NOTE ON THE INTERMITTENCY IN MULTIPLE PRODUCTION AND THE SINGULARITIES OF THE CORRELATION FUNCTIONS *

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The relationship between the "intermittent" increase of the scaled factorial moments and the singularities in the correlation functions is discussed. It is shown that for the existing data, far from the asymptotic region, the slopes measured from the increase of moments may be much smaller than the corresponding exponents in the correlation functions. Possible interpretations of some observed regularities of slope values for different processes are considered.

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1. Introduction

In the last few years the notion of intermittency in multiple production became one of the main topics in investigating the final states of high energy collisions. It has been widely popularized by the discovery that in many processes the scaled factorial moments increase (approximately linearly) for the decreasing size of the rapidity bins on double logarithmic scale. Such a behaviour has been predicted [1] in a simple model based on the analogy with the density fluctuations in a turbulent motion and is often addressed as intermittency.

In this note we intend to discuss the possible origin of the linear increase of moments in the context of corresponding singularities of the correlation functions. In the next section we discuss the simplest case of one-dimensional analysis with translational invariance for the second order moments. In Section 3 higher order moments are discussed. We conclude with Section 4.

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2. Second order moments for one-dimensional analysis

Let us consider a kinematical variable x in which there is an approximate translational invariance of distributions in a range X. Such a variable can be the (pseudo) rapidity y in the central region, or the angle Φ defining the direction in a plane perpendicular to the collision (or jet) axis.

The scaled second factorial moment of the multiplicity distribution in the bin of size δ averaged over the bin position is given by

$$F_2(\delta) = \left(\sum_{m=1}^{M} \frac{\overline{n(n-1)_m}}{M} \right) \Big/ \left(\sum_{m=1}^{M} \frac{\overline{n_m}}{M} \right)^2 , \tag{1}$$

where $M = X/\delta$. This is so-called horizontal average, in which the numerator and denominator expressions are averaged independently over the bin position (and, in fact, this average can be performed before averaging over events denoted here by a bar). For the case of exact translational invariance this procedure is fully equivalent to the "vertical" averaging

$$F_2^{\rm V} = \frac{1}{M} \sum_{m=1}^{M} \left(\frac{\overline{n(n-1)}}{\bar{n}^2} \right)_m \equiv \frac{1}{M} \sum_{m=1}^{M} (F_2)_m , \tag{2}$$

since $\bar{n}_m = \bar{n}_X/M$ and $(F_2)_m = F_2$ for all values of m.

Obviously, for each bin the factorial moments can be expressed by the integrals of inclusive distributions. Thus we have

$$F_2(\delta) = \frac{\int_\delta \rho_2(x_1, x_2) dx_1 dx_2}{(\int_\delta \rho(x) dx)^2} = 1 + \frac{\int_\delta c_2(x_1, x_2) dx_1 dx_2}{(\int_\delta \rho(x) dx)^2} . \tag{3}$$

Thus the "intermittent" behaviour of F_2 for $\delta \to 0$

$$F_2(\delta) \approx c\delta^{-\phi_2} , \tag{4}$$

i.e.

$$\ln F_2(\delta) \approx a - \phi_2 \ln \delta \tag{5}$$

corresponds to the singular behaviour of c_2 for $|x_1 - x_2| \to 0$

$$c_2(x_1, x_2) \approx \gamma |x_1 - x_2|^{-\phi_2} . \tag{6}$$

However, (5) is a good approximation of $\ln F_2$ for c_2 given by (6) only if

$$F_2 - 1 \gg 1 . \tag{7}$$

so that the second term in (3) dominates. This is definitely not the case for all the existing data. In fact, in most of the cases the reversed inequality holds

$$F_2 - 1 \ll 1 . \tag{8}$$

Obviously, if (5) is valid down to $\delta = 0$, one must eventually enter the range where (7) holds. However, one can easily show that for the real data the increase of F_2 must saturate for the values of δ below the experimental resolution in x [2]. For finite statistics and finite average multiplicities the maximal number of particles in bins is limited, which may also limit the real increase of moments, although this effect is more relevant for higher order moments [3]. Therefore, the singular term in F_2 may well never dominate. Moreover, the correlation function may have other terms than (6). Thus in all existing data the possible singular term in F_2 seems to be just a small correction in all the available range of δ. This is suggested most clearly by the fact that both for (pseudo)rapidity and Φ distributions the observed difference between the largest and smallest value of F_2 in the range of δ in which the linear form (5) is fitted is always much smaller than one.

Therefore, an interesting question arises: how can one obtain a good linear fit of the form (5) even for the cases where (7) does not hold and the observed increase of F_2 in the full range of δ is very small, as is the case *e.g.* for the UA1 data [4] for bins in Φ shown in Fig. 1 ?

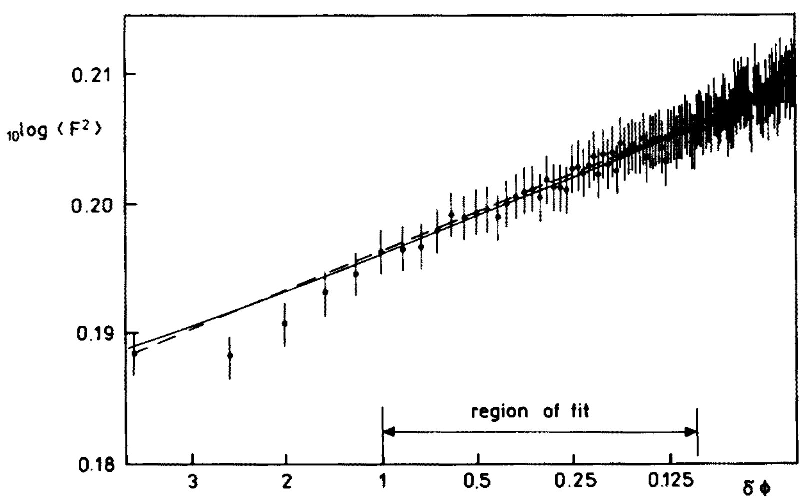

Fig. 1. ${}_{10}\log F_2$ *vs.* bin size $\delta\Phi$ on log scale [4]. The broken line is a straight line fit of the form (5) and solid line results from (11). Parameter values are quoted in the text.

Obviously, the simplest possibility is to assume that approximation (6) works well not for the correlation function, but for the two-particle density function $\rho_2(x_1, x_2)$. Then, however, one has to assume rather exotic form of the correlation function with a big constant negative term to cancel the product of single particle densities in the definition formula $\rho_2 = \rho_1\rho_1 + c_2$. Since there seems to be no model predicting such a behaviour, we restrict ourselves in the following to the more standard case of the correlation function dominated by the positive constant and singular terms.

If F_2 can be written as

$$F_2 = 1 + c + \epsilon'(\delta)\,, \tag{9}$$

where $\epsilon'(\delta) \ll 1 + c$ in the full range of δ considered, one can obviously use the approximation

$$\ln F_2 \approx \ln(1 + c) + \frac{\epsilon'(\delta)}{(1 + c)}\,. \tag{10}$$

If

$$c_2(x_1, x_2) = c' + \epsilon|x_1 - x_2|^{-\alpha} \qquad (0 < \alpha \ll 1)\,, \tag{11}$$

we find

$$c = \frac{c'}{\rho^2}\,, \qquad \epsilon'(\delta) \approx \frac{\epsilon\delta^{-\alpha}}{\rho^2} \tag{12}$$

and there seems to be no good reason for the good fit of the form (5). In fact, to get such a good fit it seems necessary to assume rather a logarithmic singularity in the correlation function [5]. However, if $\alpha \ll 1$ and the range of δ is limited, one can approximate further (12) by

$$\epsilon'(\delta) \approx \frac{\epsilon e^{-\alpha \ln \delta}}{\rho^2} \approx \frac{\epsilon(1 - \alpha \ln \delta)}{\rho^2} \tag{13}$$

and thus

$$\ln F_2 \approx \ln(1 + c) + \frac{\epsilon}{\rho^2(1 + c)} - \frac{\alpha\epsilon \ln \delta}{\rho^2(1 + c)}\,. \tag{14}$$

Thus the power-like singularity in c_2 may result in the linear dependence of $\ln F_2$ on $\ln \delta$ even in the range of δ for which the singular term is only a small correction to F_2. However, the experimentally observed slope ϕ_2 in (5) is then not equal to the exponent α of the singular term in c_2 (and, in fact, it should be much smaller). This opens quite new possibilities of the interpretation of the experimentally observed intermittency parameters. For example, the UA1 data for the Φ bins mentioned above, for which $\phi_2 = 0.01$ was fitted, can be equally well described with $c = 0.42$, $\epsilon/\rho^2 = 0.15$ and $\alpha = 0.1$, as seen in Fig. 1.

Obviously, any realistic phenomenological model of multiple production with singular correlation functions (and thus with intermittency) should provide us with the values of parameters like α, ϵ or c, and no approximations should be used to calculate F_2 to be compared with data. In the following, however, we will just consider further the results of a toy model with arbitrarily chosen values of these parameters compatible with one-dimensional data for F_2.

Let us stress here that our results do not depend crucially on the real presence of singularity in the correlation function. If we replace (11) by the non-singular form approximating (11) well for $|x_1 - x_2|$ bigger than some small scale λ (well below the available range of δ), *e.g.*

$$c_2(x_1, x_2) = c' + \epsilon(\lambda + |x_1 - x_2|)^{-\alpha} \tag{15}$$

then F_2 may be still satisfactorily approximated by (14) in the available range of $\delta \gg \lambda$.

3. Higher order moments

Obviously, the higher order moments depend on the assumed higher order correlation functions, for which no data and no reliable theoretical predictions exist. However, it is easy to see that large (and probably leading) terms in them result from the second order correlations. Using the notation

$$\int_\delta dx_1 \ldots \int_\delta dx_q \frac{c_q(x_1, \ldots x_q)}{\rho^q \delta^q} \equiv f_q \tag{16}$$

we have well known relations

$$F_2 = 1 + f_2\,, \tag{17}$$

$$F_3 = 1 + 3f_2 + f_3\,, \tag{18}$$

$$F_4 = 1 + 6f_2 + 3f_2^2 + 4f_3 + f_4\,, \tag{19}$$

$$F_5 = 1 + 10f_2 + 15f_2^2 + 10f_2 f_3 + 10f_3 + 5f_4 + f_5\,. \tag{20}$$

In general, the coefficient of the term linear in f_2 in F_q is equal to $q(q-1)/2$. Thus, for $c = 0$ and neglecting all f_q for $q > 2$ we find in the first order in ϵ a simple relation

$$\frac{\phi_q}{\phi_2} = \frac{q(q-1)}{2}\,, \tag{21}$$

where ϕ_q are defined analogously to ϕ_2

$$\ln F_q \approx a_q - \ln \delta\,. \tag{22}$$

In the real data $c \neq 0$ (and, in fact, $c > \epsilon/\rho^2$). It is easy to see that it should decrease the value of ϕ_q/ϕ_2. On the other hand, however, from the higher powers of f_2 and higher correlation terms f_q we expect terms increasing as $\delta^{-2\alpha}$, $\delta^{-3\alpha}$ *etc.*, for which the approximation (13) may be rather poor. This would increase the effective slopes ϕ_q. The final result is not easy to guess as it depends on the detailed values of parameters, and in particular on the assumed parametrization of f_q for $q > 2$. Nevertheless, it seems quite natural to expect (20) to be approximately valid, as is the case for most of the data (*e.g.* for the UA1 data mentioned above, we have $\phi_q/\phi_2 = 2.7 \pm 0.5$, 7.7 ± 1.2 and 15.2 ± 3.5 for $q = 3, 4, 5$, respectively, to be compared with $3, 6, 10$ from formula (20)). More detailed phenomenological considerations will be given elsewhere.

4. Conclusions

We have investigated the relationship between the slope parameters determining the increase of scaled factorial moments for small bins of kinematical variables and the exponents of the corresponding singularities in the correlation functions. We have shown that it is quite natural to obtain good linear fits for the log-log plots of moments as the functions of bin size even though the terms resulting from singular parts of correlation functions are only a small part of each moment in the investigated range of bin sizes. However, the fitted values of slopes are then not expected to reflect the values of exponents for singularities of the correlation functions, as should be the case for the limit of vanishing bin size. In general, the fitted slope values may be much smaller then the corresponding exponents.

This result opens quite new possibilities in interpreting the observed regularities in the intermittency parameter values. Let us mention here few of them:

(i) It is no longer necessary to assume extremely small scales for correlations in the kinematical variables, (and thus, *via* the uncertainty relation, to expect unusually large distances in space-time development of the final hadronic state).

(ii) It is natural to relate the observed differences of slope values for different processes and collision energies to the differences of the relative strengths of correlations, and not to the exponents of singular terms, which may be universal.

(iii) For the higher order moments and for processes where the correlations are large our approximations may start to break down for very small bins. Thus, if the experimental resolution and statistics are good enough, we should see the upward curvature of the log-log plots, and at least the saturation due to final resolution and statistics may be postponed.

These and other effects will be considered in more detail elsewhere. One may hope that such investigations will help in constructing more realistic phenomenological models of multiple production.

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