Vol. 22(1991) *ACTA PHYSICA POLONICA B* No 11–12

MESONS AND NUCLEI IN A NUCLEAR MEDIUM*

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(Received September 3, 1991)

The saturation properties of nuclear matter are dominated by the dynamics of a self-consistent treatment of correlations and the structure of the Dirac spinors of the nucleons in the medium. It is demonstrated that ground-state properties of finite nuclei are furthermore sensitive to the range of the interaction. This yields different compressibilities for infinite matter and finite nuclei. The medium dependence of the effective mass of nucleons and mesons are studied within the model of Nambu and Jona–Lasinio. A possible connection of this model to QCD is discussed.

PACS numbers: 21.65.+f, 12.90.+b

1. Introduction

One of the main topics of nuclear structure theory is to develop a microscopic theory for the equation of state (EOS) of nuclear matter. In particular one is interested in the EOS at high densities and temperatures. This interest is motivated by two, quite different fields of physics. One of these fields is the study of heavy ion reactions at high energies. It is the hope that such experiments lead to small areas of nuclear matter at densities very much above the saturation density of normal nuclear matter. At such high densities the confinement of quarks to baryons may be dissolved and a phase transition of normal nuclear matter to a quark gluon plasma could be obtained. Of course, in order to detect signals of such a phase transition a reliable prediction for the dynamics of heavy ion reactions within the conventional model is required. The present analysis of experimental

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* Presented at the XXXI Cracow School of Theoretical Physics, Zakopane, Poland, June 4–14, 1991.

data like the so-called "side flow" of nuclear matter and on the pion production in such heavy ion experiments seems to prefer a stiff EOS, *i.e.* a high incompressibility of nuclear matter.

Another group of physicists which is very much interested in predictions on the EOS of nuclear matter at high density works on problems of astrophysical origin. The properties of neutron stars, the maximal mass for a neutron star which is stable against a collapse to a black hole and the stability of rotating neutron stars are rather sensitive to the nuclear EOS. Also the modeling of supernova (type II) events seems to be rather sensitive to the EOS. In contrast to the analysis of heavy ion experiments, the astrophysical models tend to favor a soft EOS.

The aim of microscopic nuclear structure calculations for the EOS, however, is not only to solve this discrepancy just outlined. In order to understand *e.g.* the experimental signals from a phase transition to a quark-gluon plasma one needs very reliable predictions on the behavior of conventional nuclear matter at high densities. Such predictions cannot reliably be obtained from model calculations which employ adjustable parameters to reproduce the properties of nuclear systems at normal densities. The predictive power of such phenomenological studies for the behavior at larger densities is of course rather limited.

Therefore a more promising approach is to develop a many-body theory for nuclei as a system of interacting nucleons, for which the parameter defining the nucleon-nucleon (NN) interaction, are adjusted to describe the behavior of nucleons in the vacuum, *i.e.* the NN scattering phase shifts. If one succeeds to develop a many-body, which is able to derive the properties of nuclear matter at normal densities from the interaction of nucleons in the vacuum, one may trust this many-body theory also with respect to its prediction for the densities obtained in relativistic heavy-ion reactions or in supernovas and neutron stars.

The aim to develop such a many-body theory is of course a scientific challenge on its own and it has been one of the central topics of theoretical nuclear physics. After this introduction the Section 2 of this contribution will review some of its important aspects. In particular it will be emphasized that it is indispensable to account for the effects of nuclear correlations. If correlations are taken into account, one obtains predictions for the saturation point of nuclear matter, which are close but not yet in satisfactory agreement with the experimental data. A good agreement can only be obtained if one accounts for relativistic effects and considers the possibility that the effective mass for the nucleons, which is used to characterize the Dirac spinor, is different in the nuclear medium as compared to its value in the vacuum (Section 3). The same mechanism, however, seems not to be sufficient to describe the ground-state of finite nuclei as well. It is demonstrated that the saturation point and the compressibility of finite nuclei is different from those of nuclear matter and much more sensitive to the range of the interaction, which means the effective mass of mesons in the nuclear medium (Section 4). Possibilities to describe effective masses of mesons and nucleons in a nuclear medium within the model of Nambu and Jona-Lasinio (NJL) and the connections of the NJL model to Quantumchromodynamics (QCD) are outlined in Section 5. The last section contains some conclusions.

2. Correlations in nuclei

A characteristic feature of realistic NN interactions which are adjusted to describe NN scattering data up to medium energies, are the strong short--range components. More than 20 years ago NN potentials have been developed in which the repulsive components at short distances were even described by an infinite hard core [1]. It is clear that all attempts to evaluate the binding energy of nuclei employing such a hard-core potential in an approximation which ignores the effects of two-nucleon correlations (like the Hartree–Fock approach) or accounts for them in a perturbative way, are bound to fail because the matrix elements for such a hard-core potential using uncorrelated two-nucleon wave functions are infinite.

TABLE I

Results for the binding energy per nucleon of Nuclear Matter

k_F [fm^{-1}]	1.2	1.36
HF	119.8	176.2
BHF	−9.1	−9.8
BHF3	−12.6	−15.2
ΔE_2	−128.9	−186.0
ΔE_3	−3.5	−5.4

Results for the energy per nucleon calculated in the Hartree–Fock (HF) the Brueckner–Hartree–Fock (BHF) and with inclusion of the 3-nucleon correlations (BHF3) are listed for two densities, characterized by the Fermi momentum $k_F = 1.2$ and 1.36 fm^{-1} (the empirical saturation density), respectively. Furthermore we give the energy contribution due to two- (ΔE_2) and three-nucleon correlations (ΔE_3). All results have been obtained for the Reid soft-core potential.

The proper treatment of two-nucleon correlations is also very important if so-called soft-core potentials or One-Boson-Exchange-Potentials (OBEP) are used, which are based on the meson exchange model for the NN interaction [2]. For the purpose of illustration we list in Table I the energy per nucleon which is obtained from a Hartree–Fock calculation for nuclear matter at the empirical saturation density using the realistic Reid soft-core potential [3]. Note that the calculated energy per nucleon (176.2 MeV) is positive and far away from the empirical value of −16 MeV, deduced from the Bethe–Weizsäcker mass formula. This result demonstrates quite clearly that a many-body theory employing such realistic interactions must be flexible enough to account for 2-nucleon correlations in such a way that the amplitude of the wave function is small whenever two nucleons are so close to each other that they are exposed to the repulsive core of the NN interaction. This can be achieved either by considering such correlations in the wave function explicitly or by evaluating an effective interaction which accounts for such correlations. A first approximation for such an effective interaction is the Brueckner G-matrix which is obtained from the bare NN interaction V by solving the Bethe–Goldstone equation

$$G(W) = V + V\frac{Q}{W - QTQ}G(W). \tag{1}$$

As one can see from this equation (with T denoting the operator for the kinetic energy of the interacting nucleons), that the G-matrix is very similar to the scattering matrix for 2 free nucleons except for the fact that the Pauli operator Q prevents intermediate two-particle states which are forbidden by the Pauli principle. Also the starting energy W will typically be the energy of two bound nucleons rather than the energy of free particles. In this sense G is the solution of the 2-nucleon problem in the nuclear medium in the same way as the solution of the Lippman–Schwinger equation for the scattering matrix is the solution for 2 nucleons in the vacuum. From the G interaction one can calculate the Brueckner–Hartree–Fock (BHF) single-particle potential

$$\langle i|U|i'\rangle = \sum_{j<F}\langle ij|G(W = \epsilon_i + \epsilon_j)|i'j\rangle, \tag{2}$$

where the $\langle ij|G|i'j\rangle$ denote matrix elements of G between antisymmetrized two-body states and the sum is restricted to single-particle states below the Fermi surface or, for nuclear matter, to states with momenta below the Fermi momentum k_F. The self-consistent single-particle states $|i\rangle$ and energies ϵ_i are obtained by diagonalizing the sum of the operator for the kinetic energy and the BHF single-particle potential

$$\langle i|T + U|i'\rangle = \epsilon_i\delta_{ii'}. \tag{3}$$

One can see that one has to solve the BHF equations (1)–(3) in a self-consistent way before one can calculate the total energy

$$E = \sum_{i<F}\langle i|T|i\rangle + \tfrac{1}{2}\sum_{i,j<F}\langle ij|G(W = \epsilon_i + \epsilon_j)|ij\rangle. \tag{4}$$

This BHF approximation yields an energy for nuclear matter at the empirical density which for the Reid potential (−9.8 MeV, see Table I) and other realistic NN interaction is much closer to the empirical value than the Hartree–Fock approximation. Therefore one may try to evaluate the binding energy of nuclear matter as a function of density by repeating such a BHF calculation for various densities and determine the saturation point (*i.e.* energy and density of the stable nuclear matter) as the minimum of the energy per nucleon as a function of density.

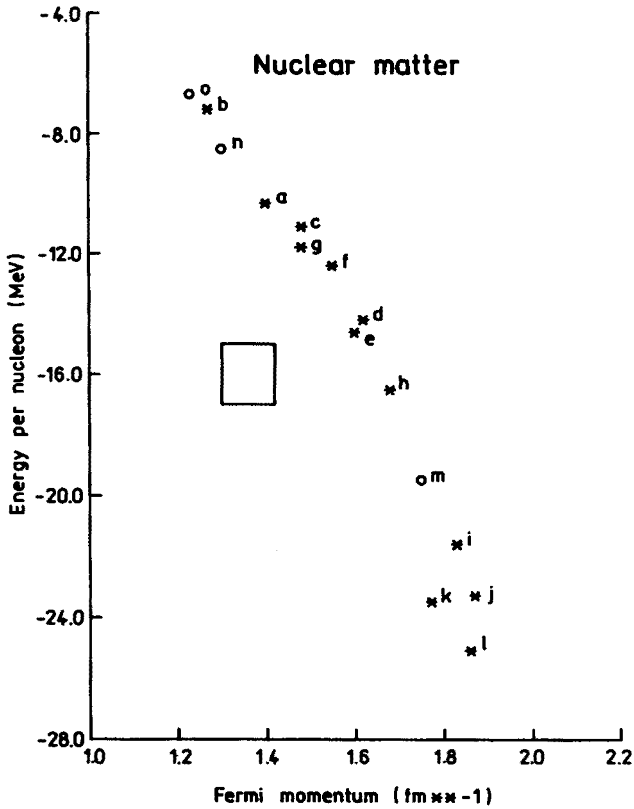

Fig. 1. Binding energy per nucleon and density of the saturation point (expressed in terms of the Fermi momentum) calculated for nuclear matter in the BHF approximation using different realistic NN interactions. Details on the individual NN interactions under consideration can be found in Ref. [9].

Such BHF calculations have been performed for various realistic NN interactions, which were all fitted to describe the free NN scattering data. It turned out, however, that all the calculated saturation points were obtained on the so-called Coester band [4,5] (see also Fig. 1 where the saturation points obtained for various NN interactions are presented and compared to the empirical value for saturation density and energy). This means that potentials, which reproduce about the correct saturation density predict a binding energy per nucleon which is to small by roughly 5 MeV, and those potentials which lead to the correct prediction for the binding energy yield a saturation density which is twice as large than the empirical one (or results in between).

Therefore attempts have been made to improve the solution of the many-body problem and account for three- and more-nucleon correlations. This can be done essentially by solving the three-particle Faddeev equation in the nuclear medium. It turns out that the effects of the three-nucleon correlations are non- -negligible and yield in the example of the Reid soft-core potential additional binding energy of about 5 MeV per nucleon [6] (see also Table I). At first sight this seems to improve the agreement with the empirical data. The density-dependence of this three-nucleon correlation effects, however, is such that the resulting saturation point is simply moved along the Coester band. Comparing the effects of two- and three-nucleon correlations one observes that the corrections due to three-nucleon correlations are much smaller. This tends to indicate a fast convergence of the many-body theory. Indeed, estimates for the effects of four-nucleon correlations [7] tend to support such a conclusion.

Since an improvement of the many-body theory does not really lead to a better agreement with the experimental data, one may try to account for sub-nucleonic degrees of freedom. In particular the possibility to excite the nucleons to the Δ resonance has been studied [8,9]. Also in this case it turned out that a consistent inclusion of Δ degrees of freedom yields a non- -negligible effect on the calculated saturation point. But again the results are essentially moved along the Coester band therefore do not really improve the agreement with the experimental data.

3. Dirac Brueckner Hartree Fock

Recently, there has been evidence that the problem how to derive the saturation properties of nuclear systems from a realistic interaction, which has just been outlined, might be resolved by taking into account some relativistic effects in the solution of the many-body problem [10–12]. At first sight relativistic effects seem to be negligible since the single-particle potential U (typically -40 MeV) is very small compared to the rest mass of the nucleons (938 MeV). One must be aware, however, that the single-particle potential U originates from a delicate balance between attractive and repulsive components in the NN interaction. In a OBE model the attractive components are mainly due to the exchange of a scalar meson, the so-called

σ meson, whereas the short-range repulsion is dominated by the exchange of the ω meson which is a vector meson. In the Hartree approximation the exchange of such mesons yield an operator for the self-energy of the nucleons with a scalar component A and a time-like component of a vector field B

$$\widehat{U} = A + B\gamma^0 . \tag{5}$$

In the Hartree approximation for nuclear matter A and B are independent of the momentum of the nucleon under consideration and depend on the density of the system, only. Including this self-energy in a Dirac equation for a nucleon with mass m and momentum p in the nuclear medium

$$(\not{p} - m - \widehat{U})\tilde{u}(p,s) = 0 \tag{6}$$

one obtains a solution for the Dirac spinor

$$\tilde{u}(p,s) = \sqrt{\frac{\widetilde{E}_p + \widetilde{m}}{2\widetilde{m}}} \begin{pmatrix} 1 \\ \frac{\sigma \cdot p}{\widetilde{E}_p + \widetilde{m}} \end{pmatrix} \chi_s , \tag{7}$$

which is identical to a solution for a free nucleon with an effective mass

$$\begin{aligned} \widetilde{m} &= m + A , \\ \widetilde{E}_p &= \sqrt{\widetilde{m}^2 + p^2} , \end{aligned} \tag{8}$$

and χ_s is a Pauli spinor. Since A is large and attractive (-300 MeV) the effective mass $\widetilde{m}$, which characterizes the ratio of large to small component in the Dirac spinor, is considerably smaller than the bare mass. This change of the Dirac spinors in the nuclear medium leads to different matrix elements for the OBE model of the NN interaction. Due to the change of the nucleon spinors in the medium, the NN interaction in nuclear matter is different from the one in the vacuum. Since the nucleon self-energy $\widehat{U}$ is determined from the NN interaction, which itself depends on the solution of the Dirac equation (6) for this self-energy, a new self-consistency problem has to be considered.

For a realistic OBE potential one has to go beyond the Hartree approximation for the self-energy and take into account the effects of Fock exchange terms and of two-body correlations by determining the self-energy $\widehat{U}$ from the G-matrix in analogy to the BHF approach outlined in the preceding Section. This leads to the so-called Dirac–Brueckner–Hartree–Fock (DBHF) approximation. The density dependence of the Dirac effects is such that the saturation points resulting from DBHF calculations are off the Coester band. It was possible to find a realistic OBE potential which fits the NN scattering data and reproduces the empirical saturation point of nuclear matter employing the DBHF approach [12].

Does this success mean that the problem of a microscopic description of the ground-state properties of nuclear systems is solved? There are several reasons which give rise to skepticisms:
1) Of course, one should take into account the effects of many-nucleon correlations and sub-nucleonic degrees of freedom. These effects are non--negligible within the non-relativistic approach to the nuclear matter problem and therefore should also be considered within a relativistic treatment.

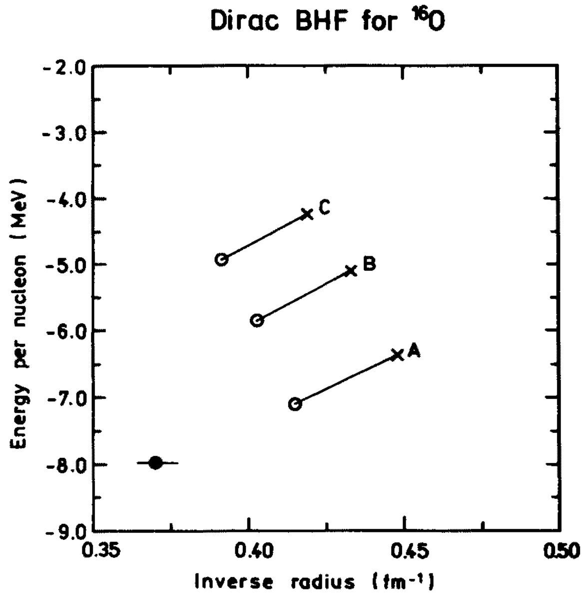

Fig. 2. BHF and DBHF calculations for ^{16}O. Using OBE potentials A, B and C as defined in Ref. [2], results for the radius of the charge distribution and the binding energy per nucleon obtained in conventional BHF (crosses) and DBHF calculations (circles) are compared to the experimental result.

2) The Dirac effects help to reproduce the properties of nuclear matter. For finite nuclei, however, the DBHF approach has not yet proven to be successful. It turns out that for light nuclei like ^{16}O or ^{40}Ca the Dirac effects improve the agreement with the experimental data for the radius of the charge distribution and the binding energy per nucleon [13] (see Fig. 2). The agreement, however, is far from satisfactory.
3) In the DBHF approach one takes into account that the solution of the Dirac equation for positive energies are modified for the nucleons in the nuclear medium. The presence of the medium, however, also modifies the solutions of the Dirac equation with negative energies. This implies that

the Dirac vacuum is modified in the medium as well. Attempts to account for such vacuum renormalization effects demonstrate that such effects are very large and cannot be ignored [14].

4. Nuclear matter and finite nuclei

Since the results obtained in DBHF calculations for finite nuclei show remarkable differences from those obtained in nuclear matter (see discussion in the previous Section) one may suspect that different saturation mechanisms must be considered for finite nuclei than for nuclear matter. To verify this we perform the following theoretical experiment [15]: In order to obtain some information on the compressibility of a finite nucleus (in our example ^{16}O) we consider wave functions which yield various expectation values for the radius of the nucleus. In our simple example we consider Slater determinants built from harmonic oscillator single-particle wave functions with various oscillator parameters. For these wave functions we calculate the binding energy using the BHF approximation and obtain the solid curve displayed in Fig. 3. In this figure the energy is shown as a function of the mean value obtained for the density of the wave function under consideration. The minimum of this curve (energy -100 MeV, density 0.12 nucleon per fm^3) is a typical result for a BHF calculation employing a potential with a weak tensor force (here potential A of Ref. [2]).

In the next step we consider the nucleus to be built by slices or shells of nuclear matter. For each of the wave functions considered above we obtain a density distribution $\rho(r)$ depending on the distance r from the center of the nucleus. For this density we can determine the energy density of nuclear matter with this very same density $\epsilon_{\rm NM}(\rho(r))$ and calculate the binding energy of ^{16}O in a kind of local density approximation as

$$E_{\rm LDA} = \int d^3r \, \epsilon_{\rm NM}(\rho(r)) \,. \tag{9}$$

This result is given by the dashed-dotted curve in Fig. 3. Comparing the LDA result with the original curve one observes that some repulsion is missing. At the minimum of the solid curve this repulsion is about 70 MeV, which is roughly equal to the surface contribution of the Bethe–Weizsäcker mass formula for this nucleus. This means that the LDA misses the dominant contribution to the surface tension.

What is the origin of this surface tension? To study this question we have compared individual contributions to the calculated energies in the two approximations. It turns out that the contribution of the kinetic energy and also the contributions due to the Fock-exchange terms are well reproduced by the LDA. The discrepancy arises essentially from the direct Hartree

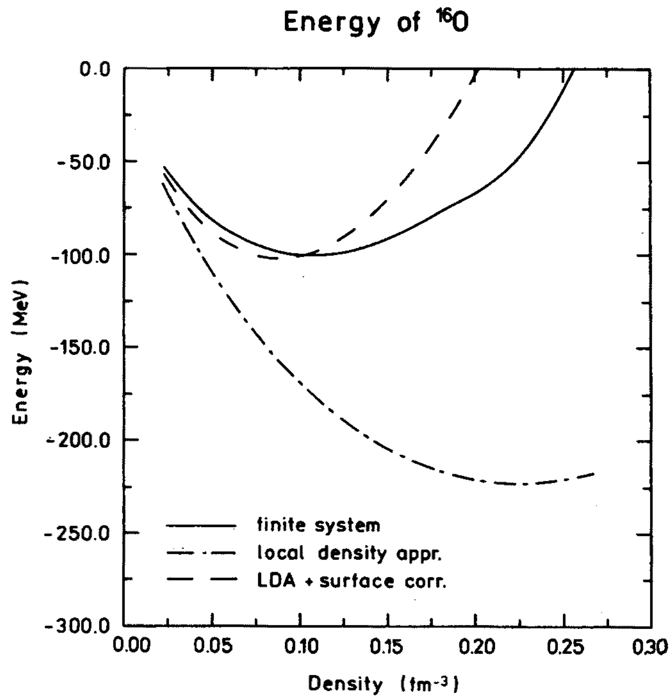

Fig. 3. Energy of ^{16}O as a function of the mean value for the density calculated in various approximations. For details see text.

contribution to the energy. In nuclear matter this direct contribution is testing the effective NN interaction at momentum transfer zero, while the calculation for finite nuclei probes the interaction at various momentum transfers. This means that the LDA presented above would be valid for effective interactions of zero range only.

On the other hand this means that in contrast to the infinite nuclear matter system, the saturation point for the finite system is very sensitive to the dependence of the interaction on the momentum transfer, respectively the range of the interaction. The non-zero range of the interaction provides a surface tension, which is not observed in the infinite system. This missing surface tension yields a softer equation of state for the infinite system as compared to the one for a finite nucleus. This may explain the differences observed in the equations of state (EOS) determined from the compression of a finite piece of nuclear matter (stiff EOS are derived from heavy ion reactions) as compared to an EOS derived from an essentially infinite system (like neutron stars or collapsing stars in supernovas).

5. Extended NJL model

Until now we have taken into account that the effective mass of the nucleons may change in the nuclear medium (see discussion of DBHF in Section 3). For the sake of consistency one may also consider a possible change of the meson masses in the nuclear medium. Such a change of the effective meson masses would modify the propagation of the mesons in the medium and give rise to a change of the range of meson exchange interaction between nucleons. Part of this change of the meson propagators can be described within the framework of the conventional many-body theory by considering a coupling of the mesons to the particle-hole excitations of the nuclear system.

In this Section, however, we would like to account for sub nucleonic degrees of freedom and try to evaluate the effective masses of baryons and mesons and their medium dependence within a phenomenological model which accounts for quark degrees of freedom. Such an effective theory which reflects the chiral symmetry as one of the basic symmetries of QCD is the one proposed long time ago by Nambu and Jona–Lasinio (NJL) [16].

In order to recall some features of this conventional NJL model we start writing down the Lagrangian density

$$\mathcal{L} = \bar{\psi}(i\nabla\!\!\!/ - M)\psi + \widetilde{G}\left[\left(\bar{\psi}1\psi\right)^2 + \left(\bar{\psi}\, i\gamma_5\vec{\tau}1\psi\right)^2\right], \tag{10}$$

where ψ is a (current) quark field with mass M, $\vec{\tau}$ the isospin matrices for the SU(2) flavor and **1** the unit operator for the color part (SU(3)) of the quark fields. The coupling constant $\widetilde{G}$, which is common for the scalar-isoscalar and the pseudoscalar-isovector part of the interaction term has dimension [mass^{-2}]. The Lagrangian of Eq. (10) is symmetric under a chiral transformation if we put $M = 0$. As a first step we would like to discuss the properties of the quarks in the vacuum. The self-energy Σ for the quarks can be evaluated in the mean-field or Hartree approximation as

$$\begin{aligned}\Sigma &= \Sigma^s + \Sigma^{ps} \\ &= 2\widetilde{G}1\langle\bar{\psi}1\psi\rangle + 2\widetilde{G}i\gamma_5 1\vec{\tau}\langle\bar{\psi}i\gamma_5\vec{\tau}1\psi\rangle .\end{aligned} \tag{11}$$

Since the vacuum expectation value $\langle\bar{\psi}i\gamma_5\vec{\tau}1\psi\rangle$ vanishes, only the scalar part of the self-energy Σ^s survives and leads in a similar way as discussed in Section 3 to an effective mass for the constituent quarks of the form

$$M^* = M - 2\widetilde{G}\langle\bar{\psi}1\psi\rangle , \tag{12}$$

with the scalar density

$$\langle\bar{\psi}1\psi\rangle = -\frac{12}{(2\pi)^3}\int_0^{\Lambda} d^3p\,\frac{M^*}{\sqrt{\vec{p}^2 + M^{*2}}} . \tag{13}$$

The two Hartree equations (12) and (13) have to be solved in a self-consistent way and it is clear that for the chiral symmetric case ($M = 0$) they have the trivial solution $M^* = 0$. If, however the interaction strength $\tilde{G}$ is strong enough (for a given cut-off momentum Λ) the Hartree equation also yield a non-trivial solution with $M^* > 0$. This solution, with the chiral symmetry spontaneously broken, shows up also for the case $M > 0$ and can be identified from the fact that the constituent quark mass M^* obtained in this case is considerably larger than the bare current quark mass M.

In order to study a system of finite density one only has to replace the expression for the scalar density given in Eq. (13) by

$$\langle\bar{\psi}1\psi\rangle = -\frac{12}{(2\pi)^3}\int_0^\Lambda d^3p\,\Theta(|\vec{p}| - k_F)\frac{M^*}{\sqrt{\vec{p}^2 + M^{*2}}}, \tag{14}$$

with the usual step function Θ and the Fermi momentum k_F for the homogeneous system of quark matter. From Eq. (14) it is clear that the absolute value for the scalar density tends to reduce with increasing k_F. As a consequence also the effective mass M^* is reduced with increasing baryonic density. The scale for this decrease is determined directly by the cut-off parameter Λ and by the assumption that this cut-off does not depend on the density under consideration. All calculated observables, however, depend very strongly on the actual value for the cut-off. Therefore this assumption has very serious consequences for the predictions obtained in this model.

The effective mass M^* for the constituent quarks can directly be related to the effective mass of the nucleons if we assume that the baryon masses are essentially three times the constituent quark mass. In a next step one may then consider the mesons as the collective modes of the quark excitations from the Dirac see. In a nuclear medium these excitations of the Dirac see interfere with the particle-hole excitation of the quarks in the Fermi see. This reflects the coupling of the mesons to the medium excitations which are present already in a non-relativistic many-body theory. The effective masses of the mesons can be determined from an inspection of the reducible response function (*i.e.* the excitation spectrum calculated in RPA approximation) [17]. Results for the effective mass for the constituent quarks (or baryons) and mesons have been determined by several groups [17,18]. It is clear, however, that this medium dependence of the effective masses is a direct consequence of the assumption of a cut-off Λ which does not depend on the density of the system under consideration.

In order to develop a model which is very similar to the NJL model, but does not require such a strict cut-off, we may consider an interaction term between the Dirac spinors in momentum representation of the form

$$\mathcal{V} = -G(P)\left[\left(\bar{\psi}\vec{\vec{\lambda}}\psi\right)^2 + \left(\bar{\psi}\,i\gamma_5\vec{\tau}\vec{\vec{\lambda}}\psi\right)^2\right]. \tag{15}$$

This interaction term is very similar to the original one of the NJL model in Eq. (10) and exhibits only two differences: First, the interaction strength is not considered any longer to be a constant, but it should depend on the momentum transfer between the interacting Fermions. Guided by the running coupling constant of QCD, we assume that it is reduced for large momentum transfers P and consider an ansatz

$$G(P) = G_0 \exp\left(-\frac{P^2}{\Delta^2}\right) , \tag{16}$$

where Δ is just a parameter. The second difference is that the interaction terms of Eq. (15) are of vector-type in color-space. For our present investigation the detailed form of the interaction is not really relevant as long as it is symmetric under chiral transformation.

The direct or Hartree contributions to the self-energy are now to be calculated with a "coupling constant" G determined at momentum transfer $P = 0$ (see also discussion in the previous Section). For a homogeneous vacuum or baryon system the expectation values for the color-vector terms occurring in (15) vanish and therefore the Hartree terms do not contribute. If, however, one would study a system with an isolated quark of a given color the Hartree terms would diverge and therefore provide a possible mechanism to obtain color-confinement. For the study of the effective mass of quarks in a homogeneous system, however, we have to investigate the exchange or Fock contributions to the self-energy. In this approximation the effective mass is calculated as

$$M^*(q) = M + \frac{4}{9} \times 2 \times \frac{12}{(2\pi)^3} \int_0^\infty d^3p\, G(P = |\vec{p} - \vec{q}\,|) \frac{M^*(p)}{\sqrt{M^*(p)^2 + \vec{p}^{\,2}}} \, . \tag{17}$$

Since the self-energy contribution to the effective mass in this equation is due to the exchange term, the coupling strength has to be determined for the momentum transfer P which is equal to the difference between the momenta of the interacting Fermions. As a consequence of this momentum--dependence of the coupling strength also the resulting effective mass $M^*(q)$ depends on the momentum of the quark under consideration. Therefore the whole self-consistency problem becomes much more complicated in this case than for the conventional NJL model.

Nevertheless one can obtain non-trivial solutions of Eq.(17) with a momentum-dependent constituent quark mass as displayed in Fig. 4. One can study the medium dependence of the constituent quark masses but also the meson spectrum of this extended NJL model [19].

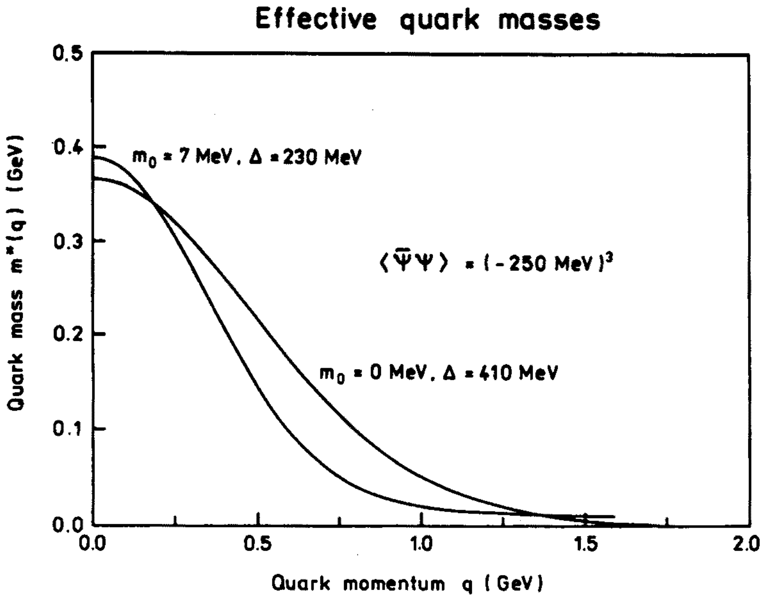

Fig. 4. Self-consistent solutions of Eq. (17) for the effective quark mass $M^*(q)$ as a function of momentum. Assuming for the current mass $M = 7$ MeV ($M = 0$) the parameters have been chosen to obtain a scalar density of $(-250\ \text{MeV})^3$ and an effective mass around 350 MeV for small momenta.

6. Conclusions

In the first part of this contribution we have demonstrated that for a realistic nuclear structure calculation several ingredients contribute to the saturation mechanism for nuclear systems. It is important to take into account the effect of two-body correlations and their medium dependence (see Section 2). The relativistic effects of the DBHF approach lead to a medium dependence of the Dirac spinors depending on the density of the nuclear system. These effects tend to improve the agreement between calculated and empirical data for the bulk properties of nuclear systems. While these two effects seem to dominate the saturation properties of infinite nuclear systems the range of the effective NN interaction seem to be a very important ingredient to describe the ground states of finite nuclei (see Section 4). This sensitivity of finite systems may be responsible for the different features derived for the equation of states as observed in heavy ion scattering experiments to those determined from astrophysical studies.

The range of the NN interaction is directly related to the propagator or the effective mass of the mesons which yield the NN interaction in the meson exchange model. Therefore we have studied simple models which try to derive the effective masses of baryons and mesons as well as their medium dependence from quark degrees of freedom. Such a model which has received a lot of attention recently is the so-called NJL model. The predictions of these models are very sensitive to a cut-off parameter. Therefore we consider a modified version of the NJL, which does not need a cut-off but rather use a running coupling constant as motivated from QCD. First studies of this model support the classical NJL model. Furthermore it gives rise to color confinement already within the mean-field approximation.

The investigations presented here have been performed in collaboration with many colleagues. I would like to list a few of them like R. Brockmann, G.E. Brown, D. Bückers, P. Czerski, H. Elsenhans, A. Faessler, R. Fritz, E. Henley, R. Machleidt, A. Polls, M. Prakash, K.W. Schmid to thank for this collaboration and many fruitful discussions. Also I would like to acknowledge the financial support by the Bundesministerium für Forschung und Technologie (Bonn, project 06 Tü 714).

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