INDUCED GAUGE POTENTIALS IN STRONG INTERACTIONS*

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The gauge structure, both abelian and nonabelian, induced in strong interaction processes is discussed in this lecture. First a simple quantum mechanical system is analyzed, followed by an extension, *via* chiral bag model, to a more realistic hadronic system. The resulting induced gauge potential, widely known as Berry potential, is then shown to describe excited states of nonstrange baryons and ground states of strange and charmed baryons. Geometric phases that emerge in such processes are relevant for spin-isospin transmutation, hyperfine splittings in the spectrum and render the skyrmion description also applicable to massive-quark baryons, exhibiting in the massive-quark limit the "Wisgur" symmetry that arises from QCD. It is proposed that the induced gauge structure *underlies all* low-energy properties of QCD in the nonperturbative regime.

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Purpose and Acknowledgments

In this lecture, I would like to develop the concept of geometric or Berry phases [1] in the strong interaction physics. Such concepts have been playing since some time a powerful role in condensed matter physics as well as in elementary particle physics [2], but surprisingly little attention has been paid to the issue in nuclear/low-energy hadron physics community in connection with unravelling QCD structure of strong interactions in the nonperturbative regime. I would like to propose in this note that a connection between the fundamental theory, QCD, and the effective Lagrangian description of nuclear/hadron physics can be made by exploiting the emergence of a hierarchy of induced (or hidden) gauge structures associated with multiple length scales of the strong interactions.

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The lecture is planned as follows. In the first Section, I will develop the concept of abelian and nonabelian Berry potentials, starting with a (0+1) dimensional field theory, *i.e.*, quantum mechanics and generalizing the notion to four dimensions through the help of a simple but nontrivial hadronic model called "chiral bag". This concept is then applied in the second Section to the description of massive-quark baryons – such as strange and charmed baryons –which exhibit an interesting layers of degrees of freedom.

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1. Induced gauge fields and geometric phases

In this Section, I will develop the concept of induced gauge fields – found to be extremely powerful in condensed matter as well as particle physics – in low-energy strong interaction physics. I will start with a simple quantum mechanical system studied by Stone [3]. I will essentially follow Stone's presentation combined with the method of Rabinovici *et al.* [4].

1.1. Quantum Mechanics: (0+1) Dimensional Field Theory

Consider a system of slowly rotating solenoid coupled to a fast spinning object (call it "electron") described by the (Euclidean) action

$$S_E = \int dt \left(\frac{\mathcal{I}}{2}\dot{\vec{n}}^2 + \psi^\dagger(\partial_t - \mu\hat{n}\cdot\vec{\sigma})\psi\right), \tag{1}$$

where $n^a(t)$, a=1,2,3, is the rotator with $\vec{n}^2 = 1$, $\mathcal{I}$ its moment of inertia, ψ the spinning object ("electron") and μ a constant. We will assume that μ is large so that we can make an adiabatic approximation in treating the slow-fast degrees of freedom. We wish to calculate the partition function

$$Z = \int [d\vec{n}][d\psi][d\psi^\dagger]\delta(\vec{n}^2 - 1)\mathrm{e}^{-S_{\mathrm{E}}} \tag{2}$$

by integrating out the fast degrees of freedom ψ and $\psi^\dagger$. Formally this yields the familiar fermion determinant, the evaluation of which is the physics of the system. In the adiabatic approximation, this can be done as follows which brings out the essence of the method useful for handling complicated situations which will interest us later.

Imagine that $\vec{n}(t)$ rotates slowly. At each instant $t = \tau$, we have an instantaneous Hamiltonian $H(\tau)$ which in our case is just $-\mu\vec{\sigma}\cdot\hat{n}(\tau)$ and the "snap-shot" electron state $|\psi_0(\tau)\rangle$ satisfying

$$H(\tau)|\psi^0(\tau)\rangle = \epsilon(\tau)|\psi^0(\tau)\rangle. \tag{3}$$

In terms of these "snap-shot" wave functions, the solution of the time-dependent Schrödinger equation

$$i\partial_t|\psi(t)\rangle = H(t)|\psi(t)\rangle \tag{4}$$

is

$$|\psi(t)\rangle = \exp\left(i\gamma(t) - i\int_0^t \epsilon(t')dt'\right)|\psi^0(t)\rangle. \tag{5}$$

Note that this has, in addition to the usual dynamical phase involving the energy $\epsilon(t)$, a nontrivial phase $\gamma(t)$ — known as Berry phase — which substituted into (4) is seen to satisfy

$$i\frac{d\gamma}{dt} + \left\langle\psi^0\middle|\frac{d}{dt}\psi^0\right\rangle = 0. \tag{6}$$

This allows us to do the fermion path integrals to the leading order in adiabaticity and to obtain (dropping the trivial dynamical phase involving ϵ)

$$Z = \text{const}\int[d\vec{n}]\delta(\vec{n}^2 - 1)\mathrm{e}^{-S^{\text{eff}}},$$

$$S^{\text{eff}}(\vec{n}) = \int\mathcal{L}^{\text{eff}} = \int\left[\frac{\mathcal{I}}{2}\dot{\vec{n}}^2 - i\vec{\mathcal{A}}(\vec{n})\cdot\dot{\vec{n}}\right]dt, \tag{7}$$

where

$$i\vec{\mathcal{A}}(\vec{n}) = -\langle\psi^0(\vec{n})|\frac{\partial}{\partial\vec{n}}\psi^0(\vec{n})\rangle \tag{8}$$

in terms of which γ is

$$\gamma = \int\vec{\mathcal{A}}\cdot d\vec{n}. \tag{9}$$

$\mathcal{A}$ so defined is known as Berry potential or connection and γ is known as Berry phase. $\mathcal{A}$ is a gauge field with coordinates defined by $\vec{n}$. That it is a gauge field can be seen as follows. Under the transformation

$$\psi^0 \to \mathrm{e}^{i\alpha(\vec{n})}\psi^0, \tag{10}$$

$\vec{\mathcal{A}}$ transforms as a gauge field, *i.e.*,

$$\vec{\mathcal{A}} \to \vec{\mathcal{A}} - \frac{\partial}{\partial\vec{n}}\alpha(\vec{n}). \tag{11}$$

The theory is gauge-invariant in the sense that under the transformation (11), the theory (7) remains unchanged. (I am assuming that the surface term can be dropped.)

It should be made clear that the gauge field we have here is first of all defined in "order parameter space", not real space like electroweak field or gluon field and secondly it is induced when fast degrees of freedom are integrated out. This is a highly generic feature we will encounter time and again. Later on we will see that the space on which the gauge structure emerges is usually the flavor space like isospin or hypercharge space in various dimensions.

We shall now calculate the explicit form of the potential $\mathcal{A}$. For this let us use the polar coordinate and parametrize the solenoid as

$$\vec{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \tag{12}$$

with the Euler angles $\theta(t)$ and $\phi(t)$ assumed to be slowly changing (slow compared with the scale defined by the fermion mass μ) as a function of time. Then the relevant Hamiltonian can be written as

$$\begin{aligned} \delta H &= -\mu\vec{\sigma}\cdot\hat{n}(t) = S(t)\delta H_0 S^{-1}(t), \\ \delta H_0 &\equiv -\mu\sigma_3 \end{aligned} \tag{13}$$

with

$$S(\vec{n}(t)) = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2}e^{-i\phi} \\ \sin\frac{\theta}{2}e^{i\phi} & \cos\frac{\theta}{2} \end{pmatrix}. \tag{14}$$

Since the eigenstates of δH_0 are $\binom{1}{0}$ with eigenvalue $-\mu$ and $\begin{pmatrix}0\\1\end{pmatrix}$ with eigenvalue $+\mu$, we can write the "snap-shot" eigenstate of $H(t)$ as

$$|\psi^0_{+\uparrow}\rangle = S\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}e^{i\phi}\end{pmatrix}, \tag{15}$$

where the arrow in the subscript denotes the "spin-up" eigenstate of δH_0 and $+$ denotes the upper hemisphere to be specified below. The eigenstate $|\psi^0_{+\downarrow}\rangle$ is similarly defined with the "down spin". Now note that for $\theta = \pi$, (15) depends on ϕ which is undefined. This means that (15) is ill-defined in the lower hemisphere with string singularity along $\theta = \pi$. On the other hand, (15) is well-defined for $\theta = 0$ and hence in the upper hemisphere. The meaning of the $+$ in (15) is that it has meaning only in the upper hemisphere, thus the name "wave section" rather than wave function.

Given (15), we can use the definition (8) for the Berry potential to obtain

$$-i\vec{\mathcal{A}}_+(\vec{n})\cdot d\vec{n} = \langle\uparrow|S^{-1}dS|\uparrow\rangle = \tfrac{i}{2}(1-\cos\theta)d\phi \tag{16}$$

given here in one-form. The explicit form of the potential is [1]

$$\vec{\mathcal{A}}_{+} = -\frac{1/2}{(1+\cos\theta)}(-\sin\theta\sin\phi, \sin\theta\cos\phi, 0) \tag{17}$$

which is singular at $\theta = \pi$ as mentioned above. This is the well-known Dirac string singularity. Since we have the gauge freedom, we are allowed to do a gauge transformation

$$\psi_{+}^{0} \rightarrow e^{-\phi}\psi_{+}^{0} \equiv \psi_{-}^{0} \tag{18}$$

which corresponds to defining a gauge potential regular in the lower hemisphere (denoted with the subscript $-$)

$$\vec{\mathcal{A}}_{-} \cdot d\vec{n} = \vec{\mathcal{A}}_{+} \cdot d\vec{n} + d\phi \tag{19}$$

giving

$$\vec{\mathcal{A}}_{-} \cdot d\vec{n} = \tfrac{1}{2}(1+\cos\theta)d\phi. \tag{20}$$

This potential has a singularity at $\theta = 0$. Thus we have gauge-transformed the Dirac string from the lower hemisphere to the upper hemisphere. This clearly shows that the string is an artifact and is unphysical. In other words, physics should not be dependent on the string. Indeed the field strength tensor, given in terms of the wedge symbol and forms,

$$\mathcal{F} = d\mathcal{A} = \tfrac{1}{2}d\theta \wedge d\phi = \tfrac{1}{2}d(\text{Area}) \tag{21}$$

is perfectly well-defined in both hemispheres and unique. A remarkable fact here is that the gauge potential or more properly the field tensor is completely independent of the fermion "mass" μ. This means that the potential does not depend upon how fast the fast object is once it is decoupled adiabatically. We will come back later to this matter in connection with applications to real systems.

Let us consider a cyclic path. We will imagine that the solenoid is rotated from $t = 0$ to $t = T$ with large T such that the parameter $\vec{n}$ satisfies $\vec{n}(0) = \vec{n}(T)$. We are thus dealing with an evolution, with the trajectory of $\vec{n}$ defining a circle C. The parameter space manifold is two-sphere S^2 since $\vec{n}^2 = 1$. Call the upper hemisphere D and the lower hemisphere $\bar{D}$ whose boundary is the circle C, *i.e.*, $\partial D = C$. Then using Stoke's theorem, we have from (9) for cyclic evolution Γ

$$\gamma(\Gamma) = \int_{C=\partial D} \vec{\mathcal{A}} \cdot d\vec{n} \equiv \int_{\partial D = C} \mathcal{A} = -\int_{D} d\mathcal{A} = -\int_{D} \mathcal{F}. \tag{22}$$

[1] If the Hamiltonian commutes for different times, then the gauge field can be made to vanish. We are considering the case where the Hamiltonians do not commute.

Since the gauge field in $\overline{D}$ is related to that in D by a gauge transform, we could equally well write γ in terms of the former. Thus we deduce that

$$\exp\Big(i\int_C \mathcal{A}\Big) = \exp\Big(i\int_D \mathcal{F}\Big) = \exp\Big(-i\int_{\overline{D}} \mathcal{F}\Big) \tag{23}$$

which implies

$$\exp\Big(i\int_{D+\overline{D}=S^2} \mathcal{F}\Big) = 1. \tag{24}$$

Thus we get the quantization condition

$$\int_{S^2} \mathcal{F} = 2\pi n \tag{25}$$

with n an integer. This just means that the total magnetic flux going through the surface is quantized. Since in our case the field strength is given by (21), our system corresponds to $n = 1$ corresponding to a "monopole charge" $g = {}^1\!/_2$ located at the center of the sphere. In general, as we will see later in real systems in (3+1) dimensions, the "monopole charge" need not be precisely ${}^1\!/_2$; it could be some quantity, say, g. What we learn from the above exercise is that consistency with quantum mechanics demands that it be a multiple of ${}^1\!/_2$. Otherwise, the theory makes no sense. It will turn out later that real systems in the strong interactions involve nonabelian gauge fields which do not require such "charge" quantization, making the consideration somewhat more delicate.

1.1.1. Level crossing

Continuing with our quantum mechanical system, we note that the singularity here is associated with the level crossing of the spin-up level with the spin-down level. The two levels become degenerate at $\mu = 0$. This degeneracy, which is not in the space we are considering since we are excluding that *trivial* point, is the cause of the presence of the monopole structure [1]. Since we are focusing on what happens in one level, either upper or lower (it does not matter which as mentioned above), the gauge field is abelian. But imagine that a level crossing occurs between an n-fold degenerate level and an m-fold degenerate level in some configuration space. In that case, if one looks at what happens in the subspace of either of the degenerate levels, one encounters an induced gauge structure which spans the degenerate subspace and hence is nonabelian. This is the situation that we shall meet in strongly interacting systems endowed with a flavor symmetry as I will indicate later.

1.1.2. The Wess–Zumino term

The key point which we will find useful later is that when the fast degree of freedom (the "electron") is integrated out, we wind up with a gauge field as a relic of the fast degree of freedom integrated out. The effective Lagrangian that results has the form (in Minkowski space)

$$L^{\mathrm{eff}} = \tfrac{1}{2}\mathcal{I}\dot{\vec{n}}^2 + \vec{A}(\vec{n})\cdot\dot{\vec{n}}. \tag{26}$$

There is another way of writing the effective Lagrangian that exhibits the relic of the integrated-out degrees of freedom, so-called Wess–Zumino term. The gauge field in (26) is, as noted above, an induced one, conducted out of the solenoid $\vec{n}$. Therefore one should be able to rewrite the second term of (26) in terms of $\vec{n}$ alone. It turns out that this cannot be done locally (because of the Dirac singularity) but the corresponding action can be written locally in terms of $\vec{n}$ by extending to one dimension higher. This is the Wess–Zumino term. When written in this way, we no longer have the gauge structure. It is "hidden" in some sense.

Let us look at the action

$$S_{\mathrm{WZ}} \equiv \int \vec{A}(\vec{n})\cdot\dot{\vec{n}}dt. \tag{27}$$

There is a standard way of expressing this in a local form. The procedure is quite general and goes as follows. First extend the space from the physical dimension d which is 1 in our case to $d+1$ dimension. This extension is possible ("no obstruction") if

$$\pi_d(\mathcal{M}) = 0, \quad \pi_{d+1}(\mathcal{M}) \neq 0, \tag{28}$$

where π is the homotopy group and $\mathcal{M}$ is the parameter space manifold. In our case

$$\pi_1(S^2) = 0, \quad \pi_2(S^2) = Z. \tag{29}$$

So it is fine. We extend the space to $\tilde{n}(s)$, $0 \leq s \leq 1$ such that

$$\tilde{n}(s=0) = 1, \quad \tilde{n}(s=1) = \vec{n}. \tag{30}$$

Now the next step is to construct a winding number density $\widetilde{Q}$ for π_{d+1} which is then to be integrated over a region of $\mathcal{M}$ ($\approx S^2$ here) bounded by d-dimensional fields n^i. The winding number density $\widetilde{Q}(x)$ (say $x_1 = t$ and $x_2 = s$) is

$$\widetilde{Q} = \frac{1}{8\pi}\epsilon^{ij}\epsilon^{abc}\tilde{n}^a\partial_i\tilde{n}^b\partial_j\tilde{n}^c \tag{31}$$

and the winding number n (which is 1 for $\pi \le \theta \le 0$, $2\pi \le \phi \le 0$)[2]

$$n = \int_{S^2} d^2x\widetilde{Q}(x). \tag{32}$$

Comparing with (25), we deduce

$$S_{\rm WZ} = 4\pi g \int_{\mathcal{D}=t\times[0,1]} d^2x\widetilde{Q}(x). \tag{33}$$

This is the familiar form of the Wess–Zumino term defined in two dimensions. As we saw before this has a "monopole charge" $g = {}^1\!/_2$. Below we will carry g as an integral multiple of ${}^1\!/_2$.

1.1.3. Quantization

There are numerous ways of quantizing the effective action (hereon we will work in Minkowski space)

$$S^{\rm eff} = S_0 + S_{\rm WZ}, \tag{34}$$

where the Wess–Zumino action is given by (33) and

$$S_0 = \oint dt\,\frac{\mathcal{I}}{2}\,\dot{\vec{n}}^2. \tag{35}$$

Here we will consider the time compactified as defined above, so the time integral is written as a loop integral. We will choose one way which illustrates other interesting properties.

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[2] This can be calculated as follows. The surface element is

$$d\Sigma^i = \left(\frac{\partial\vec{n}}{\partial s}ds \times \frac{\partial\vec{n}}{\partial t}dt\right)^i = \tfrac{1}{2}\epsilon^{ab}\epsilon^{ijk}\frac{\partial n^j}{\partial x^a}\frac{\partial n^k}{\partial x^b}d^2x$$

and hence

$$\int_{S^2}\vec{n}\cdot d\vec{\Sigma} = 4\pi.$$

The manifold has SO(3) invariance corresponding to $\sum_{i=1}^{3} n_i^2 = 1$. Consider now the complex doublet $z \in \mathrm{SU}(2)$

$$z(t) = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \tag{36}$$

$$z^\dagger z = |z_1|^2 + |z_2|^2 = 1. \tag{37}$$

Then we can write

$$n_i = z^\dagger \sigma_i z, \tag{38}$$

with σ_i the Pauli matrices. There is a redundant (gauge) degree of freedom since under the U(1) transformation $z \to e^{i\alpha} z$, n_i remains invariant. This is as it should be since the manifold is topologically S^2 and hence corresponds to the coset SU(2)/U(1). We are going to exploit this U(1) gauge symmetry to quantize our effective theory.

Define a 2-by-2 matrix h

$$h = \begin{pmatrix} z_1 & -z_2^\star \\ z_2 & z_1^\star \end{pmatrix} \tag{39}$$

and

$$a(t) = \sum_{k=1}^{2} \frac{i}{2}(z_k^\star \overleftrightarrow{\partial}_k z_k). \tag{40}$$

Then it is easy to obtain (setting $\mathcal{I} = 1$) that

$$S_0 = \tfrac{1}{2} \oint dt \mathrm{Tr}\left[\overrightarrow{D}_t^\dagger h^\dagger h \overleftarrow{D}_t \right], \tag{41}$$

where

$$\overrightarrow{D}_t = \overrightarrow{\partial}_t - ia\sigma_3. \tag{42}$$

Let

$$\tilde{a}_\mu = \frac{i}{2}(\tilde{z}_k^\star \overleftrightarrow{\partial}_\mu \tilde{z}_k), \tag{43}$$

where the index μ runs over the extended coordinate (s, t). As noted before, there is no topological obstruction to this extension. This is defined in such a way that $\tilde{z}(s = 0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\tilde{z}(s = 1) = z$. Then the Wess–Zumino action can be written in a Chern–Simons form

$$S_{\mathrm{WZ}} = 2g \int_{\mathcal{D}} d^2x \epsilon^{\mu\nu} \partial_\mu \tilde{a}_\nu = 2g \oint dt a(t). \tag{44}$$

Introducing an auxiliary function A, we can write the partition function

$$Z = \int [dz][dz^\dagger][dA] \exp\, i \oint dt \left(\tfrac{1}{2} \mathrm{Tr}(\overrightarrow{\partial}_t + iA\sigma_3) h^\dagger h(\overleftarrow{\partial}_t - iA\sigma_3) + 2gA + \tfrac{1}{4}(2g)^2 \right). \quad (45)$$

The U(1) gauge invariance is manifest in this action. Indeed if we make the (local) transformation $h \to \exp\, i\alpha(t)(\sigma_3/2)h$ and $A \to A - {}^1\!/_2\partial_t\alpha(t)$ with the boundary condition $\alpha(T) - \alpha(0) = 4\pi N$ where N is an integer, then the action remains invariant. This means that we have to gauge-fix the "gauge field" A in the path integral. The natural gauge choice is the "temporal gauge" $A = 0$. The resulting gauge-fixed action is $\oint \mathcal{L}_{\mathrm{gf}}$ with

$$\mathcal{L}_{\mathrm{gf}} = \mathrm{Tr}\big(\partial_t h^\dagger \partial_t h\big) + g^2. \quad (46)$$

Since there is no time derivative of A in (45), there is a Gauss' law constraint which is obtained by taking $\delta S/\delta A|_{A=0}$ from the action (45) *before* gauge fixing:

$$\frac{i}{2} \mathrm{Tr} \left(\sigma_3 h^\dagger \partial_t h - \partial_t h^\dagger h \sigma_3\right) + 2g = 0$$

which is

$$i\, \mathrm{Tr} \left(\partial_t h^\dagger h \frac{\sigma_3}{2}\right) = g. \quad (47)$$

The left-hand side is identified as the right rotation around third axis J_3^R, so the constraint is that

$$J_3^R = g. \quad (48)$$

Since (46) is invariant under $\mathrm{SU(2)_{L,R}}$ multiplication, we have that

$$\vec{J}^{\,2} = \vec{J}_{\mathrm{L}}^{\,2} = \vec{J}_{\mathrm{R}}^{\,2}. \quad (49)$$

The Hamiltonian is (restoring the moment of inertia $\mathcal{I}$)

$$H = \frac{1}{2\mathcal{I}} \left(\vec{J}^{\,2} - g^2\right) \quad (50)$$

which has the spectrum of a tilted symmetric top. Now adding the energy of N "electrons", the total energy is

$$E = N\epsilon + \frac{1}{2\mathcal{I}} \left(J(J+1) - g^2\right) \quad (51)$$

with the allowed values for J

$$J = |g|, |g| + 1, \cdots. \quad (52)$$

The rotational spectrum is the well-known Dirac monopole spectrum. Later we will derive an analogous formula for real systems in four dimensions.

1.1.4. Summary

When a fast spinning object coupled to slowly rotating object is integrated out, a Berry potential arises gauge coupled to the rotor. The effect of this gauge coupling is to "tilt" the angular momentum of the rotor which is still a symmetric top. It supplies an extra component to the angular momentum, along the third direction. The gauge field is abelian and has an abelian (Dirac) monopole structure. The abelian nature is inherited from one nondegenerate level crossing another nondegenerate level. When degenerate levels cross, the gauge field can be nonabelian and this is the generic feature we encounter in strong interaction physics.

1.2. Nonabelian gauge fields

In this Section, we describe the simplest possible model that is relevant to the strong interaction physics in (3+1) dimensions. We will derive — following closely Refs [5] — a generic action that catches the essence of the problem.

1.2.1. The chiral bag

Consider the two-phase picture which describes a baryon with up (u) and down (d) quarks whose masses are neglected, confined in a three-volume denoted by V coupled at the surface to (Goldstone) pions. The consistency between the two phases is assured by a boundary condition. Assuming that the quarks are the fast degree of freedom and the pion the slow degree of freedom, we wish to integrate out the quarks in the manner analogous to the integrating-out of the "electron" discussed in the previous Section.

It suffices to focus on the action of the quark sector. To do this, we simplify the boundary condition as much as possible. The nontrivial pion configuration that affects the motion of the quark in V is the hedgehog form which is time-independent

$$U_0(\vec{r}) = \exp\left(i\vec{\tau}\cdot\hat{r}\theta(r)\right), \tag{53}$$

where $\theta(r)$ is the chiral angle. Thus at classical level, the quark couples to the pion on the surface in the form

$$in^\mu\gamma_\mu\psi = U_0^5\psi, \tag{54}$$

where n^μ is the outward normal unit four-vector and

$$U_0^5 = \exp\left(i\vec{\tau}\cdot\hat{r}\gamma_5\theta(\beta)\right). \tag{55}$$

Here $r = \beta$ is a point at the surface. It is now well understood that due to an anomaly induced by the surface, the baryon charge leaks from the space V into the meson sector. Now let us imagine that the hedgehog is slowly rotating such that

$$U^5(\beta, t) = S^\dagger(t) U_0^5 S(t), \tag{56}$$

with $S(t) \in \mathrm{SU}(2)$. Now the boundary condition becomes time-dependent

$$in^\mu\gamma_\mu\psi = U^5(t)\psi. \tag{57}$$

This time dependence on the boundary condition is undesirable, so we wish to eliminate it. This can be done by the local "chiral rotation"

$$\psi' = S(t)\psi. \tag{58}$$

With this change of the quark field, the action inside the volume V becomes (suppressing primes)[3]

$$S_{\mathcal{A}} = \int_{V\times R} d^4r\psi^\dagger\left(i\partial_t + S^\dagger i\partial_t S - H_{S=1}\right)\psi, \tag{59}$$

where $H_{S=1}$ is the Hamiltonian for the quark sector in V that determines the "snap-shot" spectrum. Now the quark field obeys the "trivial" time-independent boundary condition. The four dimensional integral in this expression goes over the volume V and the time $\in R$. Thus we have reduced the chiral bag problem entirely to the problem involving quarks under the influence of an induced potential $S^\dagger i\partial_t S$ arising from the rotating hedgehog. We wish to look at the fermion degrees of freedom coupled to this rotation. In general the induced potential can couple to a hierarchy of quark excitations. The longest wavelength excitation is already subsumed in the Goldstone bosons that give rise to the hedgehog configuration. We need not worry about it anymore. The next energy scale is the excitation that involves a quark making a jump from an occupied state to an unoccupied state, *i.e.*, a "particle-hole" type excitation on top of the ground state baryon. This leads to excited baryons. Thus the physics we wish to obtain is then the effect of integrating out this "vibrational" degree of freedom,

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[3] There is no anomaly with the SU(2) group, so this axial rotation does not affect the fermion measure involved in the fermion path integral.

whose frequency is supposed to be much larger than that of the rotation that makes up the slow degree of freedom. (We will see in the next Section a beautiful application of this idea to the Callan-Klebanov skyrmion.) The collective rotation if unaffected by the vibration will describe the ground state band, namely the N and the Δ. The effect of the vibration is lodged in the Berry potential that we will obtain. This gauge field will tilt the rotational spectrum in such a way that states other than the rotator spectrum with $J = T$ will arise. Thus it remains for us to derive the coupling of the vibration and rotation modes as in the quantum mechanics example.

To do this, we expand the quark field in the basis of the hedgehog quark solution which is characterized by the grand spin K

$$\vec{K} = \vec{J} + \vec{T}. \tag{60}$$

We write the quark field as

$$\psi = \sum_{K,M} \alpha_{K,M}\phi_{K,M}, \tag{61}$$

where $\alpha_{K,M}$ is a Grassmannian c-number, $\phi_{K,M}$ is the hedgehog-quark solution and M is the projection of K on z. Both positive and negative energy solutions are understood in the sum. Substituting this into (59) and assuming that the hedgehog-quark wave function is properly normalized, we find

$$S_{\mathcal{A}} = \sum_{K,M} \int_R dt\Big(\alpha^{\dagger}_{KM}\big((i\partial_t - \epsilon_K)\delta_{MN} + g_K(\mathcal{G}_K)_{MN}\big)\alpha_{KN}\Big), \tag{62}$$

where

$$(\mathcal{G}_K)_{MN} = \int_V d^3r\phi^{\dagger}_{KM}\mathcal{G}\phi_{KN},$$
$$\mathcal{G} \equiv S^{\dagger}\partial_t S. \tag{63}$$

Here g_K is the "induced" charge associated with the "tilting" of the symmetry analogous to what we had for the abelian case. It depends on details of the quark orbits considered. I will not need it for this lecture. As defined, $\mathcal{A}_K$ is really a nonabelian object projected onto a particular K space with no off-diagonal matrix elements. Without the projection, $\mathcal{G}$ can be made to vanish. The $\mathcal{G}_K$ turns out to be related to nonabelian Berry potential. Indeed if we parametrize

$$S = a_4 + i\vec{\tau}\cdot\vec{a}, \tag{64}$$

with $a_4^2 + \vec{a}^{\,2} = 1$, then the nonabelian Berry potential one-form $\mathcal{A}_K$ is

$$\mathcal{A}_K = \mathcal{G}_K dt = -\bar{\eta}_{b\mu\nu} a_\mu \dot{a}_\nu dt T_K^b, \tag{65}$$

where T_K^b is an SU(2) generator in the $(2K+1)\times(2K+1)$ dimensional K-representation and η is the antisymmetric 't Hooft symbol. This has an instanton structure. Now $\mathcal{A}_K$ is an "invariant" gauge 1-form in the following sense. Under a local unitary rotation $D^K(t)$ of the fermionic wavefunctions in a given K band, the action (62) becomes

$$S_{\mathcal{A}}^D = \sum_{K,M} \int_R dt \Big(\alpha_{KM}^\dagger \Big((i\partial_t - \epsilon_K)\delta_{MN} + ({D_{MM'}^K}^\dagger i\partial_t D_{NM'}^K) + g_K {D_{MM'}^K}^\dagger (\mathcal{G}_K)_{M'N'} D_{NN'}^K \Big) \alpha_{KN} \Big). \tag{66}$$

This transformation can be compensated by the gauge transformation

$$\mathcal{G}_K \to D^K \left(\mathcal{G}_K + i\frac{1}{g_K}\partial_t \right) D^{K\dagger}. \tag{67}$$

Thus we have a gauge invariance in the action (62). This seems to be in a sense closely connected with the hidden gauge symmetry in the strong interaction sector emphasized by Bando, Kugo and Yamawaki [6].

The integration over the quark field is now expressed in terms of an integral over the Grassmannian α. We will now perform the fermion integral over all occupied quark states (*i.e.*, sea quarks) and leave the valence-quark states still active. In an SU(2) chiral bag, the $K=0$ level is occupied for the ground state baryons N and Δ. In order to describe excitations, we have to consider one or more quarks in the $K=0$ state being excited to the next orbital which is $K=1$ and even-parity. Now the $K=1$ level is triply degenerate. We will label this state by v, standing generically for "valence." In the case of hyperons, the additional flavor quantum number plays a particular role. How this works out is discussed in the next Section.

One can simplify the calculations by using the vielbein formulation which for our purpose goes as follows. This is purely a convenient technique and no new physics is involved other than what we already know from the standard treatment.

Let the vielbeins on S^3 be denoted as e_m^a where we use $a, b, c, \cdots$ as the internal (or, in our case, K) space index and $m, n, \cdots$ as the index for the coordinates on S^3 parametrized by X_m. The vielbein one-form is defined by

$$e^a = e_m^a dX^m. \tag{68}$$

We can think of the internal space to be the "inertial" frame with a flat metric δ_{ab} and the S^3 the "curved space" with a metric g_{mn}. The two are related by

$$\begin{aligned} g_{mn} &= \delta_{ab} e^a_m e^b_n, \\ \delta_{ab} &= g^{mn} e^a_m e^b_n. \end{aligned} \tag{69}$$

The nonabelian gauge connection induced in the way described in (65) (known as nonabelian Berry connection) can be given a simple form in vielbeins. The gauge 1-form (65) takes the simple form

$$\mathcal{A} \equiv \mathcal{A}_m dX^m = S^\dagger i dS = -iT^a e^a_m dX^m. \tag{70}$$

As before the T^a is the generator of the K space. Projected onto a K subspace, we have

$$\mathcal{A}_K = -iT^a_K e^a_m dX^m. \tag{71}$$

Since (62) is gauge invariant, one can think of this as gauge transformed form of (65).

The projected gauge field (*i.e.*, the Berry potential) has a nontrivial field strength, namely,

$$F^m_K = d\mathcal{A}^m_K - ig_K(\mathcal{A}_K \wedge \mathcal{A}_K)^m = -(1 - g_K/2)\,\epsilon^{mij} e^i \wedge e^j. \tag{72}$$

Note that for $g_K = 2$, the field tensor vanishes and the Berry potential becomes a pure gauge.

Now we are ready to write down the effective action that results when the sea quark states are integrated out. Before the sea-quark integration, we had from (62)

$$\begin{aligned} S_{\mathcal{A}} = \sum_{K,M} \int_R dt\Big(\alpha^\dagger_{KM}\Big((i\partial_t - \epsilon_K)\delta_{MN} \\ + g_K((\mathcal{G}_K)_m \dot{X}^m)_{MN}\Big)\alpha_{KN}\Big). \end{aligned} \tag{73}$$

When one integrates out the occupied quark states, one obtains a term quadratic in $\dot{X}$ involving no v state. (The term linear in $\dot{X}$ is absent as it vanishes in $K = 0$ orbit.) Ignoring higher order derivatives (in conjunction with the adiabatic approximation), we obtain

$$\begin{aligned} S^\star_{\mathcal{A}} = &\int_R dt\left(\frac{\mathcal{I}}{2}\dot{X}_\mu \dot{X}_\mu - ig\dot{X}_\mu(\mathcal{A}^\mu)_{MN}\alpha^\dagger_M \alpha_N + i\alpha^\dagger_M \dot{\alpha}_M\right) \\ &- \int_R dt\,\epsilon(t)\,\alpha^\dagger_M \alpha_M, \end{aligned} \tag{74}$$

where $\mu = 1, 2, 3, 4$ and understanding that we are explicitly dealing with quarks in the "valence" orbit, we have suppressed the index v on the Grassmannian variables. The last term is a "dynamical phase" as defined earlier and plays no role in the quantization, so we will drop it for the moment. The first term is the analog to $\dot{\vec{n}}^{\,2}$ in the quantum mechanics of the solenoid-spin system. $\mathcal{I}$ is the moment of inertia which in specific dynamical models can be calculated explicitly. The second term is the most interesting quantity describing coupling between the rotator degree of freedom with the quark excitation through a nonabelian gauge field $\mathcal{A}$. There are two ways of treating this term. One is to treat it in terms of *explicit* fermion variables which we can do for excited states of nonstrange baryons. Another possible usage of this term is to bosonize and apply it to the massive flavors such as strange hyperons. This leads to the Callan-Klebanov skyrmion structure which we will discuss later. (See Eq. (98).)

1.2.2. Canonical quantization

Let us now quantize the theory (74) canonically. To do so, we first obtain the Hamiltonian which takes the form

$$H^\star = \frac{1}{8\mathcal{I}}\left(\Pi_m - ig\mathcal{A}^m_{MN}\alpha^\dagger_M\alpha_N\right)(g^{-1})^{mn}\left(\Pi_n - ig\mathcal{A}^n_{RS}\alpha^\dagger_R\alpha_S\right), \quad (75)$$

where $\Pi_m = -i\partial/\partial\dot{X}^m$ is the momentum conjugate to X^m and $(g^{-1})^{mn} = e^m_i e^n_i$ is the inverse metric on S^3. The dynamical energy term is dropped as mentioned above. The left (L) and right (R) generators of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ are given by

$$L_i = e^m_i \Pi_m, \quad R_i = \bar{e}^m_i \Pi_m \quad (76)$$

which satisfy the commutation rules

$$[L_i, L_j] = -2i\epsilon^{ijk}L_k, \quad (77)$$

$$[R_i, R_j] = -2i\epsilon^{ijk}R_k. \quad (78)$$

On S^3, $\vec{L}^{\,2} = \vec{R}^{\,2}$. Since

$$e^m_i \mathcal{A}_m = T^j_v e^m_i e^j_m = T^i_v, \quad (79)$$

where we have put the subscript v to indicate the projection on v, the Hamiltonian then becomes

$$H^\star = \frac{1}{2\mathcal{I}}\left(L_j - g(T^j_v)_{MN}\alpha^\dagger_M\alpha_N\right)\left(L_j - g(T^j_v)_{RS}\alpha^\dagger_R\alpha_S\right). \quad (80)$$

If we ignore interactions between the fermions, then this Hamiltonian takes the form

$$H^{\star} \sim \epsilon + \frac{1}{2\mathcal{I}}\left(\left(\frac{\vec{L}}{2}\right)^2 - g\frac{\vec{L}}{2}\cdot\vec{T}_v + \frac{g^2}{4}\vec{T}_v{}^2\right). \tag{81}$$

This form has also been obtained by a standard cranking technique familiar in nuclear physics [7]. It reduces for $g = 0$ to the "tilted" spherical top corresponding to an abelian monopole obtained above. This Hamiltonian can be interpreted as follows. $L/2$ is the angular momentum stored in the soliton cloud, T_v the angular momentum carried by the "particle-hole" mode and the isospin stored in the soliton cloud R is identical to L. Now the total angular momentum is the vector sum of $\vec{L}/2$ and $\vec{T}_v$, which is conserved. Another conserved quantity is the total isospin which is given by $I = R/2$. This is a rather general form which rends itself to a variety of applications. I discuss a most intriguing case in the following Section.

2. Application to massive-quark baryons

The massive-quark baryons (by massive quark, I mean s, c, b flavor quarks) present an interesting and fascinating case of a hierarchy of length scales leading to a hierarchy of induced gauge fields. I will apply the concept developed above to this system. The discussion given here essentially rephrases Ref. [8] in terms of induced gauge fields. We will arrive at an analog of the expression (81).

The idea developed in [8] is a simple generalization of the picture proposed by Callan and Klebanov [9] for strange baryons. I shall present this in a way [10] to bring out the essence of induced gauge structure developed above. It should be noted, however, that one can proceed *without* explicitly showing gauge structure as was done by others [9,8].

Before going into detail, let me sketch the qualitative feature first. For definiteness, consider the simplest baryon with, say, a charmed quark c, namely Λ_c, which has the valence quark structure udc. In the simple quark-model picture, the u and d quarks couple to $J = I = 0$, so it is the charm flavor c quark that carries the spin $^1/_2$. The baryonic charge is $^1/_3$ for each quark, three quarks making up the required baryon charge 1.

2.1. Analogy to magnetic monopole

In modelling this in the skyrmion description, it turns out to be fruitful to exploit a close analogy to nonabelian monopole-scalar doublet system [11]: When a scalar doublet is "trapped" in an SU(2) magnetic monopole field, the isospin of the scalar is transmuted to spin $^1/_2$ with a resulting system of two bosons becoming a fermion. It is now well-known [12] that such a transmutation can also occur when a scalar doublet is trapped in an *induced* gauge field generated in a parameter space rather than in real space. In our case, we start with a skyrmion made up of the u and d quarks (*e.g.*, the flavor SU(2)) which provides a soliton background. As is well-known, the skyrmion carries the entire baryon charge 1 but it carries no massive flavor. In order to bring in, say, the charm quantum number, we thus need a charm-flavored boson field playing an analogous role of the scalar in the monopole system. Since the skyrmion is like a monopole in flavor space, a doublet of a charmed boson field bound to the skyrmion can acquire spin $^1/_2$ from its isospin $^1/_2$. Now since the skyrmion is quantized with $J = I = 0, 1, \cdots$, to describe Λ_c, for instance, we will need to couple $J = I = 0$ to $J = {}^1/_2$ of the doublet boson to give total spin $^1/_2$ and isospin 0. It was recently shown by Scoccola and Wirzba [13] that all the low-lying baryons with one or more s, c and b quarks can be correctly labelled in this way. I will make frequent use of their analysis below without elaborating on details.

Much of the essential dynamics lie then in the structure of the flavored scalar bosons. If one calls the massive quark Q and the light quark q, then the massive scalar that we need is of the form $\Phi = \bar{q}Q$. From now on Q will also label the massive flavor quantum number, *i.e.*, $Q{=}S$, C, B, *etc.* The doublet structure is in $q^T = (\mathrm{u\ d})$ which forms the isospin. Doublets can be coupled independently with their coupling differentiated only by the decay constants f_Φ and the masses m_Φ. In the limit that the massive scalars have an infinite mass, we expect that the massive baryon structure will be independent of the massive quark flavors and acquire the symmetry associated with the spin of the massive quark. This leads naturally to the "Wisgur" spin symmetry, recently discovered for massive quarks on the basis of QCD [14]. I will show how this symmetry arises in the present theory.

2.1.1. Generic structure

Let me now be more specific by taking a simplified but generic Lagrangian that illustrates the above structure. (The numerical values quoted later will be based on a more realistic Lagrangian.) Denoting the scalar doublet generically by Φ, I write the Lagrangian in the form

$$\mathcal{L} = \mathcal{L}_{\mathrm{SU(2)}} + \mathcal{L}_\Phi + \mathcal{L}_{\mathrm{WZ}}. \tag{82}$$

Here $\mathcal{L}_{\mathrm{SU(2)}}$ is the chiral Lagrangian for the (u, d) sector that supports a skyrmion

$$\mathcal{L}_{\mathrm{SU(2)}} = \frac{f_\pi^2}{4}\,\mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger] + a\,\mathrm{Tr}(MU + \mathrm{h.c.}) + \cdots \tag{83}$$

with the chiral field valued in SU(2)

$$U(x) = \exp\left(i\vec{\tau}\cdot\vec{\pi}(x)/f_\pi\right), \tag{84}$$

where M is the quark mass matrix and the ellipsis stands for higher derivative terms and low-energy spin-1 fields (such as ρ, ω,...) *etc.* $\mathcal{L}_\Phi$ describes the dynamics of the doublet scalar in the background of the skyrmion field

$$\mathcal{L}_\Phi = (D_\mu\Phi)^\dagger D^\mu\Phi - m_\Phi^2\Phi^\dagger\Phi + V(\Phi^\dagger\Phi, U, \cdots), \tag{85}$$

where D_μ is the covariant derivative $D_\mu\Phi = (\partial_\mu + v_\mu)\Phi$ with the induced vector field

$$v_\mu = \tfrac{1}{2}(\sqrt{U}^\dagger\partial_\mu\sqrt{U} + \sqrt{U}\partial_\mu\sqrt{U}^\dagger) \tag{86}$$

transforming as an SU(2)-valued gauge field, *i.e.*, $v_\mu \to g^{-1}(v_\mu + \partial_\mu)g$ for a gauge function g.[4] V stands for other (potential) terms, including the induced "axial vector" $a_\mu = {}^1\!/\!_2(\sqrt{U}^\dagger\partial_\mu\sqrt{U} - \sqrt{U}\partial_\mu\sqrt{U}^\dagger)$ which transforms covariantly, *i.e.*, $a_\mu \to g^{-1}a_\mu g$ and strong vector (*e.g.*, ρ, ω, $\cdots$) and axial-vector (*e.g.*, $A_1, \cdots$) mesons. The induced vector field v_μ, *i.e.*, (86) will turn out to have a "monopole" (or more precisely an "inside-out monopole") structure of the skyrmion. The $\mathcal{L}_{WZ}$ is a piece that comes from the Wess–Zumino term [15] that is of the from

$$\mathcal{L}_{WZ} = \frac{iN_c}{4f_\Phi^2}\left(B_\mu[(D^\mu\Phi)^\dagger\Phi - \Phi^\dagger D^\mu\Phi] + \cdots\right), \tag{87}$$

where the ellipsis stands for terms involving strong vector mesons *etc.* and B_μ is the topological baryon current. The coefficient of the Wess–Zumino term is constrained by the topological five-pseudoscalar coupling $\Phi\Phi\pi\pi\pi$ and, in the limit $m_\Phi \to 0$, by the symmetry of the massless flavors that follows from the fact that the Wess–Zumino term is independent of the mass scales involved.

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[4] This can be associated with the hidden SU(2) gauge symmetry of Bando and collaborators [6]. In fact, one can couple in the strong vector mesons ρ and have them propagate (through quantum loop effects) so as to describe short-distance physics. In the long-wavelength limit (*i.e.*, for "magnetic monopole" structure), however, one may ignore the kinetic energy term of the vector mesons in which case the ρ field can be integrated out leaving the *effective field* v_μ acting as an external field felt by the massive scalars.

2.2. The Model

To obtain the generic structure of Lagrangian (82) from an effective chiral Lagrangian, arrange the pseudoscalar doublets that we are interested in into U_5 as

$$U_5 = \exp\left\{ i2\sqrt{2}\begin{bmatrix} 0_2 & \widetilde{K} & \widetilde{D} & \widetilde{B} \\ \widetilde{K}^\dagger & & & \\ \widetilde{D}^\dagger & & 0_3 & \\ \widetilde{B}^\dagger & & & \end{bmatrix}\right\}, \tag{88}$$

with $\tilde{\Phi} = \Phi/f_\Phi$, where $\Phi = K$, D and B are the flavor doublets and 0_n is an n-dimensional null matrix. Now embed the soliton field of SU(2) flavor as

$$U_\pi = \begin{pmatrix} U_0 & 0 \\ 0 & 1_3 \end{pmatrix}_5, \tag{89}$$

where 1_3 is a 3 dimensional unit matrix and U_0 the usual SU(2) hedgehog field

$$U_0 = \exp\left(i\vec{\tau}\cdot\hat{r}F(r)\right). \tag{90}$$

Here $F(r)$ is the chiral angle of the SU(2) soliton. This enables us to write a generalized Callan–Klebanov form

$$U = \sqrt{U_\pi}\, U_5 \sqrt{U_\pi}. \tag{91}$$

With this pseudoscalar field, one can then write down effective Lagrangians in the usual manner which, when expanded in the quadratic approximation in the doublet fields (hence no coupling between different flavors), leads to the form of (82) [8,16]. Of course there will be other terms than exhibited in (82) but they are subsumed in the ellipsis.

It is instructive to compare (82) with the monopole system of [11]. The $\mathcal{L}_{SU(2)}$ and $\mathcal{L}_\Phi$ are analogous to the monopole-scalar Lagrangian. However while the monopole is a boson and the scalar field coupled to it is transmuted to a fermion, the skyrmion field of (83) is quantized as a fermion and the scalar field Φ as a boson. Thus the two systems seem to be basically different. This is not really so. The point is that in the skyrmion system there are additional velocity-dependent interactions in the potential V as well as in the Wess–Zumino term which can *transmute* the statistics. It should be possible to transform, by a gauge transformation, the velocity-dependent interactions to phases attached to the skyrmion and to the Φ such that the former behaves as a boson and the latter as a fermion. That the statistics can be either assigned to a phase in the field or equivalently delegated to a velocity-dependent long-range interaction is a well-known fact in dyon [17] and anyon physics [18].

The Wess–Zumino term has the dynamical role of binding the scalar mesons to the soliton in a way analogous to the magnetic field (in z direction) that binds charged particles in Landau levels in (x,y) plane. As I will show later, it plays a key role in generating a truly nonabelian Berry potential. It also lifts the degeneracy between the state with Φ and the state with its charge conjugate $\Phi^c = \Phi^\star$. It has the sign such that the scalar with the quantum number of $Q\bar{q}$ is bound and the one with the quantum number of $\bar{Q}q$ is unbound. The approximation that reduces the Wess–Zumino term defined on a five-dimensional disk to a two-dimensional plane with a "magnetic field" in z direction gets better, the larger the scalar mass which plays the role of the strength of the "magnetic field".[5]

When considering the skyrmions of light-quark flavors, it has proven to be essential to order the Lagrangian in the descending power of N_c or more properly in the ascending power of $\alpha_{sk} = 1/N_c$

$$\mathcal{L} = \mathcal{L}_{-1} + \mathcal{L}_0 + \mathcal{L}_{+1} + \cdots, \tag{92}$$

where the subscript stands for the power in $\alpha_{\rm sk}$. Here one is thinking of a weak-coupling expansion, with $\alpha_{\rm sk}$ being considered in some sense small. Thus one could say that the term of $O(1/\alpha_{\rm sk})$ which corresponds to the soliton component dominates, with the other terms contributing as "fluctuations". This is in conformity with the notion that solitons such as magnetic monopole make sense in the weak-coupling limit. When heavy quarks are involved, the situation is quite different: the $O(1)$ term in the mass which describes the bound Φ meson dominates over the $O(1/\alpha_{\rm sk})$ term from the skyrmion. So the literal "large-N_c" expansion is meaningless at least for the first two orders. Even so, it turns out to be convenient to arrange the Lagrangian as (92) even in the massive-Φ case.

As defined, the skyrmion lives in the flavor SU(2) sector and is given by the classical solution of (83). The structure of the skyrmion so defined is by now thoroughly studied and I have nothing new to say. We can simply assume that it is given by the most realistic SU(2) Lagrangian possible, including strong vector mesons and other degrees of freedom up to the chiral symmetry scale $\Lambda \sim 1$ GeV. The size of this skyrmion is then determined by SU(2) properties[6], and is typically of the order of 0.5-0.6 fm. Let us

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[5] This was pointed out by several authors. See, for instance, Ref. [19].

[6] The assumption that the soliton property is entirely determined by the SU(2) sector with no back-reaction from the motion of the scalar meson may be a poor approximation for such quantities as the baryon size (more generally, form factors) of massive-quark baryons. This matter requires further attention.

now look at the $O(1)$ (Φ) sector. The skyrmion provides the background potential felt by the Φ field. In particular, it defines the induced "gauge potential" v_μ and the axial-vector potential a_μ. The dynamics of the Φ field in this background field is given by (85). The gauge field playing the role of "magnetic" monopole field is gotten from (86) with the chiral field in the hedgehog form (90) satisfying the boundary conditions $F(0) = \pi$ and $F(\infty) = 0$ appropriate to baryon number $B = 1$ and has the form

$$v_i(U = U_0) = -i\tau^a A_i^a \tag{93}$$

with

$$A_i^a = \frac{\epsilon_{iak}\hat{r}^k}{r}(1 - W(r)), \tag{94}$$

$$W(r) = \cos^2 \frac{F(r)}{2}. \tag{95}$$

Note that $W = 0$ for $r = 0$ and $W = 1$ for $r = \infty$. In contrast, the non-abelian (BPS) monopole [20] has the structure (93) with $W(r) = 1$ when $r = 0$ and $W(r) = 0$ when $r = \infty$. Thus the skyrmion has an "inside-out" monopole structure as alluded above.[7] The transmutation of the "isospin" of the doublet Φ to spin is completely analogous to the monopole-scalar doublet system. To see this one has to quantize both Φ field and the skyrmion field. One can assume that the Φ has no classical component, its fluctuating field carrying the dynamical time dependence while the time dependence of the skyrmion arises through its collective coordinates associated with zero modes. To $O(\alpha_{sk}^0)$, the equation of motion for the Φ field in the skyrmion background field as given by (85) has bound-state solutions due mainly to an attraction generated by the Wess–Zumino term plus a small contribution from the potential V. In some sense, this is a vibrational mode and I will refer to it as such in what follows. In the case of the strangeness flavor $Q = S$, it is well established [9,21,22] that only one even-parity bound state exists for the p-wave kaon accounting for the octet and decuplet baryons of three flavors (u,d,s) and an odd-parity bound state describing the odd-parity hyperons like $\Lambda(1405)$. In the case of massive quarks, there may be

[7] It is perhaps instructive to comment on further "inside-out" relations between the magnetic monopole and the skyrmion. When fermions are present, chiral boundary conditions are needed. In the case of the monopole, the boundary condition is imposed at the origin $r = 0$ where the monopole sits *inside*, with the fermions lodged outside, whereas in the case of the baryon, the boundary condition is imposed at the boundary with the fermions inside and the skyrmion outside. Thus the chiral bag structure of the baryon can be viewed as an inside-out monopole with however completely different length scales.

more bound states. Let me denote by ω_Φ the eigenenergy (or vibrational frequency) of the bound Φ. To $O(\alpha_{\rm sk}^0)$, the contribution to the mass of the baryon with n_k massive flavor quanta is then $n_k\omega_{\Phi_k}$. As noted above, for a very massive Φ, this contribution, though formally subdominant to the soliton mass in N_c counting, dominates over the $O(1/\alpha_{\rm sk})$ term. The quantum number Q is defined at this order.

2.2.1. Spin-isospin transmutation

Spin and isospin are defined at $O(\alpha_{\rm sk})$. Thus to "see" the transmutation, we have to quantize the zero modes associated with the SU(2) isorotation

$$U \rightarrow A(t)U_0A^\dagger(t), \tag{96}$$

$$\Phi \rightarrow A(t)\Phi(t) \tag{97}$$

with $A(t) \in \mathrm{SU}(2)$. As usual, one makes the adiabatic assumption that the rotation is much slower than the vibration of the Φ field and obtains what corresponds to Eq. (74)

$$\delta L = \tfrac{1}{2}\mathcal{I}(\dot{\alpha}_a)^2 + \dot{\alpha}_a\mathcal{A}_a, \tag{98}$$

where $\mathcal{I}$ is the moment of inertia for the SU(2) rotation,

$$A^\dagger\dot{A} \equiv i\tau^a\dot{\alpha}_a \tag{99}$$

and $\mathcal{A}$ is the induced nonabelian gauge potential which is a space integral of complicated functions involving Φ and its time derivative, U_0 *etc.* whose explicit forms can be found in the literature but are not important for our purpose. Note that Equation (98) is an exact analog to the quantum mechanical case Eq. (26). The only difference is that the gauge field is nonabelian here. What is significant is that $\mathcal{A}$ is essentially an induced (Berry) potential [1] quite analogous to the Berry potential seen in diatomic molecules [23]. In fact, this generic form arises also in strong interaction physics whenever fast degrees of freedom are integrated out in favor of slower degrees of freedom [5]. The Berry potential here is truly nonabelian because of the following asymmetry due to the Wess–Zumino term. In the absence of the Wess–Zumino term, there would be a degeneracy with respect to rotation in Φ "isospin" space. The Wess–Zumino term breaks the rotational symmetry. If there were no symmetry breaking, then the induced gauge field would be in a Maurer–Cartan form in the SU(2) space (*i.e.*, a pure gauge) and hence would have zero field strength. We will see later on that this means the massive Φ spin becomes a good quantum number and the hyperfine structure splitting will disappear in the Q flavor direction. Thus the Wess–Zumino term plays a role quite analogous to the (rotation) symmetry breaking that lifts the Λ-π degeneracy in diatomic molecules [23].

Given the Lagrangian (92) to $O(\alpha_{sk})$, it is a straightforward matter to obtain a corresponding Hamiltonian and all Noether currents associated with the symmetries of the Lagrangian. The conserved quantum numbers of the (soliton-Φ) system are: the total angular momentum

$$J^a = J^a_{\rm rot} + j^a_{\Phi}, \tag{100}$$

where $\vec{J}_{\rm rot}$ is the angular momentum of the soliton rotor and $\vec{j}_{\Phi}$ is the spin of the Φ *transmuted* from its isospin which we might call "induced angular momentum", none of which is a good quantum number separately; the total isospin which is also the rotor angular momentum, since the isospin is entirely lodged in the skyrmion

$$I^a = J^a_{\rm rot} \tag{101}$$

and of course the massive flavor quantum number Q (*e.g.*, S, C, B *etc.*) which is, as stated above, determined at $O(\alpha^0_{\rm sk})$. In terms of these quantum number *operators*, the rotational Hamiltonian reads

$$H_{\rm rot} = \frac{1}{2\mathcal{I}}\left(\vec{J}_{\rm rot} + (1-\kappa)\vec{j}_{\Phi}\right)^2 + \cdots, \tag{102}$$

where

$$c \equiv 1 - \kappa = 2\int_0^\infty r^2 dr \lambda(r)\phi^*(r)\phi(r) + \cdots, \tag{103}$$

with ϕ a suitably normalized bound massive scalar wave function and λ the Wess–Zumino term [$\lambda(r) \propto iN_c\rho(r)/f^2_{\Phi}$ where ρ is the baryon density]. The ellipsis stands for other terms that can contribute depending upon the detail of the model Lagrangian[8]. They are not important for the main

[8] As a footnote, let me mention one thing which is not fully verified but of which I feel certain. As stated, there is a gauge invariance associated with the induced gauge field v_μ. If one chooses the temporal gauge $v_0 = 0$ before quantizing the theory, then one can show that the hyperfine coefficient is *entirely* given by the Wess–Zumino term. The works published up to now, including the one whose results I will quote below, have not used the gauge-fixing in a proper way, introducing possibly small errors in the hyperfine splitting, while unaffecting, however, other properties of the spectra or magnetic moments. A correct treatment of the induced gauge degrees of freedom is given in [7].

characteristic of the physics that we are focusing on. I should mention also that the κ here is analogous to Zygelman's κ in diatomic molecules [23]. If one denotes the bound-Φ eigenenergy as ω_Φ as before, then the normalization condition for ϕ allows us to express κ in a more transparent form

$$\kappa = 2\omega_B \int_0^\infty r^2 dr \phi^*(r)\phi(r) + \cdots . \tag{104}$$

In this form one can see immediately what is going on if one ignores the additional terms denoted by the ellipsis. If the Wess–Zumino term were absent, $\kappa \approx 1$ by normalization and hence $c \approx 0$. In addition, it is clear that as $\omega_B \to \infty$, κ tends to unity again making c go to zero. In other words, in the massive limit, we recover the SU(2) symmetry. This is precisely the "Wisgur" spin symmetry [14]. This can be seen explicitly in the spectrum that follows from (102)

$$\Delta M = \frac{1}{2\mathcal{I}}\left(cJ(J+1) + (1-c)I(I+1) + \cdots\right) \tag{105}$$

which shows that c represents hyperfine splitting within a Q sector. Now in the limit $c = 0$, the spectrum *does not* depend upon the massive-quark spin j_Φ. Another important observation is that the coefficient c or κ does not appear in the total (conserved) angular momentum although it figures explicitly in the gauge field or more properly in the field tensor which generates (*via* transmutation) the induced angular momentum from the scalar field. This is typical of the structure of the induced gauge field that belongs to the class of "invariant potential" in the sense of Jackiw [24].

2.2.2. Mass formula

To sum up to this point, we have the soliton quantized as a fermion with $J_{\rm rot} = I = 0, 1, 2, \cdots$ and the bound Φ as a boson with $\Lambda_\Phi = {}^1\!/_2$ and zero isospin when only one massive flavor is bound. Scoccola and Wirzba [13] show that with two Φ's in a baryon, the soliton should be quantized as a fermion with $J_{\rm rot} = I = {}^1\!/_2, {}^3\!/_2, \cdots$ and the angular momentum of Φ's as a sum of two bosons. Furthermore no spurious low-lying states are allowed within the scheme. The mass formula one obtains for a hyperon with n_1 mesons of species 1 (representing orbital state and flavor) with energy ω_1 and angular momentum j_1 and n_2 mesons of species 2 with energy ω_2 and angular momentum j_2 is [8]

$$
\begin{aligned}
M(I,J,n_1,n_2,J_1,J_2,J_m) &= M_{sol} + n_1\omega_1 + n_2\omega_2 \\
+\frac{1}{2\mathcal{I}}\Big(I(I+1) &+ (c_1-c_2)[c_1J_1(J_1+1) - c_2J_2(J_2+1)] \\
+c_1c_2J_m(J_m+1) &+ [J(J+1) - J_m(J_m+1) - I(I+1)] \\
\times\Big[\frac{c_1+c_2}{2} &+ \frac{c_1-c_2}{2}\,\frac{J_1(J_1+1)-J_2(J_2+1)}{J_m(J_m+1)}\Big]\Big).
\end{aligned}
\tag{106}
$$

Here I is the (iso)spin, M_{sol} the mass and $\mathcal{I}$ the moment of inertia of the soliton. The angular momentum quantum numbers J_1 and J_2 are defined as $n_1 j_1$ and $n_2 j_2$ respectively and J_m takes one of the values $|J_1 - J_2|, ...J_1 + J_2$. In the case of a single heavy flavor, the mass formula (106) reduces to that form derived for strange hyperons [21]. The appearance of the quantum number J_m, which represents the total angular momentum of the meson system, is due to the following fact. Bose statistics of the Φ's requires that when more than one of a certain kind are put on a given orbital the total mesonic wave function should be completely symmetric and hence only the maximum value of the spin is allowed. However, when different orbitals are populated and/or different kinds of mesons are considered this argument does not hold, and all different values of J_m are possible. A very good illustrative case is the cascade particles. Consider first the case of the $S = -2$ cascades. In the present approach these particles are formed by two kaons bound in the energetically lowest orbital. This orbital has $j = 1/2$. Due to the symmetry argument given above the total spin of the meson field has to be $J_m = 1$. Since for cascades the rotor (iso)spin is $I = J_{rot} = 1/2$, we find that two $S = -2$ states are predicted in the model: One with $(I, J) = (1/2, 1/2)$ and the other with $(1/2, 3/2)$. These are exactly the quantum numbers of the Ξ and Ξ^* hyperons respectively. Consider next the case of the charmed cascades. These particles are composed of a K-meson and a D-meson wrapped by the soliton. Since the flavor quantum numbers in this case are different, both $J_m = 0, 1$ are allowed. Once again we have $I = J_{rot} = 1/2$, but now *three* physical states are possible. Indeed, we predict two particles with $(I, J) = (1/2, 1/2)$ and one with $(1/2, 3/2)$. Although only one of these states has been observed experimentally ($\Xi_c(2470)$) our prediction agrees with those of the quark model, which also predicts three charmed cascades that are usually denoted as Ξ_c, Ξ_c' and Ξ_c^*.

The mass formula (106) is generic of the bound soliton-scalar meson model and follows from the presence of three scales — quark, vibrational and rotational modes – inherent in the model. Therefore we could simply determine the quantities M_{sol}, ω_i, $\mathcal{I}$ and c_i from experiments and make predictions for other masses. In the charmed (and bottom) sector, we do not yet have experimental data. But we expect the quark models to work well in heavy-baryon sectors, so the quark-model results could be used for determining those parameters. As an illustration, consider determining those parameters by fitting six experimental data, say, N, Δ, Λ, Σ, Λ_c and Σ_c. The last two are not well determined experimentally, so the parameters so determined in the charm sector may not be reliable. Be that as it may, the parameters determined as prescribed are

$$M_{sol} = 866 \text{ MeV}, \quad \mathcal{I} = 1.01 \text{ fm}, \quad \omega_K = 223 \text{ MeV}, \\ \omega_D = 1418 \text{ MeV}, \quad c_K = 0.60, \quad c_D = 0.14. \tag{107}$$

This set of parameters is found to reproduce quite well the strange baryons (compared with experiments) and reasonably well the charmed hyperons (compared with quark-model results) [8]. The burden of a particular model would then be to predict those parameters and compare with the "empirical ones".

2.2.3. Model Lagrangian

The simplest effective chiral Lagrangian model that does surprisingly well is the original Skyrme model that consists of the usual two-derivative and four-derivative terms supplemented by certain symmetry breaking terms. I will show the results of a variant of this model recently studied by Riska and Scoccola [16], which I will call Riska-Scoccola (RS) model. Implementing with vector mesons would improve the prediction, so the RS model would serve our purpose adequately. What distinguishes the RS model from the usual Skyrme model (say, for three flavors) is the additional symmetry breaking term of the type

$$\delta\mathcal{L}^{SB} = \frac{f_\pi^2 - f_K^2}{12} \mathrm{Tr}\left(\left(1 - \sqrt{3}\lambda_8\right)\left(U\partial_\mu U^\dagger \partial^\mu U + U^\dagger \partial_\mu U \partial^\mu U^\dagger\right)\right) \tag{108}$$

and similarly for other flavors. What this does effectively is to give a correct normalization as in (85) and then scale the massive scalar field with its appropriate decay constant. In other words, this procedure implements the standard skyrmion model with terms to mimic roughly the monopole-scalar field analogy sketched at the beginning of this review.

Now if one takes the empirical *ratios* of the decay constants f_K/f_π and f_D/f_π and experimental kaon and D masses, there are only two parameters in the RS model, namely, the pion decay constant f_π and the coefficient of the quartic Skyrme term e, both of which can be determined in the light-quark sector, *i.e.*, by the nucleon and Δ masses: $f_\pi = 64.5$ MeV and $e = 5.45$ which correspond to

$$M_{sol} = 866 \text{ MeV}, \quad \mathcal{I} = 1.01 \text{ fm} \tag{109}$$

as in (107). The ω's and c's are calculable without any additional parameters. The results are

$$\omega_K = 223 \text{ MeV}, \quad c_K = 0.50, \quad \omega_D = 1301 \text{ MeV}, \quad c_D = 0.20. \qquad (110)$$

TABLE I

Low-lying strange and charmed hyperon masses calculated by Riska and Scoccola [16] in the Skyrme model implemented by an additional symmetry breaking term. Comparison is made with experiments when available and with the quark model result (QM) of [25] when experiments are not available. The masses are given in MeV.

	I	J	S	C	Prediction	Experiment	QM
N	$1/2$	$1/2$	0	0	Fitted	939	
Δ	$3/2$	$3/2$	0	0	Fitted	1232	
Λ	0	$1/2$	-1	0	1107	1116	
Σ	1	$1/2$	-1	0	1205	1193	
Σ^*	1	$3/2$	-1	0	1352	1385	
Ξ	$1/2$	$1/2$	-2	0	1337	1318	
Ξ^*	$1/2$	$3/2$	-2	0	1483	1530	
Ω	0	$3/2$	-3	0	1627	1672	
Λ_c	0	$1/2$	0	1	2170	(2285)	2200
Σ_c	1	$1/2$	0	1	2326	(2453)	2360
Σ^*	1	$3/2$	0	1	2385	?	2420
Ξ_c	$1/2$	$1/2$	-1	1	2421	?	2420
Ξ_c'	$1/2$	$1/2$	-1	1	2470	(2460)	2510
Ξ_c^*	$1/2$	$3/2$	-1	1	2524	?	2560
Ω_c	0	$1/2$	-2	1	2645	(2740)	2680
Ω_c^*	0	$3/2$	-2	1	2675	?	2720
Ξ_{cc}	$1/2$	$1/2$	0	2	3510	?	3550
Ξ_{cc}^*	$1/2$	$3/2$	0	2	3569	?	3610
Ω_{cc}	0	$1/2$	-1	2	3698	?	3730
Ω_{cc}^*	0	$3/2$	-1	2	3727	?	3770
Ω_{ccc}	0	$3/2$	0	3	4784	?	4810

The predicted spectrum is listed in Table I, compared, whenever available, with experiments and, if otherwise, with the quark-model results of

De Rújula, Georgi and Glashow [25]. The agreement is quite pleasing and surprising.[9]

2.2.4. The b-quark baryons

Riska and Scoccola have also made predictions in the b-quark sector. Due to the almost total lack of empirical information in this sector, it is perhaps not yet possible to assess how well it really fares in this sector. Nonetheless, the results are quite encouraging. Their prediction for $f_B/f_\pi \approx 1.8$ is $\omega_B \approx 3729$ MeV, which puts Λ_B at 4595 MeV, to be compared with the empirical value 5425 GeV. The hyperfine splitting seems to come out even better, with the predicted splitting $m_{\Sigma_B} - m_{\Lambda_B} \approx 175$ MeV, to be compared with the empirical value ≈ 200 MeV. If one takes $f_B/f_\pi \approx 2.5$ suggested by lattice QCD calculations, one gets $m_{\Lambda_B} \approx 5342$ MeV very close to the empirical value. As noted in [8], the vector mesons — both light and massive — would improve both fine and hyperfine structure splittings. The calculations that include these vector degrees of freedom are underway and results will be available soon [26].

2.2.5. Magnetic moments

The predictive nature of this model has been further strengthened by magnetic moments of the massive baryons. The magnetic moments of both strange and charmed baryons have been calculated in the RS model by Oh, Min, Rho and Scoccola [27]. The moments of the strange baryons obtained in the RS model are close to those previously reported [28] (which are actually in satisfactory agreement with experiments). Those of the charmed baryons given (in units of the proton moment) in Table II, calculated *parameter-free*, are in quite good agreement with the quark-model predictions available in the literature. As a whole, compared with the results of quark models, the prediction for the massive baryons is just as good or perhaps even better than that for the strange hyperons. Measurement of the magnetic moments of the charmed baryons is planned at the Fermilab

[9] The centroid of the charmed (and bottom) baryons is predicted to be slightly lower in the RS model than the empirical (and quark-model) value, but much better than what was found in [8] where the universal decay constant $f_\Phi = f_\pi$ was used. I am not worried about this remaining small discrepancy since the incorporation of massive vector mesons in addition to the usual light vector mesons would certainly decrease the binding of the soliton-Φ complex, thus raising the centroid of the massive flavor and decreasing somewhat the hyperfine coefficient c.

and it would be most interesting to see whether experiments continue to support the model. It would also be desirable to have spectra and moments calculated in a quark model with a modern sophisticated quark potential.

TABLE II

Magnetic moments *relative to the proton moment* of the charmed baryons predicted *parameter free* in the RS model of Table I, compared with the quark model results of D.B. Lichetenberg (*Phys. Rev.* **D15**, 345 (1977)) and the bag model results of S.K. Bose and L.P. Singh (*Phys. Rev.* **D22**, 773 (1980)). The prediction A corresponds to taking $m_\pi = 0$ and B to $m_\pi = 138$ MeV.

Particle	PRED A	PRED B	Quark Model	Bag Model
Λ_c^+	0.11	0.10	0.13	0.18
Σ_c^{++}	0.98	0.99	0.85	0.70
Σ_c^+	0.16	0.22	0.18	0.13
Σ_c^0	− 0.65	− 0.56	− 0.49	− 0.44
$\Sigma_c^{*,++}$	1.63	1.64	1.47	1.40
$\Sigma_c^{*,+}$	0.40	0.48	0.47	0.48
$\Sigma_c^{*,0}$	− 0.82	− 0.69	− 0.53	− 0.43
Ξ_{cc}^{++}	− 0.17	− 0.17	− 0.04	0.06
Ξ_{cc}^+	0.35	0.32	0.29	0.31
$\Xi_{cc}^{*,++}$	1.14	1.13	0.93	0.91
$\Xi_{cc}^{*,+}$	− 0.42	− 0.35	− 0.07	0.07
Ω_{ccc}^{++}	0.32	0.30	0.40	0.52
Ξ_c^+	0.44	0.40	0.26	0.17
Ξ_c^0	− 0.59	− 0.57	− 0.41	− 0.39
$\Xi_c'^+$	0.11	0.10	0.13	0.18
$\Xi_c'^0$	0.11	0.10	0.13	0.18
$\Xi_c^{*,+}$	0.82	0.75	0.59	0.55
$\Xi_c^{*,0}$	− 0.72	− 0.70	− 0.41	− 0.36
Ω_c^0	− 0.31	− 0.39	− 0.32	− 0.35
$\Omega_c^{*,0}$	− 0.31	− 0.43	− 0.28	− 0.28
Ω_{cc}^+	0.21	0.22	0.25	0.30
$\Omega_{cc}^{*,+}$	0.01	− 0.06	0.06	0.14

2.3. Summary

To summarize: The bound skyrmion-Φ model is seen to work (perhaps surprisingly) well for massive-quark baryons (*e.g.*, charmed baryons) as well as not-so-massive-quark baryons such as strange hyperons. The essential ingredient is the Wess–Zumino term which controls both the fine-structure ($O(1)$) and hyperfine ($O(\alpha_{sk})$) splittings: It binds the scalar Φ to the soliton in the correct channel, lifting the degeneracy of the $\pm Q$ states and further provides just the requisite "tilting" of the rotational symmetry to induce the hyperfine splitting. Pleasingly, as the mass of the heavy-flavor quark (or the mass of Φ) increases, the binding weakens and the spin of the heavy quark decouples. As a consequence, the Wisgur spin symmetry is seen to emerge in the massive limit. A fascinating feature of this model is that even for a baryon that has no light-flavor valence quarks such as Ω_{ccc}, the soliton is essential as a "soul," carrying the entire baryon charge; in a sense, the light-quark flavor in the soliton is "cancelled" by the light antiquarks in Φ, but that this happens in just the right way to give the physical baryon is a remarkable phenomenon. Equally remarkable is that despite its intricate mechanism and apparent complexity, the structure is really very simple. It closely resembles, through nonabelian Berry phases, those atomic and molecular systems exhibiting geometric phases. Thus one can say that the model works because it has the generic feature that is shared by all other systems in condensed matter and particle physics that are described in terms of Berry phases.

I should mention that no matter how accurate it may turn out to be, this model cannot possibly compete in quantitative predictivity with the quark models based on QCD: There are still some difficult problems to resolve in the model. For instance, although, as shown in [8], there are no center-of-mass corrections at the level of approximation we are working with, the soliton-Φ complex has yet to be quantized to restore translational invariance as well as to account for other soft fluctuations (*e.g.*, pionic excitations). These are hard technical problems that plague all extended-structure models and remain to be resolved for this model to confront experimental data in a quantitative way. We will also need, for quantitative accuracy, to introduce massive vector meson degrees of freedom for short-distance physics. At present, it is not clear how to do this without guidance from experiments. In any event, the moral of the story is this: That hadronic interactions, even in massive-quark baryons, are mostly governed by the symmetries of QCD, which in terms of effective degrees of freedom manifest themselves in topologically nontrivial configurations. The understanding of these metamorphoses in degrees of freedom from QCD variables to effective variables is the ultimate challenge in hadron physics in nonperturbative regime.

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