MOTION OF CHARGED PARTICLES IN THE FIELD OF MASSIVE CHARGED FILAMENT

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Motion of a charged test particle in the field of an infinitely long massive charged filament is investigated. It is shown that the electric repulsion cannot prevent its fall on the filament. Hence, it follows that the Coulomb force cannot prevent the formation of naked linear singularity by collapse of charged matter.

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The space-time around a massive infinitely long filament with a constant mass linear density C is described by the metric [1, 2]

$$ds^2 = x^{2p_1}F(x)^{-2}dt^2 - F(x)^2\left(dx^2 + x^{2p_2}d\phi^2 + x^{2p_3}dz^2\right), \tag{1}$$

$$p_1 + p_2 + p_3 = p_1^2 + p_2^2 + p_3^2 = 1\,,\ F(x) = 1 - C^2x^{2p_1}\,. \tag{2}$$

We use the units with $c = G = 1$. The electric field is described by the potential

$$A_i = (A_0, 0, 0, 0)\,, \quad A_0 = Cx^{2p_1}F(x)^{-1} + \text{const}\,. \tag{3}$$

Let us consider motion of a test particle with charge e and mass m in this field. This problem is of importance in connection with the cosmic censorship principle, as it will be discussed further on. In addition it is interesting to examine a possibility of total compensation of gravitational and electric forces everywhere, which may take place in the Newtonian theory.

Solving the Hamilton-Jacobi equation we obtain for action the expression

$$S = -Et + M\phi + Pz + \int \Big\{x^{-2p_1}F(x)^2\left[F(x)E - eCx^{2p_1}\right]^2 - m^2F(x)^2 - M^2x^{-2p_2} - P^2x^{-2p_3}\Big\}^{\frac{1}{2}}dx \tag{4}$$

From this expression one can easily obtain the equation of motion. For their qualitative consideration let us rewrite the integral (4) in the form $\int F(x)^2 x^{-p_1}\left((E-U_1(x))(E-U_2(x))\right)^{\frac{1}{2}}\,dx$;

$$U_{1,2}(x) = x^{p_1}F(x)^{-1}\left[eCx^{p_1} \pm \left(m^2 + F(x)^{-2}(M^2x^{-2p_2} + p^2x^{-2p_3})\right)^{\frac{1}{2}}\right]. \quad (5)$$

Particle's motion can take place in the region $E \geq U_1(x)$; the region $E \leq U_2(x)$ corresponds to the Dirac sea. The behaviour of function $U_1(x)$ defining the character of motion is plotted in Fig. 1 for different values of parameters e, m, p_1, C, M and P. In diagrams 1b–e this curve is represented for $eC > 0$, which corresponds to the electric attraction to the filament. In the case of electric repulsion the boundaries of motion are defined by the curve $U_2(x)$ represented in the same figure, taken with the opposite sign.

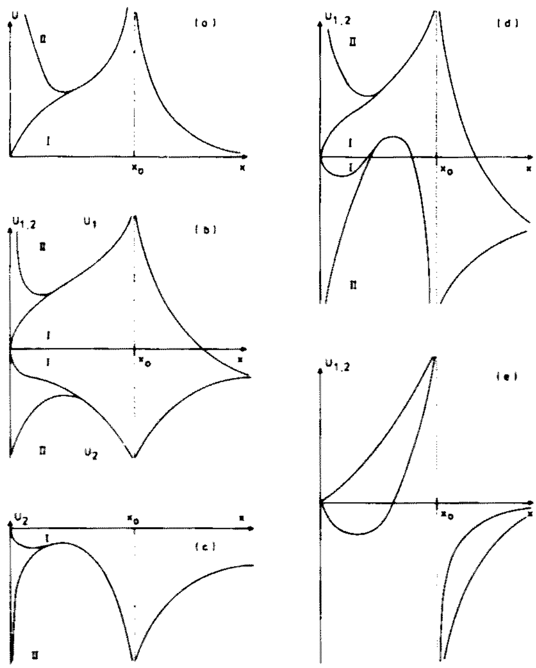

Fig. 1. Functions $U_1(x)$ and $U_2(x)$ for different values of e, M, P and p_1. Curves with sign I correspond here to the case $p_1 > 2/3$ or $M = 0$ and with sign II to the case $0 < p_1 < 2/3$, $M \neq 0$. (a) $e = 0$, $U_1(x) = -U_2(x) = U(x)$; (b) $|e| \ll m$; (c) charge increasing; (d) $|e| > m$, an arrow indicates a possible tunneling; (e) $M = P = 0$, $|e| > m$.

The particle motion for $C = 0$ was considered in [3]. The curve $U(x) = U_1(x) = -U_2(x)$ for $e = 0$ is represented in Fig. 1a. At $x = x_0$ the function $F(x)$ turns into zero and the space-time (1) has a real singularity. If the field source is situated at $x = 0$, this singularity arises due to field self-gravitation [1]. There is another possible treatment of (1) when just this singularity at x_0 is considered as the source of electric and gravitational field which in this case has an infinite negative mass density partly compensated by an infinite positive energy density of electric field [1]. The space-time is then described by (1) for $x_0 < x < \infty$.

As one can see from Fig. 1a the centrifugal force can forbid particle's fall on the source at $x = 0$ only if $0 < p_1 < 2/3$ (linear singularity) and cannot do it if $p_1 > 2/3$ (paradoxical singularity [4, 5]. The point singularity at $x = x_0$ (its type has been obtained by a diagram method proposed in [4]) can be reached only by particles with $M = P = 0$. The source at $x = x_0$ repulses particles.

Fig. 1b represents the curves $U_1(x), U_2(x)$ in the case of small particle charge $|e| \ll m, eC > 0$. Their difference from Fig. 1a is not big. At the region $0 < x < x_0$ the curve $U_1(x)$ is slightly shifted to the left in comparison with $U(x)$ and $U_2(x)$ to the right in comparison with $(-U(x))$. At $x > x_0$ both curves approach, as $x \to \infty$, the value $U_\infty = -eC^{-1}$. The increasing of charge e makes a maximum at the curve $U_2(x)$ even if $p_1 > 2/3$ or $M = 0$ (Fig. 1c). It means that there is a region where the electric repulsive force (together with a centrifugal at $p_1 > 2/3$) exceeds the gravitational one. Nevertheless, near the singularity the last force becomes a prevailing one and the particle begins to move to the source. At $p_1 > 2/3$ or $M = 0$, the same particle with the same energy can move near the filament falling on it.

By further increasing of particle charge the function $U_2(x)$ becomes positive on a certain interval of x (Fig. 1d). A condition of its positivity has a complicated form, but the inequality $|e| > m$ must be satisfied. There can be a tunneling from the Dirac sea in this case and hence a quantum creation of pairs in the strong electric field. This process leads to a decrease of filament's charge.

Since for the electron one has $|e| \gg m$, this creation occurs for each charge density C of filament, but in a very different rate. Electric repulsion at $M = 0$ and also at $p_1 > 2/3$ cannot prevent a fall of particle moving near the singularity into it.

The case $M = P = 0$ needs a special consideration. At $|e| < m$ the form of curves $U_1(x)$ and $U_2(x)$ does not differ qualitatively from the same in Fig. 1b. For $|e| > m$ it is shown in Fig. 1e. A particle in the region $0 < x < x_0$ due to the electric repulsion must move away from the filament to the singularity at x_0. Considering the motion in the region $x > x_0$ one can see that the electric attraction can force a particle to fall in the singularity $x = x_0$ despite of the gravitational repulsion. The case $|e| = m$ corresponds in the Newtonian theory to the total compensation of gravitational and electric repulsive force in the whole space. From (5) we obtain that $U_{1,2}(x) = m(\pm x^{-p_1} - x_0^{-p_1})^{-1}$. Therefore, the function $U_2(x)$ is regular at $x = x_0$. There is no nonrelativistic force compensation in this case. It arises only as a limit $x \to \infty$.

From the above consideration we see that the electric repulsion cannot prevent a fall of charged particle on a charged filament. It is easy to see that this result is valid also for the case of the most general form of linear naked singularities, which has been obtained and investigated in [4]. Thus the Coulomb force cannot prevent the formation of naked linear singularity by collapse of charged matter.

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