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AN ALGEBRAIC MODEL FOR QUARK MASS MATRICES WITH HEAVY TOP*

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In terms of an intergeneration U(3) algebra, a numerical model is constricted for quark mass matrices, predicting the top-quark mass around 170 GeV and the CP-violating phase around 75°. The CKM matrix is nonsymmetric in moduli with $|V_{\mathrm{ub}}|$ being very small. All moduli are consistent with their experimental limits. The model is motivated by the author's previous work on three replicas of the Dirac particle, presumably resulting into three generations of leptons and quarks. The paper may be also viewed as an introduction to a new method of intrinsic dynamical description of lepton and quark mass matrices.

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1. Introduction

It seems to be a common belief among theoreticians that the problem of mass spectrum of leptons and quarks overflows the physical limits of the standard model of electroweak and strong interactions. And it is, therefore, reasonable to construct phenomenological models for lepton and quark mass matrices in order to unveil the fundamental pattern of this spectrum. In particular, Fritzsch and Plankl did a lot of work in this field [1,2] emphasizing the dynamical role of numerical dominance of lepton and quark mass spectrum by the members of the third generation (*i.e.* τ as well as t and b). Such a dominance could be hopefully understood [2] by the introduction of

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"coherent states" (*viz.*, equal superpositions of three generations) as simple original states, subject to an (unfortunately strong) intergeneration mixing that leads to complicated final "mass states". Of course, the physical reason for this strong mixing and its actual form are main points to be understood (or, at least, accepted) in such an argument.

In the present paper we search directly for the mass pattern of leptons and quarks, using as a tool an intergeneration U(3) algebra spanned on the formal Gell-Mann 3×3 matrices $\hat{\lambda}_1, \hat{\lambda}_2, \ldots, \hat{\lambda}_8$ and $\hat{1}$. It turns out that in a reasonable approximation we are able to describe the mass spectrum in a concise diagonal form, subject (in the quark case) to a weak effective intergeneration mixing responsible for the Cabibbo-Kobayashi-Maskawa matrix.

The starting point of this approach is a recent work [3] where (motivated by our previous results [4] concerning the existence of three replicas of the Dirac particle) we found a semiempirical mass formula for charged leptons e, μ and τ.

When basing on the results of Ref. [4] (for the Reader's convenience they are summarized in Appendix), it is natural to make use of the column wave functions $\Psi^{(\nu)}$ and $\Psi^{(e)}$ comprising the wave functions of three generations of neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ and charged leptons (e^-, μ^-, τ^-), respectively, with the generation weight factors $\sqrt{1/29}$, $\sqrt{4/29}$ and $\sqrt{24/29}$ (cf. Eq. (A.14)). Then, on the level of field theory, assuming the minimal lepton-higgs coupling of the form

$$\left(\overline{\Psi}_{\rm L}^{(\nu)}, \overline{\Psi}_{\rm L}^{(e)}\right) \hat{f}\, \Psi_{\rm R}^{(e)} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} + \text{h.c.} \tag{1}$$

with a Hermitian diagonal 3×3 strength matrix

$$\hat{f} = \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix} \tag{2}$$

acting on three generations comprised in Ψ's, we conclude that

$$m_e : m_\mu : m_\tau = |f_1| : 4|f_2| : 24|f_3| \tag{3}$$

and $m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0$ (the Dirac masses m_e, m_μ and m_τ and, later on, those for quarks are taken as positive-definite). Here, the mass matrix for charged leptons is

$$\widehat{M} = v \hat{\varrho} \hat{f} \hat{\varrho}\,, \tag{4}$$

where $v = < \phi^0 >$ and

$$\hat{\varrho} = \frac{1}{\sqrt{29}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{4} & 0 \\ 0 & 0 & \sqrt{24} \end{pmatrix} \tag{5}$$

($\mathrm{Tr}\hat{\varrho}^2 = 1$). Here, $\widehat{M} = v\hat{f}\hat{\varrho}^2$.

Since for experimental values of m_e, m_μ and m_τ the numerical relations

$$m_e : m_\mu \simeq 1 : 207 \simeq 0.172 : 4(9 - \tfrac{1}{9}) \tag{6}$$

and

$$m_\mu : m_\tau \simeq 1 : 17 \simeq 4(9 - \tfrac{1}{9}) : 24(25 - \tfrac{1}{25}) \tag{7}$$

hold, we may try the *ansatz*

$$f_i = \mathrm{const}\left(N_i^2 - \frac{1+\varepsilon^2}{N_i^2}\right), \tag{8}$$

where $\varepsilon^2 \simeq 0.172$,while

$$N_1 = 1\,, \quad N_2 = 3\,, \quad N_3 = 5\,, \tag{9}$$

are numbers of bispinor indices appearing originally in wave functions of three generations (*cf.* Eq. (A.12)). (These indices play here a role of algebraic "intrinsic partons" of leptons and quarks.) Then, from Eqs (3) and (8) we obtain two mass relations

$$m_\mu = m_e \tfrac{4}{9}\left(\tfrac{80}{\varepsilon^2} - 1\right) \tag{10}$$

and

$$m_\tau = m_e \tfrac{24}{25}\left(\tfrac{624}{\varepsilon^2} - 1\right). \tag{11}$$

Hence, eliminating ε^2 we come to the semiempirical mass formula for charged leptons [3],

$$\tfrac{6}{125}\left(136 m_e + 351 m_\mu\right) = m_\tau\,, \tag{12}$$

which is very well satisfied by experimental masses. Indeed, its lhs and rhs is 1783.47 MeV and 1784 $^{+2.7}_{-3.6}$ MeV, respectively [5]. Our only free parameter ε^2, when evaluated from experimental m_e and m_μ by means of Eq. (10), is equal to

$$\varepsilon^2 = \frac{320 m_e}{4m_e + 9m_\mu} = 0.171590 = \tan^2 \tfrac{\pi}{7.99965}\,, \tag{13}$$

so, it takes a magic value $\tan^2(\pi/8)$ with a very good accuracy.

When turning from leptons to quarks, we face the problem of effective intergeneration mixing (and, of course, the problem of less precisely defined "experimental" masses). Also in this case we use the column wave functions $\Psi^{(u)}$ and $\Psi^{(d)}$ comprising the wave functions of three generations of up quarks (u, c, t) and down quarks (d, s, b), respectively, with the weights

$\sqrt{1/29}$, $\sqrt{4/29}$ and $\sqrt{24/29}$. Then, on the level of field theory, we assume the minimal quark-higgs coupling of the form

$$\left(\overline{\Psi}_{\rm L}^{(\rm u)}, \overline{\Psi}_{\rm L}^{(\rm d)}\right) \left[\hat{f}^{(\rm d)} \Psi_{\rm R}^{(\rm d)} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} + \hat{f}^{(\rm u)} \Psi_{\rm R}^{(\rm u)} \begin{pmatrix} \phi^{0\,\rm c} \\ -\phi^- \end{pmatrix}\right] + \text{h.c.} \tag{14}$$

with Hermitian nondiagonal 3×3 strength matrices

$$\hat{f}^{(\rm u,\,d)} = \left(f_{ij}^{(\rm u,\,d)}\right) . \tag{15}$$

Here, $\phi^- \equiv \phi^{+\,\rm c}$ and $\phi^{0\,\rm c}$ are charge conjugates of ϕ^+ and $\phi^{\rm o}$. From Eq. (14) we can conclude that the mass matrices for up and down quarks are

$$\widehat{M}^{(\rm u,\,d)} = v\hat{\varrho}\hat{f}^{(\rm u,\,d)}\hat{\varrho}\,, \tag{16}$$

where $v =< \phi^0 >=< \phi^{0\rm c} >$ and $\hat{\varrho}$ is given in Eq. (5). Hence, up to signs

$$\widehat{U}^{(\rm u)}\widehat{M}^{(\rm u)}\widehat{U}^{(\rm u)+} = \begin{pmatrix} m_{\rm u} & 0 & 0 \\ 0 & m_{\rm c} & 0 \\ 0 & 0 & m_{\rm t} \end{pmatrix},$$

$$\widehat{U}^{(\rm d)}\widehat{M}^{(\rm d)}\widehat{U}^{(\rm d)+} = \begin{pmatrix} m_{\rm d} & 0 & 0 \\ 0 & m_{\rm s} & 0 \\ 0 & 0 & m_{\rm b} \end{pmatrix}, \tag{17}$$

and the Cabibbo–Kobayashi–Maskawa mixing matrix is

$$\widehat{V} = \widehat{U}^{(\rm u)+}\widehat{U}^{(\rm d)}\,. \tag{18}$$

The unitary diagonalizing matrices $\widehat{U}^{(\rm u,d)}$ can be presented as

$$\widehat{U}^{(\rm u,\,d)} = \left(U_{ij}^{(\rm u,\,d)}\right) = \left(v_i^{(\rm u,\,d)}(j)\right), \tag{19}$$

where $v_i^{(\rm u,\,d)}(j)$ is the component j of the eigenvector $v_i^{(\rm u,\,d)}$ of Hermitian mass matrix $\widehat{M}^{(\rm u,\,d)}$. Thus

$$\widehat{V} = \left(V_{ij}\right) = \left(\sum_k v_k^{(\rm u)*}(i) v_k^{(\rm d)}(j)\right), \tag{20}$$

which is useful in practical calculations.

2. Annihilation and creation operators in an intergeneration U(3) algebra

Using a figurative language, the formal results summarized in Appendix may be expressed by saying that leptons and quarks of three generations consist of algebraic "intrinsic partons" with spins 1/2, namely of one "visible parton" and $n = 0, 1, 2$ pairs of "hidden partons". So, the total number of "intrinsic partons" is $N = 1+2n = 1, 3, 5$ for the 1^{st}, 2^{nd} and 3^{d} generation, respectively. Spins of "hidden partons" sum up to zero due to the interplay of the theory of relativity, probability interpretation and "hidden exclusion principle" (leading to only one Dirac particle of a given flavor and color in each of three generations).

Since in the case of quarks of three generations there is an effective intergeneration mixing, a U(3) algebra should be a useful tool in describing their masses and mixing parameters. Thus, let us introduce (within this intergeneration U(3) algebra) the (restricted) annihilation and creation operators

$$\hat{a} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{a}^+ = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}, \tag{21}$$

such that

$$\hat{n} = \hat{a}^+\hat{a} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad \hat{a}^3 = 0 = \hat{a}^{+3} \tag{22}$$

and

$$[\hat{a}, \hat{n}] = \hat{a}, \quad [\hat{n}, \hat{a}^+] = \hat{a}^+ . \tag{23}$$

But $[\hat{a}, \hat{a}^+] \neq \hat{1}$ and $\{\hat{a}, \hat{a}^+\} \neq \hat{1}$, in fact

$$[\hat{a}, \hat{a}^+] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \quad \{\hat{a}, \hat{a}^+\} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}. \tag{24}$$

Note that

$$\hat{a}^2 = \begin{pmatrix} 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{a}^{+2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \end{pmatrix},$$

$$\hat{a}^2\hat{a}^+ = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{a}\hat{a}^{+2} = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\hat{a}^+\hat{a}^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{a}^{+2}\hat{a} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix},$$

$$\hat{a}\hat{a}^+\hat{a} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{a}^+\hat{a}\hat{a}^+ = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2\sqrt{2} & 0 \end{pmatrix},$$

$$\hat{a}^{+2}\hat{a}^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad \hat{a}^2\hat{a}^{+2} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\hat{a}\hat{a}^{+2}\hat{a} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}^+\hat{a}^2\hat{a}^+ . \tag{25}$$

Of course, all functions of $\hat{a}$ and $\hat{a}^+$ can be expressed as linear combinations of Gell-Mann matrices $\hat{\lambda}_1, \hat{\lambda}_2, \ldots, \hat{\lambda}_8$ and $\hat{1}$. For instance,

$$\hat{a}^2 = \frac{\hat{\lambda}_4 + i\hat{\lambda}_5}{\sqrt{2}}, \quad \hat{a}^2\hat{a}^+ = \hat{\lambda}_1 + i\hat{\lambda}_2, \quad \hat{a}^+\hat{a}^2 = \frac{\hat{\lambda}_6 + i\hat{\lambda}_7}{\sqrt{2}},$$

$$[\hat{a}, \hat{a}^+] = \hat{\lambda}_8\sqrt{3}, \quad \{\hat{a}, \hat{a}^+\} = 2\,\hat{1} - \hat{\lambda}_3,$$

$$[\hat{a}^2, \hat{a}^{+2}] = \hat{\lambda}_3 + \hat{\lambda}_8\sqrt{3}. \tag{26}$$

In our figurative language, the matrix

$$\widehat{N} \equiv \hat{1} + 2\hat{n} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} \tag{27}$$

is the operator of the number of all "intrinsic partons", while $\hat{n} \equiv \hat{a}^+\hat{a}$ is the operator of the number of pairs of "hidden partons". So, $\hat{a}$ and $\hat{a}^+$ are the (restricted) annihilation and creation operators of a pair of "hidden partons". In the next two Sections we use these operators to construct both a diagonal model and a non-diagonal model for quark mass matrices $\widehat{M}^{(\mathrm{u})}$ and $\widehat{M}^{(\mathrm{d})}$. The first model may be considered as a diagonal approximation of the second (in the limit, when the effective intergeneration mixing is neglected).

3. A diagonal model for quark mass matrices

First, we will construct a diagonal model for quark mass matrices following the numerically successful model for charged-lepton mass matrix (4) with the *ansatz* (8) for the strength matrix. This *ansatz* can be rewritten as

$$\hat{f} = \frac{C}{\varepsilon^2}\left(\widehat{N}^2 - \frac{1+\varepsilon^2}{\widehat{N}^2}\right), \tag{28}$$

where $\widehat{N} = \hat{1} + 2\hat{a}^+\hat{a}$ is the operator of the number of "intrinsic partons" whose eigenvalues are $N_1 = 1$, $N_2 = 3$ and $N_3 = 5$ for three generations (*cf.* Eq. (27)).

When passing from leptons to quarks one can observe the following qualitative features of quark mass spectrum:

(i) for up quarks

$$m_c : m_u \simeq m_\mu : m_e ,$$
$$m_c \gg m_\mu ,$$
$$m_t : m_c \gg m_\tau : m_\mu , \quad m_t : m_u \gg m_\tau : m_e ,$$

(ii) for down quarks

$$m_s : m_d \ll m_\mu : m_e ,$$
$$m_s \gtrsim m_\mu ,$$
$$m_b : m_s > m_\tau : m_\mu , \quad m_b : m_d < m_\tau : m_e .$$

Additionally, $m_u \lesssim m_d$, $m_c \gg m_s$ and $m_t \gg m_b$.

In order to describe those features we will try an *ansatz* analogical to (28), but with the operator $\widehat{N}$ replaced by an effective operator $\widehat{N}_{\rm eff}$ that should emphasize the numerical dominance of t and b (especially t) over c and s, respectively, in comparison with the dominance of τ over μ. Moreover, the value of ε^2 for down quarks is supposed to be much larger than that for up quarks, the second value being expected similar to the value $\varepsilon^2 = \tan^2(\pi/8)$ for leptons (the rest C of the constant C/ε^2 at the front of $\hat{f}$ is presumed to be not very different in both cases). Thus, we take the *ansatz*

$$\hat{f}^{(\rm u,d)} = \frac{C^{(\rm u,d)}}{\varepsilon^{(\rm u,d)2}} \left(\widehat{N}_{\rm eff}^{(\rm u,d)2} - \frac{1+\varepsilon^{(\rm u,d)2}}{\widehat{N}_{\rm eff}^{(\rm u,d)2}} \right) \tag{29}$$

(with $C^{(\rm u)}$ and $C^{(\rm d)}$ not very different) and consider *for example* the model where

$$\widehat{N}_{\rm eff}^{(\rm u,d)} = \hat{1} + 2\hat{a}^+\hat{a} + \xi^{(\rm u,d)}\hat{a}^{+2}\hat{a}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5+2\xi^{(\rm u,d)} \end{pmatrix} \tag{30}$$

with

$$\xi^{(\rm u)} = 4 , \quad \xi^{(\rm d)} = 1 . \tag{31}$$

The quartic term in Eq. (30) is intended to introduce a pairing interaction between two pair of "hidden partons" in t and b. Its appearance for quarks (in contrast to charged leptons) may be somehow related to their nonzero baryon number $B = 1/3$. Note that formally $\xi^{(\rm u,d)} = (B + Q^{(\rm u,d)} + 1)^2$, where $Q^{(u)} = 2/3$ and $Q^{(d)} = -1/3$ are the quark electric charges. For charged leptons a counterpart of such $\xi^{(u,d)}$ is zero because then $B + Q + 1 = 0$.

The numerical results of the diagonal model defined by Eqs (16), (5), (29) and (30) with (31) are the following:

(i) for up quarks, taking as an input

$$5\ \mathrm{MeV} \lesssim m_u \lesssim 8\ \mathrm{MeV}, \quad m_c \simeq 1.5\ \mathrm{GeV}, \tag{32}$$

one gets

$$0.118 \lesssim \varepsilon^{(u)2} \lesssim 0.189, \quad 171\ \mathrm{GeV} \lesssim m_t \lesssim 172\ \mathrm{GeV} \tag{33}$$

(in particular for $\varepsilon^{(u)2} = \tan^2(\pi/8) = 0.171573$ and $m_c \simeq 1.5$ GeV one obtains $m_u \simeq 7.25$ MeV and $m_t \simeq 171$ GeV),

(ii) for down quarks, taking as an input

$$7\ \mathrm{MeV} \lesssim m_d \lesssim 10\ \mathrm{MeV}, \quad m_b \simeq 5\ \mathrm{GeV}, \tag{34}$$

one gets

$$1.64 \lesssim \varepsilon^{(d)2} \lesssim 2.35, \quad 148\ \mathrm{MeV} \gtrsim m_s \gtrsim 147\ \mathrm{MeV} \tag{35}$$

(in particular for $\varepsilon^{(d)2} = 2$ and $m_b \simeq 5$ GeV one obtains $m_d \simeq 8.5$ MeV and $m_s \simeq 148$ MeV).

Additionally, $m_u : m_d = C^{(u)} : C^{(d)}$, so these constants are really not very different.

4. A nondiagonal model for quark mass matrices

In order to switch on the effective intergeneration mixing we consider *for example* the model where

$$\hat{f}^{(u,d)} = C^{(u,d)} \left(\frac{1}{\varepsilon^{(u,d)2}} \left(\hat{N}_{\mathrm{eff}}^{(u,d)2} - \frac{1 + \varepsilon^{(u,d)2}}{\hat{N}_{\mathrm{eff}}^{(u,d)2}} \right) + \eta^{(u,d)} 2 \left(\hat{a} \exp(i\varphi^{(u,d)}) + \hat{a}^{+} \exp(-i\varphi^{(u,d)}) \right) \right) \tag{36}$$

with

$$\eta^{(u)} = 4, \quad \eta^{(d)} = 1 \tag{37}$$

and $\hat{N}_{\mathrm{eff}}^{(u,d)}$ as given in Eq. (30). Here, the previous diagonal strength matrices (29) are perturbed by the simplest off-diagonal "hidden parton" Yukawa-

-type interaction having a known factor $\eta^{(u,d)}$ and two unknown phases $\varphi^{(u)}$ and $\varphi^{(d)}$. Note that formally $\eta^{(u,d)} = (B+Q^{(u,d)}+1)^2 = \xi^{(u,d)}$ (*cf.* Eq. (31)). The counterpart of such $\eta^{(u,d)}$ for charged leptons is zero.

The quark mass matrices (16) with the nondiagonal *ansatz* (36) take explicitly the form

$$\widehat{M}^{(u,d)} = \frac{vC^{(u,d)}}{29\varepsilon^{(u,d)2}} \times \begin{pmatrix} \mu_1^{0(u,d)} & 4\eta^{(u,d)}\varepsilon^{(u,d)2}e^{i\varphi^{(u,d)}} & 0 \\ 4\eta^{(u,d)}\varepsilon^{(u,d)2}e^{-i\varphi^{(u,d)}} & \mu_2^{0(u,d)} & 16\sqrt{3}\eta^{(u,d)}\varepsilon^{(u,d)2}e^{i\varphi^{(u,d)}} \\ 0 & 16\sqrt{3}\eta^{(u,d)}\varepsilon^{(u,d)2}e^{-i\varphi^{(u,d)}} & \mu_3^{0(u,d)} \end{pmatrix} \tag{38}$$

where $\mu_i^{0(u,d)}$ are defined by the diagonal matrices

$$\hat{\varrho}\left(\widehat{N}_{\rm eff}^{(u,d)2} - \frac{1+\varepsilon^{(u,d)2}}{\widehat{N}_{\rm eff}^{(u,d)2}}\right)\hat{\varrho} = \frac{1}{29}\begin{pmatrix} \mu_1^{0(u,d)} & 0 & 0 \\ 0 & \mu_2^{0(u,d)} & 0 \\ 0 & 0 & \mu_3^{0(u,d)} \end{pmatrix} \tag{39}$$

and so have readily calculable values (from Eqs (5) and (30) with (31)). They are -0.1716, 35.48, 4056 and -2, 34.67, 1175, respectively, if the values (40) are used. The only parameters beside the $vC^{(u,d)}/29$ are here $\varepsilon^{(u,d)2}$ and $\varphi^{(u,d)}$.

To set a *numerical example* let us put

$$\varepsilon^{(u)2} = \tan^2\frac{\pi}{8} = 0.171573\,, \quad \varepsilon^{(d)2} = 2 \tag{40}$$

(in the diagonal model of the previous Section these values correspond to $m_u \simeq 7.25$ MeV and $m_d \simeq 8.5$ MeV, if $m_c \simeq 1.5$ GeV, $m_b \simeq 5$ GeV). Then, carrying out numerically the exact diagonalization of the mass matrices (38), we obtain (up to the overall constants $vC^{(u,d)}/29\varepsilon^{(u,d)2}$ at the front) the following eigenvalues:

$$\mu_1^{(u)} = -0.3823\,, \quad \mu_2^{(u)} = 35.60\,, \quad \mu_3^{(u)} = 4056 \tag{41}$$

and

$$\mu_1^{(d)} = -3.785\,, \quad \mu_2^{(d)} = 33.76\,, \quad \mu_3^{(d)} = 1177 \tag{42}$$

(independently of $\varphi^{(u,d)}$). The corresponding unitary diagonalizing matrices (*cf.* Eq. (17)) are

$$\widehat{U}^{(u)} = \begin{pmatrix} 0.9971e^{i\varphi^{(u)}} & 0.0765e^{i\varphi^{(u)}} & 0.000003e^{i\varphi^{(u)}} \\ -0.0765 & 0.9971 & 0.0047 \\ 0.00036e^{-i\varphi^{(u)}} & -0.0047e^{-i\varphi^{(u)}} & 1.0000e^{-i\varphi^{(u)}} \end{pmatrix} \tag{43}$$

and

$$\hat{U}^{(d)} = \begin{pmatrix} 0.9759e^{i\varphi^{(d)}} & 0.2180e^{i\varphi^{(d)}} & 0.00033e^{i\varphi^{(d)}} \\ -0.2178 & 0.9748 & 0.0485 \\ 0.0102e^{-i\varphi^{(u)}} & -0.0474e^{-i\varphi^{(u)}} & 0.9988e^{-i\varphi^{(d)}} \end{pmatrix} . \tag{44}$$

They lead to the Cabibbo–Kobayashi–Maskawa matrix $\hat{V}$ (*cf.* Eq. (18)) involving one unknown phase $\varphi \equiv \varphi^{(u)} - \varphi^{(d)}$.

In the extreme case of $\varphi \to 0°$, when there were no CP violation caused by $\hat{V}$, one would get

$$\hat{V} = \begin{pmatrix} 0.990 & 0.143 & -0.003 \\ -0.143 & 0.989 & 0.044 \\ 0.009 & -0.043 & 0.999 \end{pmatrix} . \tag{45}$$

In the realistic case of φ fitted to the experimental $|V_{us}| = 0.217 \div 0.223$ [5] one obtains $\varphi = \mp(79.8° \div 84.5°)$ and then

$$\hat{V} = \begin{pmatrix} 0.976\,e^{\pm i\,79°} & 0.217\,e^{\pm i\,100°} & 0.0036\,e^{\mp i\,180°} \\ 0.217\,e^{\pm i\,160°} & 0.975\,e^{\pm i\,1°} & 0.048\,e^{\pm i\,6°} \\ 0.010\,e^{\mp i\,86°} & 0.047\,e^{\pm i\,95°} & 0.999\,e^{\mp i\,80°} \end{pmatrix}$$
$$\div \begin{pmatrix} 0.975\,e^{\pm i\,84°} & 0.223\,e^{\pm i\,104°} & 0.0036\,e^{\mp i\,179°} \\ 0.223\,e^{\pm i\,161°} & 0.974\,e^{\pm i\,1°} & 0.048\,e^{\pm i\,6°} \\ 0.010\,e^{\mp i\,90°} & 0.047\,e^{\pm i\,90°} & 0.999\,e^{\mp i\,85°} \end{pmatrix} , \tag{46}$$

where the phases α_{ij} satisfy the relations

$$\alpha_{11} + \alpha_{22} + \alpha_{33} = 0 , \quad \alpha_{12} + \alpha_{21} + \alpha_{33} = \pm 180° , \quad \alpha_{23} + \alpha_{32} + \alpha_{11} = \pm 180° . \tag{47}$$

We include the value

$$\varphi = \mp(80° \div 85°) , \tag{48}$$

as an element of our *numerical example.* Note that this is a highly nontrivial feature of our numerical model that in its framework φ can be fitted to experimental $|V_{us}|$.

Performing in this case the rephasing for up and down quarks of three generations

$$u_j \to u_j\, e^{i\varphi_j^{(u)}} , \quad d_j \to d_j\, e^{i\varphi_j^{(d)}} , \tag{49}$$

where

$$\sum_i \left(\varphi_i^{(u)} - \varphi_i^{(d)}\right) = 0 , \tag{50}$$

and putting

$$\varphi_i^{(u)} - \varphi_i^{(d)} = \alpha_{ii} ,$$

$$\varphi_1^{(u)} - \varphi_2^{(d)} = \alpha_{12} ,$$
$$\varphi_2^{(u)} - \varphi_3^{(d)} = \alpha_{23} , \tag{51}$$

we eliminate from the matrix (46) by means of Eq. (47) all phases but those of V_{ub} and V_{td}. The result is

$$\widehat{V} = \begin{pmatrix} 0.976 & 0.217 & 0.0036\, e^{\pm i\, 76^\circ} \\ -0.217 & 0.975 & 0.048 \\ 0.010\, e^{\pm i\, 20^\circ} & -0.047 & 0.999 \end{pmatrix}$$
$$\div \begin{pmatrix} 0.975 & 0.223 & 0.0036\, e^{\pm i\, 72^\circ} \\ -0.223 & 0.974 & 0.048 \\ 0.010\, e^{\pm i\, 19^\circ} & -0.047 & 0.999 \end{pmatrix} , \tag{52}$$

where due to the unitarity of $\widehat{V}$ we can write the triangle relation

$$V_{td} = \frac{0.217\times 0.048}{0.999} - \frac{0.976}{0.999} V^*_{ub}$$
$$\div \frac{0.223\times 0.048}{0.999} - \frac{0.975}{0.999} V^*_{ub} . \tag{53}$$

We can see from Eq. (52) that the magnitudes of all elements of $\widehat{V}$ are consistent with their experimental estimates [5] ($|V_{ub}|$ is close to its lower experimental limit as given in Ref. [5], while $|V_{cb}|$ lies nearly in the middle of its experimental range). In Eq. (52) the CP-violating phase δ [5] (invariant under the equal rephasing of up and down quarks) is $\mp(76^\circ \div 72^\circ)$. Since $\sin\delta > 0$ from measurements of the CP-violation parameter ε [5] (if the B_K parameter is really positive [6]), the positive value $\delta = 76^\circ \div 72^\circ$ corresponding to $\varphi = 80^\circ \div 85^\circ$ should be chosen.

Of course, when switching on the effective intergeneration mixing, we change the quark masses in comparison with the diagonal model of Section 3. The new masses are

(i) for up quarks (*cf.* Eq. (41))

$$m_u = m_c \frac{|\mu_1^{(u)}|}{\mu_2^{(u)}} \simeq 16 \text{ MeV} ,$$
$$m_t = m_c \frac{\mu_3^{(u)}}{\mu_2^{(u)}} \simeq 171 \text{ GeV} , \tag{54}$$

if $\varepsilon^{(u)2} = \tan^2(\pi/8) = 0.171573$ and $m_c \simeq 1.5$ GeV,

(ii) for down quarks (*cf.* Eq. (42))

$$m_d = m_b \frac{|\mu_1^{(d)}|}{\mu_3^{(d)}} \simeq 16 \text{ MeV} ,$$
$$m_s = m_b \frac{\mu_2^{(u)}}{\mu_3^{(d)}} \simeq 143 \text{ MeV} , \tag{55}$$

if $\varepsilon^{(d)2} = 2$ and $m_b \simeq 5$ GeV.

Here, m_u and m_d are approximately equal and probably too large (though the problem is still open).

It is perhaps interesting to point out that the Cabibbo–Kobayashi–Maskawa matrix $\hat{V}$ has in our model nonsymmetric moduli. As follows from a recent analysis of the CP-violation parameters ε, ε'/ε and $B-\overline{B}$ mixing within the standard model (and under the assumption that the $\hat{V}$ matrix is the only source of the observed CP violation) [8], there seem to be two favorable regions for m_t: $m_t = 0$ (100 GeV) (miliweak case: $\varepsilon'/\varepsilon = 0(10^{-3})$) and

$m_t = 0$ (200 GeV) (superweak case: $\varepsilon'/\varepsilon \simeq 0$ and $\lesssim 0$). A Cabibbo–Kobayashi–Maskawa matrix with symmetric moduli may be consistent only with the second region [7]. This region is dynamically possible in a recently proposed "minimal breaking scheme" for the standard model [9]. Our numerical model favours rather the second region, but with $|V_{ub}|$ being very small, though lying within its experimental limits [5,10].

5. Conclusions

We constructed an algebraic numerical model for quark mass matrices defined by Eqs (16), (5), (36) with (37) and (30) with (31). Then, we predict the top-quark mass $m_t \simeq 171$ GeV and CP-violating phase $\delta \simeq 76° \div 72°$. The CKM matrix comes out nonsymmetric in moduli with $|V_{ub}|$ being very small. All moduli are consistent with their experimental limits [5,10].

In this model, the pattern of quark mass spectrum is coded in the formulae (16) and (36). They are analogical to Eqs (4) and (28) appearing in the numerically successful charged-lepton case, but in the quark case Eq. (28) becomes perturbed by a diagonal intrinsic pairing force $\sim \hat{a}^{+2}\hat{a}^2$ and an off-diagonal intrinsic Yukawa-type interaction $\sim \hat{a}\exp\left(i\varphi^{(u,\,d)}\right) + \hat{a}^+\exp\left(-i\varphi^{(u,d)}\right)$ (both are absent in the charged-lepton case). The strength-matrix formula (36) provides kernel for the mass-matrix formula (16).

Note that, except for $\varepsilon^{(u)}$, $\varepsilon^{(d)}$ and $\varphi^{(u)} - \varphi^{(d)}$, our numerical example contains no more parameters *sensu stricto* (*i.e.* fitted to experimental data), since here other numerical factors are chosen a priori in Eqs (31) and (37). The parameter $\varepsilon^{(u)}$ is taken the same as for charged leptons ($\varepsilon^{(u)} = \tan(\pi/8)$). The masses m_u and m_d come out probably too large (this is, however, still an open question)[1].

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[1] Adding to the quark matrices (16) some corrections $vx^{(u,\,d)}\hat{1}$ proportional to the unit matrix, we do not change the diagonalizing matrices $\hat{U}^{(u,\,d)}$

The main motivation of the model originates from a previous work of the author on three replicas of Dirac particle, presumably resulting into three generations of leptons and quarks.

The results of this paper may be also viewed as an introduction to a new method of intrinsic dynamical description of lepton and quark mass matrices.

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APPENDIX
Three replicas of the Dirac particle

In Ref. [4] three statements formulated below were proved.

(i) The Dirac algebra

$$\{\Gamma^\mu, \Gamma^\nu\} = 2g^{\mu\nu}, \tag{A.1}$$

admits the sequence of representations[2]

$$\Gamma^\mu = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\gamma_i^\mu \quad (N = 1, 2, 3, \ldots), \tag{A.2}$$

where

$$\{\gamma_i^\mu, \gamma_j^\nu\} = 2\delta_{ij}g^{\mu\nu} \quad \begin{matrix}(i, j = 1, 2, \ldots, N \\ N = 1, 2, 3, \ldots)\end{matrix} \tag{A.3}$$

define a sequence of Clifford algebras. The representation (A.2) may be realized in the form

$$\Gamma^\mu = \gamma^\mu \otimes \underbrace{\mathbf{1} \otimes \ldots \otimes \mathbf{1}}_{N-1 \text{ times}}, \tag{A.4}$$

where γ^μ and $\mathbf{1}$ are the usual Dirac 4×4 matrices. Thus, the Dirac Equation

$$[\Gamma \cdot (p - eA) - m]\psi = 0, \tag{A.5}$$

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nor the CKM matrix $\widehat{V}$, but we shift the quark masses:
$m_{\rm ud} \to |-m_{\rm ud} + vx^{(\rm u,\, d)}|$, $m_{\rm c,\, s} \to m_{\rm c,\, s} + vx^{(\rm u,\, d)}$, $m_{\rm t,\, b} \to m_{\rm t,\, b} + vx^{(\rm u,\, d)}$. For instance, if $vx^{(\rm u)} = 11 \div 8$ MeV, and $vx^{(\rm d)} = 9 \div 6$ MeV, the new masses are: $m_{\rm u} \simeq 5 \div 8$ MeV, $m_{\rm c} \simeq 1.5$ GeV, $m_{\rm t} \simeq 171$ GeV and $m_{\rm d} \simeq 7 \div 10$ MeV, $m_{\rm s} \simeq 152 \div 149$ MeV, $m_{\rm b} \simeq 5$ GeV.

[2] All combinations $\sum_{i=1}^{N} c_i\gamma_i^\mu$, where $\sum_{i=1}^{N} c_i^2 = 1$, are representations of (A.1). For any fixed N they are unitary equivalent.

leads to the sequence of wave functions

$$\psi = (\psi_{\alpha_1\alpha_2\ldots\alpha_N}) \quad (N = 1, 2, 3, \ldots), \tag{A.6}$$

where α_i ($i = 1, 2, \ldots, N$) are bispinor indices (each $\alpha_i = 1, 2, 3, 4$) of which α_1 describes magnetically "*visible*" spin 1/2 coupled to A_μ and $\alpha_2, \ldots, \alpha_N$ are responsible for $N - 1$ magnetically "*hidden*" spins 1/2 decoupled from A_μ [11].

(ii) If the relativistic Lorentz covariance is imposed on *all* bispinor indices $\alpha_1, \alpha_2, \ldots, \alpha_N$ and, at the same time, the quantal probability interpretation — on *each* wave function of the sequence (A.6), then only odd N's are admissible,

$$N = 1, 3, 5, \ldots, \tag{A.7}$$

and $N - 1$ "hidden" spins 1/2 in each wave function (A.6) must add up to zero. Otherwise both requirements cannot be reconciled.

(iii) If in addition *each* wave function (A.6) is required to be antisymmetric in all "hidden" bispinor indices $\alpha_2, \ldots, \alpha_N$ (what may be called the "hidden exclusion principle"), then N must terminate at 5,

$$N = 1, 3, 5, \tag{A.8}$$

and, moreover, *there exist three and only three replicas of the Dirac particle.* They correspond to three nonzero bispinor wave functions $\psi^{(1)}_{\alpha_1}$, $\psi^{(3)}_{\alpha_1}$ and $\psi^{(5)}_{\alpha_1}$ (where α_1 is the "visible" bispinor index) involved in three ψ's corresponding to $N = 1$, 3 and 5, respectively, *viz.*

$$\psi = (\psi_{\alpha_1}) \equiv \left(\psi^{(1)}_{\alpha_1}\right), \tag{A.9}$$

$$\psi = (\psi_{\alpha_1\alpha_2\alpha_3}) \equiv \begin{pmatrix} 0 & \psi^{(3)}_{\alpha_1} & 0 & 0 \\ -\psi^{(3)}_{\alpha_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi^{(3)}_{\alpha_1} \\ 0 & 0 & -\psi^{(3)}_{\alpha_1} & 0 \end{pmatrix} \tag{A.10}$$

and

$$\psi = (\psi_{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5}) \equiv \left(\varepsilon_{\alpha_2\alpha_3\alpha_4\alpha_5}\psi^{(5)}_{\alpha_1}\right). \tag{A.11}$$

Then, with the notation

$$\Psi = \frac{1}{\sqrt{29}} \begin{pmatrix} \psi_{\alpha_1} \\ \psi_{\alpha_1\alpha_2\alpha_3} \\ \psi_{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5} \end{pmatrix}, \tag{A.12}$$

one gets

$$\Psi^+\Psi = \tfrac{1}{29}\left(\psi^{(1)+}\psi^{(1)} + 4\psi^{(3)+}\psi^{(3)} + 24\psi^{(5)+}\psi^{(5)}\right), \tag{A.13}$$

where the "visible" bispinor index α_1 is supressed in $\psi^{(1)}$, $\psi^{(3)}$ and $\psi^{(5)}$. Notice the weights 1/29, 4/29 and 24/29 appearing in the product (A.13). Changing a bit the definition (A.12) of Ψ we may write

$$\Psi = \frac{1}{\sqrt{29}} \begin{pmatrix} \psi^{(1)} \\ \sqrt{4}\psi^{(3)} \\ \sqrt{24}\psi^{(5)} \end{pmatrix} , \tag{A.14}$$

also consistently with Eq. (A.13). It is tempting to conjecture that the three replicas of Dirac particle should correspond to three observed generations of leptons and quarks.

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