry is broken by the chiral anomaly. In addition,  $SU(1, 1) \times U(1)_R$  includes the scale transformation  $t \rightarrow a^2 t$ , under which the cut-off for the theory (which at energies above the scale of hidden gaugino condensation is just  $\Lambda_{GUT}$ , Eq. (18)) scales as

$$\Lambda_{GUT}^2 \propto \langle (\text{Re } t)^{-1} \rangle \rightarrow a^{-2} \Lambda_{GUT}^2, \quad (25)$$

so the symmetry is further broken at the quantum level by the conformal anomaly.

The dominant effect of these anomalies arises from the highest mass scale at which nonperturbative effects come into play. In the context of the effective 4-d field theory, these are associated with instantons and gaugino condensation in the hidden Yang-Mills sector. Just as one can construct low energy effective Lagrangians for pseudoscalar mesons that are  $q\bar{q}$  bound states using the symmetries of QCD and the chiral and conformal anomaly, one can use [1]  $SU(1, 1) \times U(1)$  and its anomalies, together with supersymmetry [22], to construct an effective Lagrangian for the lightest hidden sector chiral multiplet, denoted  $H = (h, \chi^H)$ , which is a bound state, with mass  $m_H$ , of the hidden gauge supermultiplet. Retaining loop corrections from these additional degrees of freedom, whose couplings explicitly include the anomalous symmetry breaking, one finds [1] that gaugino masses are generated in the observable sector that are of order

$$m_{\tilde{g}} \sim \frac{1}{(16\pi^2 m_p^2)^2} m_{\tilde{G}} m_H^2 \Lambda_c^2. \quad (26)$$

The factor  $(4\pi)^{-4}$  appears in (26) because the effect arises first at two-loop order in the effective theory, the factor  $m_{\tilde{G}}$  is the necessary signal of SUSY breaking, the factor  $m_H^2$  is the signal of  $SU(1, 1) \times U(1)$  breaking, and  $\Lambda_c^2$  is the effective cut-off. This last factor arises essentially for dimensional reasons: the couplings responsible for transmitting the knowledge of symmetry breaking to the observable sector are nonrenormalizable interactions with dimensionful coupling constants proportional to  $m_p^{-2}$ .

Solving [20] the minimization conditions for the effective theory at the one-loop level yields, for vacua with broken supersymmetry, the values

$$m_{\tilde{G}} \simeq \frac{1}{3} m_H \simeq \frac{1}{3} \Lambda_c \simeq (10^{-2} - 10^{-1}) \frac{m_p}{\sqrt{b_{0c}}}, \quad (27)$$

where the parameter  $c$  is proportional to the VEV of  $H_{LMN}$ . The quantization condition (14) and dimensional analysis suggest [20]  $c > 10^3 n$  if  $c \neq 0$ , or

$$m_{\tilde{g}} < 10^{-15} m_P \simeq 2 \text{ TeV} . \quad (28)$$

Once gauginos acquire masses, gauge nonsinglet scalars (in particular the Higgs particles) will acquire masses  $m_\varphi \sim \frac{\alpha}{4\pi} m_{\tilde{g}}$  at the next loop order in the renormalizable gauge interactions.

The superstring context used here is not the most general one, but there is a broad class of models with similar features, so these results suggest that there is hope, after all, of extracting meaningful physics from the superstring T.O.E.. I now turn to a more technical description of the results described above.

## 7. The structure of the supergravity Lagrangian

When expressed in Planck mass units, so that the Einstein curvature term reads  $\mathcal{L}_E = \frac{1}{2} \sqrt{g} R$  (which can always be achieved by a Weyl transformation), the classical Lagrangian for a general supergravity in four dimensions is determined [23,24] by three functions of the chiral superfields:

$$\tilde{\Phi}^a = \Phi^i, S, T, \dots \quad (29)$$

These are

(i) A gauge field normalization function,  $f(\Phi) = \bar{f}(\bar{\Phi})^\dagger$ . In the superfield formulation [25] the Yang–Mills part of the Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{\text{YM}} &= \frac{1}{4} \int d^2 \Theta f(\Phi) W_\alpha^a W_\alpha^a + \text{h.c.} \\ &= -\frac{1}{4} \{ \text{Re } f(\varphi) F_{\mu\nu}^a F_a^{\mu\nu} + \text{Im } f(\varphi) \tilde{F}_{\mu\nu}^a F_a^{\mu\nu} \} + \dots \end{aligned} \quad (30)$$

Here  $\Theta$  is a complex two-component fermionic variable in superspace:  $x \rightarrow x, \Theta, \bar{\Theta}$ , and we have indicated some of the terms that appear after  $\Theta$  integration when the superfields are expanded in terms of their component fields. The first term in this expansion implies that the gauge coupling constant is determined by the VEV  $\langle \text{Re } f(\varphi) \rangle = g^{-2}$ .

(ii) The Kähler potential  $K(\Phi, \bar{\Phi}) = K(\Phi, \bar{\Phi})^\dagger$ , which determines, for example, chiral multiplet kinetic energy terms:

$$\mathcal{L}_{\text{KE}}(\Phi) = K_{a\bar{b}} \partial_\mu \varphi^a \partial^\mu \varphi^{\bar{b}} + \dots, \quad K_{a\bar{b}} = \frac{\partial^2 K}{\partial \varphi^a \partial \varphi^{\bar{b}}} . \quad (31)$$

(iii) The superpotential  $W(\Phi) = \overline{W}(\overline{\Phi})^\dagger$ , which determines the Yukawa couplings and the scalar potential:

$$\mathcal{L}_{\text{pot}} = \int d^2\Theta e^{K/2} W(\Phi) + \text{h.c.} = -e^{\mathcal{G}} [\mathcal{G}_a (\mathcal{G}^{-1})^{a\bar{b}} \mathcal{G}_{\bar{b}} - 3] + \dots, \quad (32)$$

where on the right hand side I have introduced the generalized Kähler potential

$$\mathcal{G} = K + \ln |W|^2 \quad (33)$$

of Cremmer *et al.* [24]. In fact, the theory defined above is classically invariant [24,25] under a Kähler transformation that redefines both the Kähler potential and the superpotential in terms of a holomorphic function  $F(\Phi) = \overline{F}(\overline{\Phi})^\dagger$ :

$$K \rightarrow K' = K + F + \overline{F}, \quad W \rightarrow W' = e^{-F} W, \quad (34)$$

provided one also transforms the fermions by a chiral rotation; for example

$$W_\alpha^a \rightarrow e^{-\text{Im } F/2} W_\alpha^a, \quad \lambda_\alpha^a \rightarrow e^{-\text{Im } F/2} \lambda_\alpha^a. \quad (35)$$

This last transformation is anomalous at the quantum level, a point that will be important in the discussion below. One can fix the “Kähler gauge” by a specific choice of the function  $F$ . In particular, choosing  $F = -\ln W$  casts the Lagrangian in a form [24] that depends on only two functions of the scalar fields,  $f$  and  $\mathcal{G}$ .

Here I will describe a prototype [19] supergravity model from superstrings, with nonperturbative SUSY breaking included [17]. The functions (30)–(32) are given in terms of the superfields (29) by

$$f = S, \quad (36a)$$

$$K = -\ln(S + \overline{S}) - 3\ln(T + \overline{T} - |\Phi|^2), \quad |\Phi|^2 = \sum_{i=1}^N \Phi^i \overline{\Phi}^{\bar{i}}, \quad (36b)$$

$$W(\Phi) = c_{ijk} \Phi^i \Phi^j \Phi^k + c + \tilde{h} e^{-3b_0 S/2}. \quad (36c)$$

The last two terms in the superpotential  $W$  are parametrizations of nonperturbative SUSY breaking effects [17]. The parameter  $c$  is proportional to the VEV of the antisymmetric tensor field strength (14), and the last term represents the gaugino condensate (13). The form of this term can be understood in terms of the standard R.G.E. result (19), together with the relation (20), implied by (30) and (36a).

The structure of the condensate term in  $W$  is further justified by symmetry considerations [17,26]. For  $c = \tilde{h} = 0$ , the theory is formally invariant under the Kähler transformation (34) with

$$F = i\alpha, \quad K \rightarrow K, \quad W \rightarrow e^{i\alpha}W, \quad \lambda \rightarrow e^{i\alpha}\lambda, \quad \alpha \text{ real.} \quad (37)$$

This symmetry, which is just the "R-symmetry" of renormalizable SUSY models, is broken at the quantum level (which cannot be ignored for the strongly interacting hidden Yang-Mills sector) due to the chiral anomaly; under (37)

$$\delta\mathcal{L} = -\frac{i\alpha}{6b_0}(F\tilde{F})_{\text{hid}}. \quad (38)$$

However, because of the coupling (36a), (30) of the Yang-Mills supermultiplet to the  $S$ -supermultiplet, the variation (38) can be cancelled by a shift in  $S$ :

$$S \rightarrow S - \frac{2i\alpha}{3b_0}. \quad (39)$$

The combined transformations (37) and (39) are an exact (neglecting the  $c$ -term and quantum corrections in the observed gauge sector) invariance of the theory; this is reflected by the transformation property

$$W(S) = e^{-3b_0/2\tilde{h}} \rightarrow e^{i\alpha}W(S) \quad (40)$$

of the superpotential for  $S$  in (36c).

The general features of the theory defined by (36), first obtained by Witten [19] for the case of a simple torus compactification, are common to a broad class of more realistic models [27]. These possess the classical properties described above, namely the vanishing of the cosmological constant and of observable SUSY breaking effects even when local supersymmetry is broken ( $m_{\tilde{G}} \neq 0$ ). As discussed above, these features are unchanged at the one loop level [20].

## 8. Classical symmetries of the theory

The class of 4-d theories considered possesses [21] a classical nonlinear global symmetry under the noncompact group  $SU(1,1)$  or  $SL(2, R)$ :

$$T \rightarrow T' = \frac{aT - ib}{icT + d}, \quad \Phi^i \rightarrow \Phi^{i'} = \frac{\Phi^i}{icT + d}, \quad S \rightarrow S' = S,$$

$$ad - bc = 1, \quad a, b, c, d \text{ real.} \quad (41)$$

For  $c = h = 0$ , Eqs (41) in fact represent a Kähler transformation (34), with

$$F = 3 \ln(icT + d), \quad (42)$$

under which the full Lagrangian is invariant provided the fermion fields undergo a chiral transformation (35). The group of transformations (41) includes the subset (21), with  $a = d = 0$ ,  $bc = -1$ . In addition, the theory is invariant [21] under R-symmetry, Eq. (37).

When we allow  $c, \tilde{h} \neq 0$ , the  $SU(1, 1)$  symmetry can be formally maintained by allowing these parameters to transform like a superpotential, Eq. (34):

$$c \rightarrow c' = e^{-F} c, \quad \tilde{h} \rightarrow \tilde{h}' = e^{-F} \tilde{h}. \quad (43)$$

This makes sense when one recalls that  $c$  and  $\tilde{h}$  are actually the VEVs of underlying dynamical variables; therefore their values will relax to those that minimize the total vacuum energy density; and the relevant symmetries are those of the full parameter space. This was precisely the attitude taken in [20], where it was found that observable SUSY breaking vanishes at the overall ground state of the one-loop corrected effective theory. Moreover it can be seen [1] that (43) corresponds to the correct transformations of the fields in (13) and (14).

## 9. Construction of the effective composite Lagrangian

The noncompact symmetry (41) and the R-symmetry (37) are broken at the quantum level by the chiral anomaly (see (37), (38)) and also by the conformal anomaly, as indicated in (25) ( $c = b = 0$ ,  $ad = 1$  in (41)). More generally, under a Kähler transformation (34), (35) we have

$$\Lambda_{\text{GUT}}^2 = 4g^2 \langle e^{K/3} \rangle \rightarrow e^{\frac{2}{3} \text{Re } F} \Lambda_{\text{GUT}}^2. \quad (44)$$

Then under  $SU(1, 1) \times U(1)_R$

$$\begin{aligned} \delta \mathcal{L} &= \frac{2}{3} b_0 \left\{ \text{Re } F(t) F_{\mu\nu}^a F_a^{\mu\nu} + \text{Im } F(t) \tilde{F}_{\mu\nu}^a F_a^{\mu\nu} \right\} + \dots \\ &= -\frac{2}{3} b_0 \int d^2 \Theta F(T) W_a^\alpha W_a^\alpha + \text{h.c.}, \end{aligned} \quad (45)$$

where  $F(T)$  is the function defining the Kähler transformation (41) or (37).

The transformation property (45) may be used [22,1] to construct an effective Lagrangian for the composite multiplet  $U$ :

$$\frac{1}{4} W_\alpha^a W_\alpha^a \rightarrow U = \lambda H^3 e^{K/2} e^{-3S/2b_0}, \quad (46)$$

or equivalently the chiral multiplet  $H$ , which is the lightest composite state, with mass  $m_H$ , of the (confined) hidden gauge sector. The Kähler transformation property of  $H$

$$H \rightarrow H' = e^{-F/3} H \quad (47)$$

can be inferred from those of  $\Phi^a$  and  $W_\alpha^a$ . With this transformation property, the anomalies are correctly reproduced [22,28,1] by the following effective potential Lagrangian for the composite chiral fields:

$$\begin{aligned} \mathcal{L}_{\text{pot}}^{\text{eff}} &= \int d^2 \Theta e^{K/2} 2b_0 \lambda e^{-3S/2b_0} H^3 \ln(H/\mu) \equiv \int d^2 \Theta e^{K/2} W(H, S) \\ &= \int d^2 \Theta [SU + U \ln(4Ug^2/\Lambda_{\text{GUT}}^3 \lambda \mu^3)], \end{aligned} \quad (48)$$

which is also invariant [28] under the nonanomalous transformation (37) + (39). Aside from the numerical parameters (or order unity)  $\mu$  and  $\lambda$ , the logarithmic term in (48) is precisely what is expected from the one-instanton contribution [29]. Note that  $\Lambda_{\text{GUT}}$  is the physical cut-off for the theory above the condensate scale, and that the gauge multiplets  $W_\alpha^a$  are normalized with a factor  $g^{-2} = \langle \text{Re } s \rangle$  relative to the canonical normalization. In addition, the ground state configuration is determined by the minimum with respect to  $H$  of the potential (48). This gives

$$\langle H \rangle = h_0 = \mu e^{-1/3}, \quad \langle \bar{\lambda} \lambda \rangle_{\text{hid}} = 4 \langle U \rangle = \frac{\lambda h_0^3}{g^2} \Lambda_c^3. \quad (49)$$

Again (19) corresponds exactly to the one-instanton contribution [29].

It remains to specify the  $H$ -dependence of the Kähler potential. The symmetries of the theory dictate [1] the form

$$K = -\ln(S + \bar{S}) - 3 \ln(T + \bar{T} - |\Phi|^2 - |H|^2). \quad (50)$$

The effective classical theory below the scale of condensation is determined by “integrating out” the  $H$ -supermultiplet, that is, by the sum of tree diagrams with “light” particles ( $m < \Lambda_c$ ) on external legs only. It turns out that there are no such diagrams with  $H$ -exchange, because vertices with a single  $H$  leg vanish at the  $H$  ground state, and one recovers exactly the theory defined by (36), with the parameter  $\hbar$  in (36c) determined as



$$\tilde{h} = -\frac{2}{3}b_0\lambda\mu^3e^{-1}. \quad (51)$$

Retaining one-loop corrections from the  $H$  degrees of freedom, whose couplings explicitly include the anomalous symmetry breaking, one finds [1] that the effective low energy theory defined in this way is no longer totally  $SU(1,1)$  invariant, although no observable SUSY breaking appears at the “classical” level of this effective theory. However, at the one loop level of this effective theory, gaugino masses are generated in the observable sector that are of order (26)–(28).

The numerical estimate in (28) is obtained using the results of [20], where the vacuum configuration was determined at the minimum of the potential with respect to all parameters, including  $c$  and  $\tilde{h}$  of the effective superpotential (36). With  $\tilde{h}$  now determined by (51), minimization with respect to the parameter  $\tilde{h}$  is equivalent to minimization with respect to the parameter  $\mu$ . The presence of the parameter  $\mu$  in fact reflects [1] an additional degree of freedom of the underlying theory, namely the gauge field strength  $F_{\mu\nu}$ . In the superfield formulation, the composite superfield  $U(\Theta)$  defined in (46) has (like all chiral superfields) three components: the complex scalar  $u$ , the fermion  $\chi^U$  and the auxiliary field  $F^U$ , which has no kinetic energy term and can therefore be eliminated by the equations of motion as a function of the other fields. Specifically

$$u = U|_{\Theta=0} = \frac{1}{4}(\bar{\lambda}_R\lambda_L)_{\text{hidden}},$$

$$\begin{aligned} F^U &= -\frac{1}{4}D^\alpha D_\alpha U|_{\Theta=0} = -\frac{1}{8}\left(F_{\mu\nu}F^{\mu\nu} - iF_{\mu\nu}\tilde{F}^{\mu\nu}\right) + \dots \\ &= \lambda e^{K/2}e^{-3s/2b_0}\left(h^3K_aF^a + 3h^2F^H - \frac{3}{2}b_0h^3F^S\right) + O(\chi^2), \end{aligned} \quad (52)$$

where  $D^\alpha$  is the Kähler covariant spinorial derivative, and in the last equality we have evaluated the derivative in terms of the components of the superfields  $\Phi^a(\varphi^a, \chi^a, F^a)$  using the functional form (46) of  $U$  in terms of these superfields. Although in the effective composite theory  $F_{\mu\nu}$  does not appear as an independent dynamical variable, it is one in the underlying theory. Therefore the VEV  $\langle F_{\mu\nu}F^{\mu\nu} \rangle$  should relax to a value that minimizes the vacuum energy. Variation of this physical parameter is reflected in the variation of  $\mu$  — or of  $\tilde{h}$ , Eq. (51).

Defining the ground state as the minimum of the potential with respect to all parameters, it was found [20] that most of the degeneracy of the classical vacuum is lifted by one-loop corrections, but the vacuum energy vanishes at one loop if the potential is bounded from below. Moreover, if there is a nontrivial SUSY breaking (i.e.  $m_{\tilde{G}} \neq 0$ ) vacuum there remains

one degenerate, zero-energy direction in parameter space. Along that direction the ratios of physical scales are determined as indicated in (27). The degeneracy with respect to the overall scale is lifted once a value for  $c$  is chosen.

### 10. Restoration of $SL(2, Z)$ symmetry

The results reported here may be modified by the inclusion of a  $T$ -dependence in the superpotential  $W(S, H)$  defined by (48):  $\mu \rightarrow \mu(T)$ . Such a modification is expected, so as to restore [30,31] the discrete subgroup  $SL(2, Z)$  ( $a, b, c, d$  integers in (41)) of  $SL(2, R)$ , which is known [32] to be an exact symmetry of string perturbation theory, and also to break the residual Peccei–Quinn  $U(1)$  subgroup of  $SL(2, R)$ ,  $T \rightarrow T - ib$ , to its discrete form. Such a term has recently been found [33] as a loop correction to the function  $f$ , Eq. (36a), from the heavy string modes:

$$f(Z) = S \rightarrow f(Z) = S + \Delta(T),$$

$$\Delta(T) = Ab_0 \ln [\mu' \eta^2(T)], \quad (53)$$

where  $\eta(T)$  is the Dedekind function, and the constants  $A$  and  $\mu'$  depends on the topology of the compact manifold. From the point of view of the four dimensional effective supergravity theory, a term like this is expected [1] by analogy with QCD, where the chiral anomaly induces a pion coupling to  $(F\tilde{F})_{\text{QED}}$ , inducing the decay  $\pi \rightarrow \gamma\gamma$ , *via* the pion coupling to the axial quark current:

$$\mathcal{L}_{\text{eff}} \ni \partial_\mu \vec{\pi} \vec{q} \vec{\tau} \gamma^\mu \gamma_5 q \Rightarrow \mathcal{L}_{\text{eff}} \ni \text{constant} \times \int \pi \left( F^{\mu\nu} \tilde{F}_{\mu\nu} \right)_{\text{QCD}}. \quad (54)$$

Since  $T$  couples to a fermionic axial current through the Kähler connection

$$\Gamma_\mu = -\frac{1}{4}i(\partial_\mu z^a K_a - \text{h.c.}), \quad (55)$$

(thus assuring Kähler invariance; *cf.* Eqs (34,35)) anomalous triangle diagrams will induce the corresponding Wess–Zumino term:

$$\begin{aligned} \mathcal{L} \ni \Gamma_\mu J_{\text{axial}}^\mu &\Rightarrow L_{\text{eff}} \ni \frac{1}{4} \int d^2 \Theta \Delta(T) W^\alpha W_\alpha + \text{h.c.} \\ &= -\frac{1}{4} \Delta(t) (F^2 - iF\tilde{F}) + \text{h.c.} + \dots \end{aligned} \quad (56)$$

This contribution modifies the superpotential (48) according to

$$W_{\text{eff}} \rightarrow W(\Phi) + c + \lambda e^{-3S/2b_0} H^3 [2b_0 \ln(H/\mu) + \Delta(T)] . \quad (57)$$

The string loop calculation [33] of (53) gives  $A = 1$  for a particular compactification, so

$$W_{\text{eff}} \rightarrow W(\Phi) + c + 2b_0 \lambda e^{-3S/2b_0} H^3 \ln \{ H \eta^2(T) / \mu'' \} . \quad (58)$$

The result (58) has been obtained [31] directly from the requirement of covariance under  $\text{SL}(2, \mathbb{Z})$  of the effective potential for the composite superfield  $U$ .

An immediate consequence of the above modification is the that the continuous vacuum degeneracy is reduced to a discrete degeneracy. If the parameter  $c$  quantized, the issue arises as to whether both quantizations conditions can be satisfied at the overall minimum of the effective (quantum corrected) potential.

A second consequence is related to the noninvariance of  $\eta(T)$  under the global Heisenberg transformations [34]

$$\delta \Phi = \alpha^i , \quad \delta T + \bar{\alpha}_i \Phi^i , \quad (59)$$

that leaves the Kähler potential (36b) invariant. Together with the Peccei-Quinn transformation  $\delta T = -ib$ , these form a subgroup of  $\text{SU}(N+1, 1)$  which is an invariance of the full Lagrangian (for  $\Delta(T) = 0$ ) in the absence of the superpotential  $W(\Phi)$  and of gauge couplings in the observable sector. This symmetry, if exact, assures [34] the vanishing of gauge nonsinglet scalar masses. Since these masses are also protected by SUSY, they are therefore generated only at one loop order higher than that at which the other observable SUSY breaking effects (gaugino masses or A-terms) first appear. This feature could be modified for  $\Delta \neq 0$ .

Finally, the effective theory defined by (36a,b) and (58) is not positive definite [30,31]. In fact, when considered over the full space of VEVs, it is unbounded from below. However it is, actually becomes unbounded only outside the region of parameter space for which it is expected to be applicable. Moreover, other one loop effects, such as the renormalization of the Kähler potential [35], should be included and could modify this unwanted feature.

## 11. Summary

To summarize, there is a class of models from superstrings that are invariant, at the classical level, under a continuous global  $\text{SL}(2, \mathbb{R})$  symmetry. Global supersymmetry breaking in the observable sector is generated at the quantum level by anomalous symmetry breaking, which, if the contribution

$\Delta(T)$  in (57) is neglected, results in a hierarchy with respect to local SUSY breaking of order

$$m_{\text{Higgs}}/m_{\tilde{G}} \sim 10^{-13} - 10^{-12}. \quad (60)$$

In this case the requirements of phenomenology can be satisfied with a relatively mild hierarchy for local SUSY breaking:

$$m_{\tilde{G}}/m_{\text{Planck}} \sim 10^{-3} - 10^{-2}. \quad (61)$$

An alternative viewpoint is that

$$m_{\text{Higgs}}/m_{\tilde{G}} \sim 1 \Rightarrow m_{\tilde{G}}/m_{\text{Planck}} \sim 10^{-15}. \quad (62)$$

In fact, such a low gravitino mass could be in conflict with standard Big Bang cosmology, but this can be avoided with  $m_{\text{Higgs}}/m_{\tilde{G}}$  hierarchy of just a few orders of magnitude. In this picture one requires

$$m_{\tilde{G}} = \langle e^{K/2} W(z) \rangle \ll m_{\text{Planck}}. \quad (63)$$

At the classical level of the effective low energy theory defined by (36) one has

$$\langle W(z) \rangle = \left\langle c + \tilde{h} e^{-3s/2b_0} \right\rangle, \quad (64a)$$

$$\langle V \rangle = \left\langle e^K |c + \tilde{h} \left(1 + \frac{3}{2b_0}\right) e^{-3s/2b_0}|^2 \right\rangle, \quad (65b)$$

so, since  $\langle V \rangle$  is minimized by  $\langle V \rangle = 0$ ,  $|c| \sim |\tilde{h}|$ , for realistic values of the gauge coupling constant (20) and the GUT scale (18),  $m_{\tilde{G}} \sim |c| g \Lambda_{\text{GUT}}^{3/2} / m_{\text{Planck}}$  cannot be small (see, *e.g.*, the estimate (28)).

One way to evade this problem is to replace the quantized VEV (14) by an alternative second source of SUSY breaking (in addition to the hidden gaugino condensate). Suggested mechanisms are the VEV of some scalar [36] that has been integrated out of the effective low energy theory, gravitino condensation [18], and more than one hidden gaugino condensate [37,38].

Another approach is to appeal to quantum effects in string theory, such as world sheet instantons [39,38], that can induce a  $T$ -dependence in the effective potential. In this case there may be the possibility of inducing SUSY breaking with just a single gaugino condensate. One such example studied [30,31] is the modification (53)–(58). Assuming that the effective potential is bounded in the direction  $\text{Re } s \rightarrow 0$ , Ferrara *et al.* [31] found a minimum with  $c = 0$  in the strong coupling regime,  $\alpha_{\text{GUT}} > 1$ , and with negative cosmological constant. One may worry in this case whether

the unspecified mechanism that must be invoked to drive the cosmological constant to zero might not affect the other parameters of the effective theory.

Thus no entirely satisfactory picture has yet emerged with  $m_{\tilde{G}} \ll m_{\text{Planck}}$ . It will be important to study whether the scenario (60) is still tenable when the correction (53)–(58) is included.

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