AN APPLICATION OF QCD SUM RULES IN NUCLEAR PHYSICS — THE NOLEN–SCHIFFER ANOMALY*

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We discuss the possibility that the Nolen–Schiffer anomaly, an old problem in nuclear physics, might be a signal that the neutron-proton mass difference is smaller in nuclei than in vacuum.

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The Nolen–Schiffer anomaly [1], (first remarked by Okamoto [2]), is an almost thirty year old problem in nuclear physics. Opinions are still divided when it comes to offering explanations of it. For a particle physicist the anomaly might be very interesting as it opens up possibility of solving this problem invoking the elusive quark degrees of freedom in nuclei.

The anomaly is seen in analogue states or mirror nuclei, the simplest of which are made up of a common core where in one case a neutron, in the other case a proton is added. Examples of these are ^{3}He and ^{3}H, ^{17}O and ^{17}F and the pair ^{41}Sc and ^{41}Ca.

The mass $M(\mathrm{n})$ of the nucleus with a neutron outside the core is of course not the same as the mass $M(\mathrm{p})$ of the nucleus with a proton outside the same core. The different Coulomb energies, electromagnetic spin-orbit interactions and the neutron–proton mass difference $\delta M = 1.3$ MeV ensures that. In addition comes possible effects of charge symmetry breaking of nuclear forces.

Nuclear physicists think that they have the calculations of electromagnetic effects well in hand: they have realistic wavefunctions for the nucleons that reproduce electron scattering very well.

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With these wavefunctions, however, one calculates a theoretical value $\Delta_{np}(\text{th}) = M(\text{n}) - M(\text{p})$ which is bigger than the observed $\Delta_{np}(\text{exp})$. This is the Nolen–Schiffer anomaly: It runs through the whole periodic table and increases with nucleon number to something like 900±200 keV in the lead region. The difference $\delta_{\text{ONS}} = \Delta_{np}(\text{th}) - \Delta_{np}(\text{exp})$ is the Nolen–Schiffer anomaly.

It is obvious that the nuclear theorists first turned to charge symmetry breaking (CSB) of nuclear forces to explain the anomaly. A long list of effects of CSB can be found in the review article by Shlomo [3]. He states there that if CSB is made to work for ^{3}He-^{3}H it will give too small effects higher up in the periodic system to explain the anomaly. On the other hand there has been a recent proposal invoking mixing effects between ρ and ω exchanges that is claimed to explain about 75% of the anomaly [4].

Personally, I would have liked the conventional nuclear physics to have been better under control, but I will in the following assume that the Nolen–Shiffer anomaly is a real problem. Many respectable nuclear physicists have told me so.

There is one explanation of the anomaly that I, for one, find particularly elegant, which highlights the anomaly as a striking signal of chiral restoration in nuclei. In a way you could take this explanation to be orthogonal to CSB, as it asserts that nucleons bound in nuclei are more similar than when they are free.

Its essence is that the neutron-proton mass difference decreases from the value of 1.3 MeV when nucleons are put into a nuclear medium. This is brought about because the covariant condensates (the vacuum expectation values of the normal ordered produsts) of fundamental fields decrease in absolute magnitude in the medium relative to what they are in vacuum.

The first to exploit this possibility were Henley and Krein [5]. By choosing a specific model for chiral symmetry breaking which is much used in nuclear theory these days, namely the Nambu Jona-Lasinio (NJL) model for quark–quark interactions and combining it with a non-relativistic quark model for the nucleons, they showed inside this model that the neutron-proton mass difference Δ_{np} indeed decreased sufficiently much in a nuclear medium to account qualitatively for a phenomenon like the Nolen–Schiffer anomaly.

A closer study shows however that relativistic corrections strongly decrease the variations of Δ_{np} in the medium [7]. It would clearly be desirable to link Δ_{np} in the medium with vacuum parameters in a better way.

The QCD sum rules approach is a relativistic formulation of fundamental origin that can be used to analyze how baryon masses [9–11] change in the medium. The neutron proton mass difference can be related to the variation of the quark and gluon condensates in the nuclear medium. Reviews of the QCD sum rules method can be found elsewhere in proceedings from this school [10].

For the application of QCD sum rules $\Delta_{\rm np}$, the operators relevant for the proton (neutron) with broken iso-spin symmetry are

$$\Psi_{\rm p} = \epsilon_{abc}\left[(\mathrm{u}^a C \mathrm{u}^b)\gamma_5 \mathrm{d}^c + t(\mathrm{u}^a C\gamma_5 \mathrm{u}^b)\mathrm{d}^c\right],$$

$$\Psi_{\rm p} = \epsilon_{abc}\left[(\mathrm{d}^a C \mathrm{d}^b)\gamma_5 \mathrm{u}^c + t(\mathrm{d}^a C\gamma_5 \mathrm{d}^b)\mathrm{u}^c\right], \tag{1}$$

where C denotes charge conjugation, a, b and c are colour indices of the u and d quarks, and t is the mixing strength of the two independent operators having the appropriate symmetry.

The parameter t above represent the mixing between two independent operators that have the same quantum number as the nucleon we consider. Ioffe's original choice for the nucleon correlator was $t = -1$, a best fit of the masses inside the $S = {}^1\!/_2{}^+$ baryon octet gives $t = -1.15$.

The QCD sum rules [8] are based on the use of the operator product expansion (OPE) of the correlation function for Ψ as defined above

$$\pi^{\alpha\beta}(q) \equiv \int d^4x e^{iqx}\mathrm{T}\left[\Psi^\alpha(x), \overline{\Psi}(0)^\beta\right] = \sum_{\rm n} C_{\rm n}(q) O_{\rm n}, \tag{2}$$

where α and β are spinor indices.

The short range part of the correlation function is given by the Wilson coefficients $C_{\rm n}$, the long-range part enters the local composite operators $O_{\rm n}$. In vacuum we take the expectation value of $O_{\rm n}$ in the vacuum, in a medium the expectation value in the ground state of the medium.

In this way the medium correlation to the sum rules are obtained by the density dependence of the expectation values of the composite operators. This approach is the most simple in the low density (or temperature) regime.

One should note that when one takes the expectation values in a medium one has to choose a specific reference frame — the most convenient is the rest frame of matter. Non-covariant condensates enter the game and they generate contributions to the vector self energy $\Sigma^{\rm V}$ as the covariant condensates give the scalar self energy $\Sigma^{\rm S}$.

The phenomenological nucleon propagator in the medium takes the form

$$\pi^{\rm phen}(q) = \frac{\lambda_{\rm N}}{(q_\mu - \Sigma^{\rm V}_\mu)\gamma^\mu - \Sigma^{\rm S}}, \tag{3}$$

where $\lambda_{\rm N}$ is the coupling of the operator in Eq. (2) to the nucleon and the OPE expansion entering sum rules read

$$\pi^{\rm OPE}(q) = q_\mu\gamma^\mu\pi_1 + \pi_2 + \alpha_\mu\gamma^\mu\pi_3 \tag{4}$$

when we treat

$$\alpha_\mu \equiv \langle \bar{q}\gamma_\mu q\rangle \tag{5}$$

as a background field.

A nice thing is that the noncovariant condensates that appear, lead to pole positions in the nucleon propagator that vary very little with the density of the medium when covariant condensates decrease. On the other side it can be shown that the mass difference $\Delta_{\rm np}$ is mainly given by the isospin breaking in the covariant condensates of $\langle\bar{\rm u}{\rm u}\rangle$ and $\langle\bar{\rm d}{\rm d}\rangle$.

The noncovariant condensates can therefore be neglected when we make a semiquantitative [7, 12–13] discussion of the in-medium dependence of $\Delta_{\rm np}$.

It is well known since the first work by Ioffe [9] that the quantities which are the most important in determining baryon masses are the values of the quark condensates $\langle\bar{\rm q}{\rm q}\rangle$ and the quark masses. Gluon and mixed condensates are rather unimportant.

Then it follows that the quantities with dimension that are the most important for the neutron-proton mass difference are [14]

1. The difference between the quark (current) masses. $\delta m = m_{\rm d} - m_{\rm u}$ will always be taken to be 4 MeV.
2. The difference between quark condensates $\langle\bar{\rm u}{\rm u}\rangle - \langle\bar{\rm d}{\rm d}\rangle = x$.

For convenience we can instead of choosing δm and x choose δm and $\langle\bar{\rm u}{\rm u}\rangle$ as our variables with dimension, leaving the variable

$$\gamma = \frac{\langle\bar{\rm d}\,{\rm d}\rangle}{\langle\bar{\rm u}\,{\rm u}\rangle} - 1 \tag{6}$$

as a dimensionless quantity.

We have now made plausible the result which emerges from more detailed studies that the neutron proton mass difference can be expressed in terms of $\langle\bar{\rm u}{\rm u}\rangle$, δm and the three dimensionless coefficients

$$\Delta_{\rm np} \simeq D_1 \cdot \gamma\langle\bar{\rm u}{\rm u}\rangle^{1/3} + D_2 \cdot \delta m\,. \tag{7}$$

The surprising thing in this formula is that QCD sum rules give — for all values of the mixing parameter t that are successful in spectroscopy — a negative sign for the coefficient $D_2 \cdot D_1$ and the first term in (7) is positive.

This is the same as what happens when one calculates [9–11, 15] SU(3)-flavour breaking with QCD sum rules by giving a mass to the strange quark. Then one also finds for the mass difference between Ξ and Σ a formula where the strange quark mass enters explicitly with a sign contrary to what one would guess.

It is this property that enables us to give an explanation of the Nolen–Schiffer effect.

On the face of it our explanation of the effect through a decreasing neutron–proton mass difference in the nuclear medium looks a bit strange. The Nolen–Schiffer effect in Pb amount to something like 0.9±0.2 MeV and this is around 70% of the neutron-proton mass difference in vacuum.

On the other hand the covariant condensates $\langle \bar{q}q \rangle^{1/3}$, that according to Ioffe give the dominant contribution to the nucleon mass, decreases only by 10–20% in absolute magnitude when the density varies from zero to nuclear matter densities. But according to formula (7) the neutron-proton mass difference is made up of two relatively big terms with opposite sign. Only one of these shows an appreciate variation with density in the low density regime.

Formula (7) is, therefore, quite interesting. Let us look at the first term: $\langle \bar{u}u \rangle$ is negative, in vacuum $\langle \bar{u}u \rangle \approx (-250\ \text{MeV})^3$. (-250 MeV)3.

Now

$$\gamma = \frac{\langle \bar{d}d \rangle}{\langle \bar{u}u \rangle} - 1 = \frac{\langle \bar{d}d \rangle - \langle \bar{u}u \rangle}{\langle \bar{u}u \rangle} < 0$$

is clearly also implicitly a function of $\delta m = m_d - m_u$ such that Δ_{np} has the right sign because of the positivity of the first term in (7). As an example we take the Ioffe choice for the correlator, neglect the continuum contribution and find

$$\Delta_{np} = -A_1 \gamma M_0 - A_2 \delta m\,. \tag{8a}$$

M_0 which is the nucleon mass in the chiral limit is proportional to $\langle \bar{q}q \rangle^{1/3}$ and

$$A_1 = 1 + \frac{4aM_0}{3y^2}\,, \qquad A_2 = 2 + \frac{M_0^2}{y} - \frac{2M_0 a}{y^2} - \frac{bM_0}{ay}\,. \tag{8b}$$

Here $a \equiv -(2\pi)^2 \langle \bar{u}u \rangle$, $b \equiv \pi^2 \langle \frac{\alpha_S}{\pi} G^2 \rangle$ and y is the Borel parameter. α_S is the fine structure constant for colour forces and $\langle G^2 \rangle$ is the gluon condensate.

In vacuum M_0 and $\sqrt{y}$ are around 1 GeV and with values of condensates determined by fitting the spin $^1/_2$baryon octet one finds

$$\langle \bar{q}q \rangle \simeq (-250\ \text{MeV})^3\,, \qquad \left\langle \frac{\alpha_S}{\pi} G^2 \right\rangle \approx (330\ \text{MeV})^4\,.$$

A_1 and A_2 are therefore both positive. You also see that $b/a \approx 0.19$ GeV, showing that the gluon condensate is quite unimportant for what follows.

The value of the parameter γ that contains the isospin breaking effects in the quark condensates can be estimated from the value it has in the broken SU_3^F sector. Using the simple scaling relation $\frac{\langle \bar{d}d \rangle}{\langle \bar{S}S \rangle} \simeq \frac{m_d}{m_S}$ we find $\gamma =$

$-7.8 \cdot 10^{-3}$, a value consistent with other [17] independent determination which give values in the range -0.006 to -0.009 and also consistent with values found in the NJL model.

The same model shows that when the $\langle\bar{q}q\rangle$ condensates change in a nuclear medium γ changes little, something to be expected as γ involve the ratio of $\langle\bar{d}d\rangle$ and $\langle\bar{u}u\rangle$. We shall, therefore, keep γ as a constant for baryon densities in the medium much lower than the nuclear matter equilibrium density $\rho_{\rm nm} \simeq 0.16\ {\rm fm}^{-3}$.

The essential point is now that as the baryon density increases $|\langle\bar{u}u\rangle^{1/3}|$ decreases, so does M_o and the same happen to Δ_{ur}.

Looking at the theoretical expression of Eq. (7) or (8) we realize the importance of the negative factor multiplying the current quark mass difference $\delta m = m_{\rm d} - m_{\rm u}$ in the expression for $\Delta_{\rm np}$. A relatively small decrease in $\langle\bar{u}u\rangle$ is magnified by the size of $|D_1\gamma|$ in Eq. (7).

The particular form for $\Delta_{\rm np}$ in Eq. (8) comes from a very simplified version of the QCD sum rules. The inclusion of the continuum, of noncovariant condensates and nucleon correlators different from Ioffe (as long as the correlators will work in baryon spectroscopy) all give the generic form (7) although the coefficients D_1 and D_2 vary somewhat.

To see the possible relevance of Eq. (7) for nuclear physics we need to know how $\langle\bar{q}q\rangle$ varies with nuclear density. For small baryon densities ρ one can always write

$$\left|\frac{\langle\bar{u}u\rangle}{\langle\bar{u}u\rangle_0}\right|^{1/3} = 1 - C\frac{\rho}{\rho_{\rm nm}} = R(\rho) \tag{9}$$

and this is a good approximation when nucleons are close to the nuclear surface. The constant C is in the range 0.1 to 0.2.

Suppose now that a neutron or a proton move in the field of a common core of A nucleons with density $\rho_A(r)$ such that we compare two mirror nuclei with nucleon number $A+1$.

To the extent that the last nucleon has a flavour independent probability distribution $P(\vec{r})$ for moving in a medium of density $\rho(r)$ it is clear that (9) together with (7) leads to an effect in the mass difference between the two mirror nuclei $M(\rm n)-M(\rm p)$ that is precisely of the Okamoto-Nolen-Schiffer anomaly type

$$\begin{aligned}\delta_{\rm ONS} &= \int (\Delta_{\rm np}(0) - \Delta_{\rm np}(\rho))P(\vec{r})d^3r \\ &= \langle\bar{u}u\rangle_0 C\int D_1\gamma\frac{\rho(\vec{r})}{\rho_{\rm nm}}P(r)d^3r \simeq CD_1\gamma\langle\bar{u}u\rangle_0\int\frac{\rho(r)}{\rho_{\rm nm}}P(r)d^3r\,. \end{aligned} \tag{10}$$

Note that the value of D_2 in (7) is irrelevant in the formula.

Explicite calculations have been done where we use a density distribution of the Woods-Saxon form [12], or by densities calculated using a Woods–Saxon potential with parameters chosen to fit the experimental values for the core densities and single particle energies of the valence particles, [13]. There is not much of a difference. They all show a gentle increase with increasing A and can easily accommodate the calculations of the Nolen–Schiffer anomaly by Sato [18]. In his calculations there are the systematic feature that the anomaly is bigger for nuclei when we have a filled shell minus one nucleon than when we have a filled shell plus one nucleon.

This is exactly the feature that we would expect from Eq. (10), *i.e.*that the hole in the shell feels a bigger density than the particle outside the filled shell [19]. The calculations by Shlomo [3] do not, however, show this effect so it is clear that professionals in theoretical nuclear physics have to do some work to get a consensus on what the anomaly really is.

I hope to have been able to show that if QCD-sum rules as we know them can be used to compute properties of baryons, they can put the Nolen–Schiffer anomaly in a completely new light. Instead of being a consequence of broken charge symmetry they are a striking signal of the restoration of symmetry when density increases.

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